Optimal Policy under Dollar Pricing*

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Abstract

Recent empirical evidence shows that most international prices are sticky in dollars. This paper studies the optimal policy implications of this fact in the context of an open economy model, allowing for an arbitrary structure of asset markets, general preferences and technologies, time- or state-dependent price setting, a rich set of shocks, and endogenous currency choice. We show that although monetary policy is less efficient and cannot implement the flexible-price allocation, inflation targeting remains robustly optimal in non-U.S. economies. The implementation of this non-cooperative policy results in a "global monetary cycle" with other countries partially pegging their exchange rates to the dollar and importing the monetary stance of the U.S. In spite of the aggregate demand externality, capital controls cannot unilaterally improve the allocation and are useful only when coordinated across countries. The optimal U.S. policy, on the other hand, deviates from inflation targeting to take advantage of its effects on global product and asset markets, generating negative spillovers on the rest of the world. International cooperation benefits other countries by improving global demand for dollar-invoiced goods, but may be hard to sustain because it is not in the self-interest of the U.S. At the same time, countries can still gain from local forms of policy coordination — such as forming a currency union like the Eurozone.

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1 Introduction

What is the optimal monetary policy in an open economy? According to the standard Mundell-Fleming view, central banks should focus on domestic targets, such as price stability, leaving the burden of external adjustment to freely floating exchange rates. Yet, in practice, this prescription is rarely followed with policymakers referring to international spillovers as a rationale for responding to foreign shocks and anchoring the exchange rate. Among other potential channels that might be responsible for this discrepancy between conventional wisdom and actual policies, recent literature has emphasized the asymmetric use of currencies in international trade with most import and export prices sticky in dollars (Gopinath 2016, Goldberg and Tille 2008). While a lot of progress has been made in understanding the positive consequencies of dollar currency pricing (DCP) (see Gopinath, Boz, Casas, Diez, Gourinchas, and Plagborg-Møller 2018), much less is known about its implications for optimal policy and the welfare.[1] These questions are, however, at the heart of policy debates: Should countries peg exchange rates or let them float (Friedman 1953)? Can capital controls insulate economies from foreign spillovers? Are there gains from cooperation and/or forming a currency union (Mundell 1961)? Should the Fed be concerned about international spillovers of its policy (Bernanke 2017, Obstfeld 2019)? Does the U.S. enjoy an "exorbitant privilege" from DCP (Eichengreen 2011)?

We answer these questions in the context of a generalized version of a conventional sticky-price open economy model by Gali and Monacelli (2005), augmented with a more realistic structure of the international price system. In particular, firms use producer currency pricing (PCP) in domestic markets and DCP in exports. In addition, we assume that only a fraction of imports is used for final consumption, while the rest is used as inputs in production. This is consistent with the fact that intermediates account for most of international trade (Johnson and Noguera 2012) and that final goods contain a significant local distribution margin (Burstein, Neves, and Rebelo 2003). Combined together, these two assumptions allow the model to simultaneously reproduce a high pass-through of the U.S. exchange rate into import prices at the dock and a low pass-through at the retail level (Auer, Burstein, and Lein 2018), and as we show, have important implications for the optimal policy. We keep the rest of the model quite general — allowing for arbitrary (in)complete asset markets, a rich set of shocks, flexible functional forms of utility and production, and either Rotemberg or Calvo price friction — and solve for the optimal non-cooperative policy in the U.S. and other economies.

Our central finding is that even though the actual allocation depends on the particular assumptions, such as completeness of asset markets and the degree of price stickiness, the optimal monetary policy in non-U.S. economies can be summarized with one "sufficient statistic", namely, domestic price stability. It is robust to all details of the environment, is independent from the values of model parameters, and is potentially measurable in the data. Moreover, this simple form of the optimal policy allows us to solve the planner's problem in a fully non-linear stochastic environment without using the second-order ap-

[1] The literature review below discusses in detail a few important exceptions and contrasts them with our analysis.

proximations, which are usually employed in such cases. More precisely, the optimal non-cooperative policy stabilizes the average price of all goods that are sticky in domestic currency. Importantly, this includes the retail prices of imported products, so in practice, the policy target might be closer to the consumer rather than the producer price index. The optimal policy is time consistent, and is therefore the same with and without commitment.

