Precautionary Saving and Aggregate Demand

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Precautionary saving and the business cycle

How does time-varying precautionary saving propagate business cycle shocks?

Micro evidence suggests precautionary saving matters:


What are the macroeconomic/general-equilibrium effects of this?
Precautionary saving and the business cycle

2 potential effects: **aggregate supply** vs. **aggregate demand**

- **AS**: smoother investment, capital and ultimately output  
  (Krusell Smith 1998...)

- **AD**: through consumption + feedback loop through labour markets

  
  Aggregate demand falls $\downarrow$ Precautionary saving rises $\uparrow$

  Unemployment rises $\Rightarrow$ Unemployment *risk* rises

We would like to extract these two forces from the data and assess how they alter the impact of structural shocks
What do we do?

Construct + estimate tractable incomplete-insurance macro model with nominal rigidities and labour market frictions

- Labour market frictions ⇒ time-varying idiosyncratic risk
- Incomplete insurance + debt limit ⇒ precautionary saving
- Nominal rigidites ⇒ AD effects

Model has both AS effects and AD-precautionary saving feedback loop

Use aggregate + cross-sectional data
How?

Typical incomplete-insurance models generate **large-dimensional cross-sectional heterogeneity** (Aiyagari 1994, Krusell-Smith 1998...)

We construct a model with **limited cross-sectional heterogeneity**
⇒ retains flexibility of RA models + key features of HA models

Simple state-space representation

- accommodates large number of state variables (21) and shocks (8)
- solved under rational expectations
- amenable to likelihood-based estimation (with, e.g., Dynare)

Time-varying precautionary saving significantly alter the effect of some of the structural shocks

Framework useful in any context where incomplete insurance / households heterogeneity matters
Model outline

Basic ingredients and spices

Basic frictions

- incomplete insurance
- labour market search
- nominal rigidities

Additional features

- consumption habits
- investment adj. costs
- variable K utilisation

Shocks

- Supply: TFP, investment, markup
- Demand: monetary policy, impatience
- Labour market: separation, wage
- Measurement error: captures NIPA-CEX discrepancy
Model outline

Assets and agents

2 assets

- capital
- nominal bonds

2 household types

- Workers: labour income (net wage or UI) + bond income
- Firm owners: same + capital income + monopolistic profits

4 firm types

- final goods
- wholesale goods
- intermediate goods
- labor intermediaries

Central Bank
Firm owners

- $1 - \Omega$ families of firm owners of size 1

- **Patient**: discount factor $\beta^F$ high (steady state interest rate)

- Participate in labor and both asset markets, and **own the firms**

- Enjoy **full income insurance** (within every family)
  
  Basic idea: *wealthy people* who would be almost perfectly self-insured anyway (hence would behave almost like PI consumers)

$\Rightarrow$ Firm owners behave like the RA of the standard NK model
Firm owners

\[ V^F(\bar{n}^F, k, a^F, i, X) = \max_{a^F', i', u, k'} e^{\phi c} u(c^F - h c^F) + \beta^F \mathbb{E}[V^F(\bar{n}'^F, k', a^F', i', X')] \]

s.t.

\[ c^F + i' + a^F' = w^F n^F + [r_k u - \eta(u)] k + (1 + r) a^F + Y, \]

with \( 1 + r' = \frac{1 + R}{1 + \pi'} \)

and

\[ k' = (1 - \delta) k + e^{\phi i} (1 - S(i'/i)) i' \]

\( \Rightarrow \) Results in usual equilibrium conditions:

- Bond Euler equation
- optimal investment function
- common SDF that prices all future profits
Workers

- Measure $\Omega$

- Discount factor $\beta^W < \beta^E$

- Incomplete income insurance:
  \[
  \begin{cases}
    (1 - \tau)w & \text{if employed} \\
    b^u e^z < (1 - \tau)w & \text{if not}
  \end{cases}
  \]

- Hold nominal bonds subject to borrowing constraint

Remarks:

- With full income insurance, workers would be at the constraint (Becker-Foias, Kiyotaki-Moore, Iacoviello...)

- Incomplete insurance + possibility that constraint be binding in the future motivates buffer-stock saving ex ante
Workers

Incomplete insurance usually generates large-dimensional cross-sectional heterogeneity. How to get limited cross-sectional heterogeneity?