Though similar to the case of a closed economy and an open economy under PCP, this result is arguably much more surprising in the context of DCP when, in contrast to the former two cases, the monetary policy *cannot* implement the first-best allocation. Indeed, because domestic and export prices are sticky in different currencies, the planner faces a trade-off between stabilizing domestic demand and exports. The result showing that the optimal policy focuses exclusively on local prices and does not target inefficient terms-of-trade and trade balance seems to be counterintuitive and inconsistent with a standard second-best logic (Lipsey and Lancaster 1956). The reason for this "corner solution" is that conditional on the optimal value of marginal costs in local currency, the prices set by exporters are *constrained efficient*, i.e. a planner who has an additional instrument to manipulate export prices, but is subject to the same price-adjustment friction as private firms, would decide not to intervene. Given that the optimal policy ensures the efficiency of both domestic prices and the marginal costs of exporters, export prices are socially optimal and there is no room for further improvement. Thus, monetary policy that stabilizes only local prices already achieves the best possible terms of trade and does not need to target them directly.

Even though the optimal non-U.S. policy can be formulated in terms of a domestic target, it is generically outward-looking, responds asymmetrically to U.S. shocks, and under plausible assumptions, partially pegs the exchange rate to the dollar. To see this, consider the tightening of U.S. monetary policy that appreciates the dollar and hence, under DCP, increases the prices of imported intermediates in other economies. This puts an inflationary pressure on foreign central banks and requires them to tighten their monetary policy as well, giving rise to a "Global Monetary Cycle" — a situation when monetary stance is correlated across countries even if exogenous shocks are purely idiosyncratic. The pass-through of U.S. monetary shocks into local interest rates is higher for more open countries with a larger share of DCP in imports, consistent with the recent empirical findings of Zhang (2018). This "leaning against the wind", however, is optimal more broadly in response to any local or U.S. shock and effectively implies a partial peg to the dollar. Thus, the widespread "fear of floating" and the anchoring of exchange rates to the dollar in the data (Calvo and Reinhart 2002, Ilzetzki, Reinhart, and Rogoff 2018) can be due not only to the global financial cycle (Rey 2013), but also to the dominant status of the dollar in international trade. This analogy between the two sources of international spillovers applies also to the Trilemma-Dilemma debate: dollar pricing implies that a floating exchange rate cannot fully insulate countries from foreign spillovers and worsens the trade-off faced by policymakers. This, however, does not mean that the planner should give up on exchange rates altogether, as they still constitute an important margin of adjustment (*cf.* Gourinchas 2018, Kalemli-Ozcan 2019).

Motivated by the recent arguments in favor of macroprudential instruments to deal with international spillovers (see e.g. Blanchard 2017), we next study the optimal mix of monetary policy and capital controls and find, surprisingly, that the latter do *not* insulate economies from foreign spillovers and are set to zero by individual countries. Again, this outcome seems to be inconsistent with the second-best intuition, which prescribes intervening in asset markets to improve the allocation in goods markets: as shown forcefully by Farhi and Werning (2016), the laissez-faire risk sharing is generically inefficient when monetary policy cannot implement the first-best allocation. This discrepancy comes from the fact that there are two sources of the aggregate demand externality in our setup — a local and a foreign one. On one hand, the optimal monetary policy ensures that domestic demand is efficient and eliminates the local externality. On the other hand, the foreign demand for exported goods is in general suboptimal due to dollar pricing, but cannot be changed by the domestic economy via capital controls. As a result, under the optimal monetary policy and arbitrary structure of asset markets, the private risk sharing is constrained efficient, leaving no room for macroprudential policies. This shows that it is not just the number of available instruments, but also the nature of the international spillovers that is important for the optimal policy: while capital controls might be efficient in curbing financial spillovers (see e.g. Bianchi 2011), they are unlikely to help with the spillovers arising from DCP.

In contrast, the state-contingent export tariffs combined with production subsidies can be quite efficient in restoring the first-best allocation. Intuitively, as long as the tariffs are imposed on top of the export prices, the planner can implement the optimal terms of trade, while production subsidies ensure that firms keep their dollar-invoiced prices constant, saving on price-adjustment costs. Because the Lerner symmetry does not hold under DCP (Barbiero, Farhi, Gopinath, and Itskhoki 2019), the export tariff is crucial and cannot be substituted for other border taxes or the VAT. At the same time, the optimal monetary policy remains robust and still targets domestic prices.

Three extensions of the model clarify our results for non-U.S. economies. First, the optimal policy remains remarkably robust to both state-dependent (Rotemberg) and time-dependent (Calvo) price settings, a non-CES (Kimball) demand, and heterogeneity across firms. Intuitively, in all these cases, the private incentives of exporters are perfectly aligned with the social interests of a small open economy and the policy that implements the optimal local demand achieves a constrained efficient allocation leaving, in particular, no room for capital controls. Second, we allow firms to optimally choose the currency of invoicing and show that our results withstand the Lucas critique. Interestingly, the currency choice of exporters is also constrained optimal: despite being the key source of inefficiency in world economy, individual countries have no incentive to deviate from the DCP equilibrium. Finally, consistent with the evidence from several emerging economies (Drenik and Perez 2019), we allow some *domestic* prices to be set in dollars and show that the optimal monetary policy stabilizes the prices of goods invoiced in local currency. Because demand for dollar-invoiced goods is suboptimal in this case, the local aggregate demand externality is non-zero, and the equilibrium allocation can be improved with capital controls. Interestingly, extending the results from Farhi and Werning (2016), we show that the same externality implies that the price setting of exporters is also constrained inefficient and should be corrected with production subsidies because the firms do not internalize the stimulating effect of export revenues on local demand.

We then argue that due to its global spillovers, optimal U.S. policy deviates from a pure inflation targeting and is likely to achieve higher welfare than in other countries. On one hand, the fact that both domestic and export prices are set in producer currency makes U.S. monetary policy more efficient in simultaneously stabilizing local and foreign demand. As a result, from the *ex-post* perspective, the Fed does not need to worry about the negative international spillovers of its policy and can focus exclusively on targeting domestic inflation. On the other hand, this policy is suboptimal from the *ex-ante* perspective. First, even if the U.S. is a small economy, because of DCP, its monetary policy affects the global stochastic discount factor and hence, the asset prices at which the country can borrow and save. This gives rise to the "dynamic terms-of-trade" motive (Costinot, Lorenzoni, and Werning 2014) and contributes to U.S. "exorbitant privilege" (Gourinchas and Rey 2007): to borrow cheaply and earn high interest on savings, U.S. policy stimulates the world economy when it runs a negative current account, and depresses the world economy when its current account is positive. Second, the prices preset by foreign exporters depend on expectations about U.S. monetary shocks. By partially stabilizing the dollar value of marginal costs of world producers, the optimal policy dampens their markups and makes foreign goods cheaper for the U.S. and other economies (*cf.* Devereux and Engel 1998). The welfare of the U.S. depends on the relative strength of these motives and in general, can be higher or lower than the welfare of other economies. We show that the first motive dominates under a standard parametrization, and the U.S. gains from DCP because of a higher efficiency of its monetary policy in stabilizing foreign demand. In a globalized economy with firms likely to coordinate on a single currency, this naturally leads to a competition between nations for the status of the dominant-currency provider.

Finally, we show that the optimal *cooperative* policy stabilizes global demand using monetary instruments, fights the aggregate demand externality with capital controls, and although a full-scale coordination might not be in the self-interest of the U.S., more local forms of cooperation, such as a currency union, can still improve welfare. More precisely, just as in a non-cooperative case, the global planner uses monetary policy in non-U.S. economies to target domestic prices and demand for local goods. In contrast, U.S. monetary policy switches to stabilizing world export prices and global demand for dollar-invoiced goods. Since one U.S. monetary instrument is not sufficient to implement the optimal demand for all exported goods, the planner also uses capital controls against the aggregate demand externality. Importantly, because it is production of *exported* rather than local goods that is inefficient, the optimal policy redistributes demand towards *importers* of depressed goods, not to their *exporters*. While beneficial for the world economy, it might, however, be hard to sustain the cooperative policy: to stabilize global demand, U.S. policy needs to sacrifice domestic objectives, which may have detrimental effects on the country's welfare. At the same time, we show that there are gains from local coordination, even if the U.S. is not part of it. In particular, our analysis reveals a new source of gains from forming a currency union such as the Eurozone. While the standard critique — that a member of a currency area loses its independent monetary policy and cannot use it to stabilize the economy — still applies in our setting, forming a currency union can boost the welfare of its members by internalizing the aggregate demand externality and improving demand for dollar-invoiced goods.

Related literature This paper contributes to a vast literature on the optimal policy in New-Keynesian open-economy models. The seminal papers by Obstfeld and Rogoff (1995), Clarida, Gali, and Gertler (2001, 2002), Gali and Monacelli (2005) and their extensions by De Paoli (2009) and Dmitriev and Hoddenbagh (2014) focus on monetary policy under PCP, formalizing the Friedman (1953) argument in favor of floating exchange rates. Motivated by the fact that at the retail level, import prices are sticky in consumer currency, Devereux and Engel (2003, 2007) and Engel (2011) contrast this result with the desirability of the peg under LCP. The gains from monetary cooperation are the focus of Obstfeld and Rogoff (2002), Benigno and Benigno (2003, 2006) and Corsetti and Pesenti (2005), while Corsetti, Dedola, and Leduc (2010, 2018) make an important contributions regarding the optimal policy under incomplete markets. Our main departure from this literature is that we assume a more realistic structure of the international price system with global trade invoiced in one dominant currency.