**Tight borrowing limit**

- borrowing limit tighter than natural limit
- hence binding in finite time for workers remaining unemployed

**Partial risk sharing**

- Every *employed* workers belongs to a “family” with full risk sharing
- Unemployed taken charged of by unemployment insurance scheme
- Family’s wealth split across members before idiosyncratic shock hits
  ⇒ Workers falling into unemployment leave the family with their fair share of assets
Workers

Unemployed workers

\[ V^u(a^u, X) = \max_{a^{u'}, c^u} \left\{ e^{\psi c} u(c - h c^W) + \beta W E \left[ (1 - f') V^u(a^{u'}, X') + f' \frac{V^e(\tilde{\mu}' , a^{e'}, X')}{n^{W'}} \right] \right\} \]

s.t.

\[ a^{u'} + c^u = b^u e^z + (1 + r) a^u \]

\[ a^{u'} \geq ae^z \]

and where \( c^W \) is the relevant habit level
Workers

Employed workers

\[
V^e(\tilde{\mu}, a^e, X) = \max_{\tilde{A}^e, c^e, a^e'} \{ e^{qc} n^W u(c^e - h c^W) \\
+ \beta^W E[V^e(\tilde{\mu}', a^e', X') + s' n^W V^u(a^e', X')] \}
\]

s.t.

\[
n^W (a^e' + c^e) = (1 - \tau) wn^W + (1 + r) A^e
\]

\[
A^e = (1 - s) \tilde{n}^W a^e + B
\]

\[
B = f \int_a ad\tilde{\mu}^u(a)
\]

\[
a^e' \geq a^e z
\]
Limited cross-sectional heterogeneity

Cross-sectional distributions

Proceed by construction:

1. Guess form of the equilibrium/number of household types
2. Derive sufficient existence conditions
3. Verify that the existence conditions hold empirically

Here: debt limit is binding after one quarter of unemployment (think of liquid wealth, see e.g. Kaplan & Violante 2013, 2014)

Results in 4 household types (1 for firm owners, 3 for workers):

- **Firm owners**: common weath & consumption $a^{F'}$, $k'$, $c^{F'}$
- **Employed workers**: common wealth & consumption $a^{e'}$, $c^{e}$
- **Unemployed workers**: common wealth $a^{u'} = a^{e^u}$ and:
  
  $$c^{e u} = b^{u} e^z + (1 + r) a^{e} - a^{u'}$$
  $$c^{u u} = b^{u} e^z + (1 + r) a^{u} - a^{u'}$$

Cross-sectional distributions: $\{0, a^{e'}, a^{F}\}, \{c^{F}, c^{e}, c^{e u}, c^{u u}\}$
Limited cross-sectional heterogeneity

Existence conditions

1. Debt limit **binding** for *eu* workers (hence for all unemployed workers):

\[
u'(c^{eu} - hc^{eu}) - \\
\mathbb{E} \left[ \left( (1 - f')u'(c^{uu} - hc^{uu}) + f'(u'(c^e - hc^e)) \right) (1 + r') \right] > 0
\]

2. Debt limit **not binding** for employed workers:

\[
\mathbb{E} \left[ M^{e'} (1 + r') \right] = 1 \iff a^{e'} > ae^z
\]

These conditions can be checked empirically after the estimation.
Time-varying precautionary saving at the first order

Ignore consumption habits for a moment

Princing kernel of employed workers:

\[ M^{e'} = \beta^W e^{\Delta \varphi'_c} \frac{(1 - s') u'(c^{e'}) + s' u'(c^{eu'})}{u'(c^e)}, \]

For \( s' \) small, we have:

\[ \hat{M}^{e'} \simeq \Delta \varphi'_c - \sigma \left( \frac{c^{e'} - c^e}{c^e} \right) + \sigma \left( \mathbb{E} \left[ \frac{c^{e'} - c^{eu'}}{c^e} \right] \right) s' \]

\[ \begin{array}{c}
\text{full-insurance SDF} \\
\text{impact of incomplete insurance}
\end{array} \]
# Production structure

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<th>Inputs &amp; output</th>
<th>Firms</th>
<th>Frictions</th>
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<tr>
<td>vacancy, fixed and</td>
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</table>
| capital adj. costs |      |            | ❯ final goods
**Wage, interest rate and aggregate state**

Nominal wage stickiness (Hall 2005):

\[
w = \left(\frac{w - 1}{1 + \pi}\right)^{\gamma_w} \left(\bar{w}e^{z + \varphi_w} \left(\frac{n}{\bar{n}}\right)^{\psi_n}\right)^{1 - \gamma_w}
\]