Our work is most closely related to Corsetti and Pesenti (2007), Devereux, Shi, and Xu (2007), Goldberg and Tille (2009), and Casas, Díez, Gopinath, and Gourinchas (2017) who solve for the optimal monetary policy under DCP. Despite the apparent similarity, there are three key differences that distinguish our paper from this literature (see Table 1 for detailed comparison). First, in contrast to these papers, we allow for incomplete pass-through of border prices into retail prices, which is pronounced in the data and, as we show, has important implications for the optimal policy. Second, although our central result about the optimality of price stabilization in non-U.S. economies resembles the findings of the previous literature, the intuition is qualitatively different. Indeed, the papers mentioned above make strong assumptions about fully sticky prices, log-linear preferences and/or complete asset markets, under which the terms of trade are *exogenous* to monetary policy and, unsurprisingly, are not targeted by a planner. In contrast to these knife-edge cases, we show that it is *generically* optimal for monetary authorities to stabilize domestic prices even when the terms of trade are endogenous to the policy. Finally, several of our results are completely new to the literature. This includes the suboptimality of capital controls, the robustness of optimal policy to endogenous currency choice, and the new motives of U.S. monetary policy that are absent in special cases analyzed in the previous models. Our analysis is complementary to Mukhin (2018), who however, focuses exclusively on the *cooperative* policy and hence, cannot speak to the strategic interactions between countries that arise under DCP.

This paper is also related to recent literature about the role of fiscal instruments when the monetary policy alone is not sufficient to implement the optimal allocation. Our analysis of optimal fiscal policy follows Adao, Correia, and Teles (2009), Farhi, Gopinath, and Itskhoki (2014) and Chen, Devereux, Xu, and Shi (2018) who show how the effects of exchange rate depreciation can be replicated with taxes and

Table 1: Comparison to the literature

<table>
<tr><th></th><th>DSX</th><th>CP</th><th>GT</th><th>CDGG</th><th>EM</th></tr>
<tr><td colspan="6">Environment:</td></tr>
<tr><td># of countries</td><td colspan="2">two</td><td>three</td><td>SOE</td><td>continuum</td></tr>
<tr><td>preferences</td><td colspan="4">log-linear</td><td>general</td></tr>
<tr><td>intermediates</td><td colspan="4">no</td><td>yes</td></tr>
<tr><td>asset markets</td><td colspan="4">complete</td><td>arbitrary</td></tr>
<tr><td>prices</td><td colspan="3">fully sticky</td><td>Calvo</td><td>Rotemberg/Calvo</td></tr>
<tr><td>terms-of-trade</td><td colspan="4">exogenous to MP</td><td>endogenous</td></tr>
<tr><td>currency choice</td><td>rationalized</td><td colspan="3">exogenous</td><td>endogenous</td></tr>
<tr><td colspan="6">Non-U.S. policy:</td></tr>
<tr><td>optimal target</td><td colspan="5">price stabilization</td></tr>
<tr><td>allocation</td><td colspan="5">inefficient</td></tr>
<tr><td>implementation</td><td colspan="4">inward-looking</td><td>outward-looking</td></tr>
<tr><td>exchange rates</td><td colspan="4">floating</td><td>partial peg</td></tr>
<tr><td>capital controls</td><td colspan="4">—</td><td>inefficient</td></tr>
<tr><td>trade policy</td><td colspan="4">—</td><td>efficient</td></tr>
<tr><td colspan="6">U.S. policy:</td></tr>
<tr><td>ToT motive</td><td colspan="3">yes</td><td>—</td><td>yes</td></tr>
<tr><td>dynamic ToT motive</td><td colspan="3">no</td><td>—</td><td>yes</td></tr>
<tr><td>gains from DCP</td><td>negative</td><td>—</td><td>negative</td><td>—</td><td>ambiguous</td></tr>
<tr><td>cooperative policy</td><td colspan="3">monetary</td><td>—</td><td>monetary+fiscal</td></tr>
<tr><td>currency union</td><td colspan="4">—</td><td>potential gains</td></tr>
</table>

Note: DSX stands for Devereux, Shi, and Xu (2007), CP for Corsetti and Pesenti (2007), GT for Goldberg and Tille (2009), CDGG for Casas, Díez, Gopinath, and Gourinchas (2017), EM for this paper.

tariffs. Lorenzoni (2008), Jeanne and Korinek (2010), Bianchi (2011) and Dávila and Korinek (2017) study the implications of the pecuniary externality for the macroprudential policy, while Farhi and Werning (2012, 2013, 2016, 2017) and Fornaro and Romei (2019) derive the optimal capital controls in the presence of aggregate demand externality. We complement this literature with the analysis of optimal capital controls under DCP.

Finally, our results go beyond the normative analysis, as they also have important positive implications. The optimality of partial peg to the dollar under DCP complements other explanations for the "fear of floating" by Calvo and Reinhart (2002), Rey (2013), Fanelli (2017) and Hassan, Mertens, and Zhang (2019). It also generates additional complementarities between the dominant status of the dollar as an international currency of invoicing, funding, anchoring, and reserves as discussed by Gopinath and Stein (2017), Mukhin (2018), Maggiori, Neiman, and Schreger (2018), and Gourinchas (2019).

2 Environment

This section describes the baseline setup, which builds on a conventional sticky-price open economy model by Gali and Monacelli (2005) that has been extensively used in the recent normative literature (see e.g. Farhi and Werning 2012, 2013). We augment this model with two additional ingredients to allow for a more realistic structure of the international price system. First, all international prices are sticky in dollars rather than in the currency of the producer or buyer, while domestic products are invoiced in local currency. While an extreme assumption, the empirical literature shows that it provides a good first-order approximation to the real world (Gopinath 2016). For simplicity, we take the currency of invoicing as given for now and show that our results remain true under an endogenous currency choice in Section 4.2. Second, only a fraction of imports is used for final consumption, while the rest is used as intermediate goods in production. This is consistent with the fact that intermediates account for most of global trade (see e.g. Johnson and Noguera 2012) and final goods contain a significant local distribution margin (Burstein, Neves, and Rebelo 2003). Combined together, these two assumptions allow the model to simultaneously reproduce a high pass-through of the U.S. exchange rate into import prices at the dock and a low pass-through at the retail level (Auer, Burstein, and Lein 2018) and have important implications for the optimal policy.

The rest of the model is standard. Time is discrete, and the horizon is infinite. The world consists of a continuum of symmetrical small open economies $i \in [0, 1]$. To disentangle the role of the dominant currency, we assume that the U.S. is a small economy indexed by $i = 0$ and is symmetric to other countries in all respects except for the use of the dollar in international trade. Each country is populated by identical households that consume local and imported goods, supply labor and make savings decisions. Monopolistic firms produce domestic and export goods and are subject to price-adjustment frictions. We generalize the model of Gali and Monacelli (2005) in several dimensions to demonstrate the robustness of our results — allowing for flexible preferences and technologies, complete and incomplete asset markets, and a rich set of exogenous shocks — while also trying to keep the model tractable enough to convey the main mechanisms.

2.1 Households

A representative household in country i has preferences over consumption C_{it} and labor N_{it}:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t U\big(C_{it}, N_{it}, \xi_{it}\big),$$

where ξ_{it} includes both intra-temporal (labor supply) and inter-temporal (discount) shocks and $U(\cdot)$ satisfies the standard regularity conditions. Consumption is an aggregator of home and foreign goods:

$$C_{it} = \left[(1-\gamma)^{\frac{1}{\theta}} C_{iit}^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} C_{it}^{*\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},$$

where $1-\gamma$ reflects the home bias in consumption that can arise due to trade costs or preferences for locally produced goods. The import bundle aggregates products from all other countries

$$C_{it}^{*} = \left(\int C_{jit}^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}j\right)^{\frac{\varepsilon}{\varepsilon-1}},$$

while each bilateral trade flow includes a continuum of unique products $\omega \in [0,1]$:

$$C_{jit} = \left(\int C_{jit}(\omega)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}\omega\right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Thus, following the previous literature, we allow for a nested CES structure to distinguish the "macro elasticity" of substitution θ between home and foreign goods from the "micro elasticity" $\varepsilon > 1$ governing the substitution between individual products (as e.g. in Feenstra, Luck, Obstfeld, and Russ 2018). These functional forms — with constant elasticities and symmetric import shares across countries — are standard and simplify the notation, but are not essential for our results as shown in Section 4.1.