Taylore rule:

\[
\log \left(\frac{1 + R}{1 + \bar{R}}\right) = \rho_R \log \left(\frac{1 + R - 1}{1 + \bar{R}}\right)
+ (1 - \rho_R) \left[ a\pi \log \left(\frac{1 + \pi}{1 + \bar{\pi}}\right) + ay \log \left(\frac{1 + g}{1 + \bar{g}}\right)\right] + \varphi_R
\]

Aggregate state:

\[
X = \left\{ k, a^F, a^e, i, c^F, c^e, c^eu, c^{uu}, R_{-1}, \Lambda_{-1}, \pi_{-1}, y_{-1}, w_{-1}, \Phi \right\}
\]

with

\[
\Phi = \left\{ z, \varphi_i, \varphi_c, \varphi_s, \varphi_R, \varphi_w, \varphi_p \right\}
\]
Empirical analysis

Procedure

Detrend (\(\times e^{-z}\)) + log-linearise + solve for state-space representation

Transition equation:

\[ \hat{X}_t = F(\vartheta) \hat{X}_{t-1} + G(\vartheta) \epsilon_t \] (1)

Measurement equation:

\[ \begin{pmatrix} 
\Delta \log(c_t) \\
\Delta \log(\bar{i}_t) \\
\log(c_{\Omega,t}/c_t) \\
\pi_t \\
R_t \\
\Delta \log(W_t) \\
s_t \\
f_t 
\end{pmatrix} = M(\vartheta) + H(\vartheta) \hat{X}_{t-1} + J(\vartheta) \epsilon_t \] (2)

Let \(\vartheta = (\vartheta_1, \vartheta_2)\), with \(\vartheta_1\) calibrated and \(\vartheta_2\) estimated

Sample period: 1985Q1–2007Q1
Empirical analysis

Calibrated parameters

Population and preferences:
- share of workers $\Omega = 0.6$
- CRRA with $\sigma = 2$
- $\beta^F$ to match average real interest rate
- $\beta^W$ s.t. $\bar{c}^{eu} / \bar{c}^e = 0.8$

Labor market and insurance:
- matching elasticity $\chi = 1/2$
- unit vacancy cost $\kappa_v$ s.t. total cost =1% of output
- $\bar{w}$ so as to match job-finding rate $\bar{f}$
- UI benefit $b^u$ s.t. replacement ratio =1/2
- skill premium $\psi$ to match consumption share of bottom 60%
- zero debt limit (i.e., the unemployed can't borrow)
- $\bar{\rho}$ to match quarter-to-quarter separation rate $\bar{s} = \bar{\rho} (1 - \bar{f})$

Production:
- depreciation rate $\delta$ s.t. =6% annually
- capital elasticity $\phi$ s.t. labor share =64%
- TFP growth $\mu_z$ to match average growth
- cross-partial elasticity btw goods $\theta$ s.t. markup =20%
## Empirical analysis

### Estimated parameters

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<tr>
<th>Parameter</th>
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<th>Prior s.d.</th>
<th>Posterior Mean</th>
<th>Posterior s.d.</th>
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<td>9.91</td>
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Does precautionary saving matter?

Compare with responses in economies with full insurance:

- **Representative Agent** \((\Omega^{RA} = 0, \psi^{RA} = \Omega + (1 - \Omega) \psi)\)
- **Hand-to-Mouth** \((a^{e'} = a^{u'} = 0 \Rightarrow c^W = w)\)

The 3 economies (baseline, RA, HtM) have

- same steady state interest rate \(1/\beta^F\) and net wealth \(\bar{K}\)
- different cross-sectional distributions
Monetary policy shock

Consumption – Monetary Policy Shock

Investment – Monetary Policy Shock

Output – Monetary Policy Shock
Job separation shock
A comparison of peak responses

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Conclusion

- Formulation + estimation of DSGE model with incomplete insurance

- Competition of AS and AD effects of time-varying precautionary saving

- Precautionary saving alter the economy’s responses to macro shocks, sometime by a large amount (e.g., monetary policy, job separation)

- Framework bridges RA/HA macro; useful in other contexts, e.g., aggregate demand effects of redistributive policies
Appendix A: More on the model

- SDF $M^{F'} = \beta^E e^{\Delta \phi'_c} \frac{u'(c^{F'} - h c^{F'})}{u'(c^F - h c^F)}$

- Bond Euler equation $\mathbb{E} \left[ M^{F'} (1 + r') \right] = 1$, with $1 + r' = \frac{1 + R}{1 + \pi'}$