Each period t, households face a flow budget constraint:

$$P_{it}C_{it} + \mathcal{E}_{it}\sum_{h\in H_t} \mathcal{Q}_t^h B_{it+1}^h + \frac{\mathcal{B}_{it+1}^i}{R_{it}} = W_{it}N_{it} + \Pi_{it}^f + T_{it} + \mathcal{E}_{it}\sum_{h\in H_{t-1}} (\mathcal{Q}_t^h + D_t^h) B_{it}^h + \mathcal{B}_{it}^i,$$

where consumer price index P_{it} and nominal wages W_{it} are in local currency, H_t is the set of internationally traded assets in period t with prices $\mathcal{Q}_t^h$ and payouts D_t expressed without loss of generality in dollars, and $\mathcal{E}_{it}$ is the nominal exchange rate of country i against the dollar. Households receive transfers T_{it} from the government and profits Π_{it}^f from local firms. In addition to globally traded securities B_{it}^h, the agents can also invest in local government bonds $\mathcal{B}_{it}^i$ with the nominal rate of return R_{it} set by local monetary authorities.

Households choose consumption, labor and asset portfolio to maximize expected utility subject to the budget constraint. The resulting optimal labor supply is given by

$$-\frac{U_{Nit}}{U_{Cit}} = \frac{W_{it}}{P_{it}}, \tag{1}$$

where U_C and U_N are respectively the marginal utilities of consumption and work. As usual, the inter-temporal Euler equation for nominal bonds is:

$$\mathbb{E}_t \Theta_{it,t+1} R_{it} = 1, \quad \text{where} \;\; \Theta_{it,t+\tau} \equiv \beta^\tau \frac{U_{Cit+\tau}}{U_{Cit}} \frac{P_{it}}{P_{it+\tau}} \tag{2}$$

is the nominal stochastic discount factor (SDF). The optimal portfolio of households is characterized by the system of no-arbitrage conditions:

$$\mathbb{E}_t \Theta_{it,t+1} \frac{\mathcal{E}_{it+1}}{\mathcal{E}_{it}} \frac{\mathcal{Q}^h_{t+1} + D^h_{t+1}}{\mathcal{Q}^h_t} = 1, \;\; \forall h \in H_t. \tag{3}$$

Moving to the goods market, demand for domestic and foreign products is

$$C_{iit}(\omega) = (1-\gamma) \left(\frac{P_{iit}(\omega)}{P_{iit}} \right)^{-\varepsilon} \left(\frac{P_{iit}}{P_{it}} \right)^{-\theta} C_{it}, \qquad C_{jit}(\omega) = \gamma \left(\frac{P_{jit}(\omega)}{\mathcal{E}_{it} P^*_t} \right)^{-\varepsilon} \left(\frac{\mathcal{E}_{it} P^*_t}{P_{it}} \right)^{-\theta} C_{it},$$

where $P_{iit} = \left(\int P_{iit}(\omega)^{1-\varepsilon} \mathrm{d}\omega \right)^{\frac{1}{1-\varepsilon}}$ is the price index for home goods in the local currency and $P^*_t = \left(\int P^{*\,1-\varepsilon}_{it} \mathrm{d}i \right)^{\frac{1}{1-\varepsilon}}$ is the import price index in dollars, which due to the DCP assumption is the same for all countries and is the aggregate of dollar export prices of all economies P^*_{it}. The consumer price index (CPI) can then be written as

$$P_{it} = \Big[(1-\gamma) P_{iit}^{1-\theta} + \gamma (\mathcal{E}_{it} P^*_t)^{1-\theta} \Big]^{\frac{1}{1-\theta}}. \tag{4}$$

2.2 Firms

Production In each country i, there is a continuum of firms, each producing a unique variety ω from labor L_{it} and a bundle of intermediate goods X_{it}:

$$Y_{it} = A_{it} F(L_{it}, X_{it}).$$

Though not essential for our results, we assume that technology $F(\cdot)$ is constant returns to scale, which in particular, allows us to abstract from the issue whether different markets are served by the same or different firms and simplifies the aggregation. Producers use a roundabout technology with the same bundle of intermediates as in final consumption. While it is a hardly realistic assumption that import share γ is the same for consumers, domestic producers and exporters, we start with this symmetric case as it implies that the relevant price indices are the same for all agents and greatly simplifies the notation. Importantly, our main results remain true when we relax the assumption in Section 4.1.

Solving the firm's cost minimization problem, we get the relative demand for inputs

$$\frac{X_{it}}{L_{it}} = g \left(\frac{W_{it}}{P_{it}} \right), \tag{5}$$

where function $g(\cdot)$ is implicitly defined by $\frac{F_L(1,g(z))}{F_X(1,g(z))} = z$ with $g'(\cdot) > 0$. The resulting real marginal costs of production are

$$\frac{MC_{it}}{P_{it}} = \frac{h\left(\frac{W_{it}}{P_{it}}\right)}{A_{it}},$$

where we define $h(z) \equiv \frac{1}{F_X(1,g(z))}$.

Domestic prices Firms are monopolistic competitors, are subject to price-adjustment friction, and set local prices in domestic currency and export prices in dollars. While there is little consensus in the literature about the best way of modelling sticky prices, it is encouraging that our results hold for both time-dependent and state-dependent models of nominal rigidities. For this reason, in our baseline specification, we adopt the Rotemberg (1982) model with labor adjustment costs, which results in a tractable *non-linear* New-Keynesian Phillips curve, while Appendix A.5 extends the results to a setup with the Calvo (1983) friction.[2]

In the domestic market, firms set prices in local currency to maximize expected profits net of price-adjustment costs:

$$\max_{\{P_t\}} \mathbb{E} \sum_{t=0}^{\infty} \Theta_{i0,t} \left[\left(P_t - \tau_i MC_{it}\right) \left(\frac{P_t}{P_{iit}}\right)^{-\varepsilon} Y_{iit} - (1-\gamma)\frac{\varphi}{2}\left(\frac{P_t}{P_{t-1}} - 1\right)^2 W_{it} \right],$$

where $Y_{iit} \equiv C_{iit} + X_{iit}$ is the local demand shifter, menu costs are normalized by the size of the domestic market $1-\gamma$, and τ_i is the time-invariant production subsidy to local sellers.[3] Taking the first-order condition and imposing symmetry across firms allows us to express the domestic Phillips curve as

$$\pi_{iit}\left(\pi_{iit}+1\right)W_{it} = -\kappa\left(P_{iit} - \frac{\varepsilon\tau_i}{\varepsilon-1}MC_{it}\right)\frac{Y_{iit}}{1-\gamma} + \mathbb{E}_t\Theta_{it,t+1}\pi_{iit+1}\left(\pi_{iit+1}+1\right)W_{it+1}, \qquad (6)$$

where $\kappa \equiv \frac{\varepsilon-1}{\varphi} > 0$ and $\pi_{iit} \equiv \frac{P_{iit}}{P_{iit-1}} - 1$ is the PPI inflation rate.

As usual, the dynamic price setting equation can be interpreted as an error-correction model: if prices were flexible, the firms would charge a constant markup over marginal costs, i.e. $P_{iit} = \frac{\varepsilon\tau_i}{\varepsilon-1}MC_{it}$ in all states of the world. In the presence of nominal rigidities, the price can temporarily deviate from the optimal level: e.g. when it is too high, the bracket on the right-hand side is positive and, other things equal, firms decrease their prices, i.e. $\pi_{iit} < 0$. The speed of adjustment is higher when prices are more flexible (large κ) and when price deviations are associated with larger profit losses (high demand Y_{iit}). As is standard in the New-Keynesian literature, the Phillips curve is forward-looking with firms trying

[2]The previous literature has explored several alternative assumptions about adjustment costs in the Rotemberg model (*cf.* e.g. Schmitt-Grohé and Uribe 2004, Faia and Monacelli 2008, Kaplan, Moll, and Violante 2018). Our results hold independently whether the costs are set in labor units or product units and whether they are scaled by firms' output or not.

[3]In line with the standard timing under commitment, the state-contingent policy is announced before firms set their prices, which is equivalent to assuming zero costs of price adjustment in the initial period $t = 0$.

to smooth the marginal costs $\pi_{iit}\left(\pi_{iit}+1\right)W_{it}$ of price adjustment across periods.