- Optimal investment:

$$q = \mathbb{E} \left[ M^{F'} (r'_k + (1 - \delta) q') \right]$$

$$1 = q e^{\phi_i} \left[ 1 - S \left( \frac{i'}{i} \right) - S' \left( \frac{i'}{i} \right) \frac{i'}{i} \right] + \mathbb{E} \left\{ M^{F'} q' e^{\phi'_i} S' \left( \frac{i''}{i'} \right) \left( \frac{i''}{i'} \right)^2 \right\}$$
Appendix A: More on the model

Market clearing and symmetric equilibrium

**Labor services:**

\[(\Omega + (1 - \Omega)\psi)n = \tilde{n}\]

**Assets:**

\[(1 - \Omega)uk = \tilde{k}\]

\[(1 - \Omega)a^{F'} + \Omega n a^{e'} + \Omega \sum_{\eta \geq 1} \int_a a^{u'} \, d\mu(a, \eta) = 0\]

**Goods:**

\[(1 - \Omega)(c^F + i' + \eta(u)k) + \Omega n^W c^e + \Omega \sum_{\eta \geq 1} \int_a c^{u} \, d\mu(a, \eta) + \kappa_y e^z \nu = y\]

\[\int_0^1 x_\zeta \, d\zeta = y_m = \tilde{k}^\phi (e^z \tilde{n})^{1-\phi}\]

\[\Lambda y = \tilde{k}^\phi (e^z \tilde{n})^{1-\phi} - \kappa_y e^z\]

**Symmetric eq.:**

\[\tilde{\mu}(a, \eta) = \tilde{\mu}(a, \eta), \quad \mu(a, \eta, X) = \mu(a, \eta, X)\]

\[\tilde{n}^W = \tilde{n}^F = \tilde{n}^W = \tilde{n}^F = \tilde{n}, \quad n^W = n^F = n^W = n^F = n\]
Appendix A: More on the model

Time line

Labor Market Transitions Stage  Production Stage  Consumption and Saving Stage

Exogenous state is revealed  Vacancies are created, matches are formed  Production takes place  Asset holding choices are made

$t$  $t+1$

Matches are destroyed  Income components are paid (wages, interests, rents)  Family head allocates goods and assets across members
Appendix A: More on the model

Existence conditions for the 2-wealth state model

Figure 2: Equilibrium Reduction Condition

Equilibrium Reduction Condition, Part 1

Equilibrium Reduction Condition, Part 2

Note: The thick red line is the posterior mean path, the grey area delineated by thin, black lines is the 90 percent HPD interval, and the pink bars indicate the NBER recession dates present in our sample.
Appendix B: Data

- Sample period: 1985Q1–2007Q1
- "Investment" = gross private investment + durables from NIPA
- "Consumption" = Personal + government cons. from NIPA
- GDP deflator from NIPA
- Average weekly nominal earnings from CES
- Consumption share of poorest 60% from CEX (with "nondurables" defined as in Heathcote et al RED 2010, and with HH sorted by income levels)
- Labor market transition rates constructed from CPS as in Shimer (2005, 2012), then made quarterly by multiplying monthly transitions matrices
- Effective Fed funds rate
Appendix C: Effect of nominal rigidities

Figure 6: Comparison of IRFs in the Baseline Model with or without Flexible Prices and Wages

Note: Impulse response functions of Output, Consumption and Investment, after a monetary shock (first row), a preference shock (second row), a technology shock (third row) and a shock on the job separation rate (fourth row). The black solid line is the baseline, the dashed line is the HM-economy, the grey line is the RA economy.
Appendix D: Related literature

- Labor market frictions + nominal rigidities
  Walsh 2005; Gertler et al. 2008; Trigari 2009; Blanchard Gali 2010; Gali 2011...

- Labor market frictions + incomplete insurance
  Krusell et al. 2011; Nakajima 2012; Kehoe et al. 2014

- Incomplete insurance + nominal rigidities
  Guerrieri Lorenzoni 2013; Oh Reis 2012; McKay Reis 2014

- All 3 frictions
  Gornemann et al. 2012; Ravn Sterk 2013

- Hand-to-mouth economies

- Estimation of Krusell-Smith model
  McKay 2014

- (Recent) models of aggregate demand effects
  Michaillat Saez 2014; Beaudry Portier 2014; Rendhal 2014; Chamley 2014; Heathcote Perri 2013