Export prices We assume that exporters set a uniform dollar price for all markets of destination:

$$\max_{\{P_t\}} \mathbb{E}\sum_{t=0}^{\infty} \Theta_{i0,t}\left[\left(\mathcal{E}_{it}P_t-\tau_i^* MC_{it}\right)\left(\frac{P_t}{P_{it}^*}\right)^{-\varepsilon}Y_{it}^*-\gamma\frac{\varphi}{2}\left(\frac{P_t}{P_{t-1}}-1\right)^2 W_{it}\right],$$

where $Y_{it}^* \equiv \int\left(C_{ijt}+X_{ijt}\right)\mathrm{d}j$ is the global demand shifter for exported goods and τ_i^* denotes the time-invariant production subsidy to exporters. The first-order optimality condition can be rewritten as an export Phillips curve:

$$\pi_{it}^*\left(\pi_{it}^*+1\right)W_{it}=-\kappa\left(\mathcal{E}_{it}P_{it}^*-\frac{\varepsilon\tau_i^*}{\varepsilon-1}MC_{it}\right)\frac{Y_{it}^*}{\gamma}+\mathbb{E}_t\Theta_{it,t+1}\pi_{it+1}^*\left(\pi_{it+1}^*+1\right)W_{it+1}, \tag{7}$$

where $\pi_{it}^* \equiv \frac{P_{it}^*}{P_{it-1}^*}-1$ is the inflation rate for the dollar export price index. Equation (7) is a direct counterpart of (6) with inflation going up when dollar export price P_{it}^* is below the optimal flexible-price level $\frac{\varepsilon\tau_i^*}{\varepsilon-1}\frac{MC_{it}}{\mathcal{E}_{it}}$ and the speed of adjustment is increasing in global demand shifter Y_{it}^*.

2.3 Market clearing

The goods market clears when total production equals the sum of local and foreign demand for final and intermediate goods:

$$A_{it}F(L_{it},X_{it})=(1-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}(C_{it}+X_{it})+\gamma\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}\int\left(\frac{\mathcal{E}_{jt}P_t^*}{P_{jt}}\right)^{-\theta}(C_{jt}+X_{jt})\,\mathrm{d}j, \tag{8}$$

where we have used the symmetry across producers within a country. The labor market clearing condition aggregates labor used in production and on price adjustments:

$$N_{it}=L_{it}+\frac{\varphi}{2}(1-\gamma)\pi_{iit}^2+\frac{\varphi}{2}\gamma\pi_{it}^{*2}. \tag{9}$$

Because the Ricardian equivalence holds in the model, it is without loss of generality to assume that the government balances its budget every period using the lump-sum taxes to finance production subsidies. Combing the budget constraint of households with firms' profits, we arrive at the country's budget constraint:

$$\begin{aligned}\sum_{h\in H_t}\mathcal{Q}_t^h B_{it+1}^h-\sum_{h\in H_{t-1}}(\mathcal{Q}_t^h+D_t^h)B_{it}^h\\ =\gamma\left[P_{it}^*\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}\int\left(\frac{\mathcal{E}_{jt}P_t^*}{P_{jt}}\right)^{-\theta}(C_{jt}+X_{jt})\,\mathrm{d}j-P_t^*\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{-\theta}(C_{it}+X_{it})\right]+\psi_{it},\end{aligned} \tag{10}$$

where the right-hand side is the dollar value of the country's net exports and the left-hand side combines the change in the net foreign asset position with the "valuation effects" (Gourinchas and Rey 2007). Notice that we allow for an idiosyncratic wealth shock ψ_{it} with a zero global mean $\int \psi_{it} \mathrm{d}i = 0$. This shock is meant to capture several additional (unmodelled) sources of volatility that previous literature has found to be important drivers of business cycles and exchange rates in open economies: e.g. fluctuations in commodity prices (as in Casas, Díez, Gopinath, and Gourinchas 2017), terms-of-trade shocks (Mendoza 1995), taste shocks for foreign versus domestic goods (Pavlova and Rigobon 2007) as well as shocks to capital flows (Gabaix and Maggiori 2015). Importantly, the shock ensures that the model can replicate the exchange rate disconnect from the data (see Itskhoki and Mukhin 2019a).

Finally, given that households earn the profits of local firms, we can assume without loss of generality that internationally traded assets are in zero net supply:

$$\int B^h_{it+1} \mathrm{d}i = 0, \;\; \forall h \in H_t, \qquad \mathcal{B}^i_{it} = 0. \tag{11}$$

2.4 Equilibrium

As usual, for given monetary policies, the equilibrium is such that households maximize expected utility subject to the sequence of budget constraints, firms maximize expected profits, the government's budget constraint is satisfied, and the markets clear. Following the primal approach, we let the planner choose the optimal allocation subject to the relevant equilibrium conditions. In particular, we substitute out nominal interest rates R_{it} using the Euler equation (2) and nominal wages W_{it} using the labor supply condition (1), while keeping other prices that are either subject to adjustment frictions (P_{iit}, P^*_{it}) or are not directly chosen by an individual country ($\mathcal{Q}^h_t$). The next lemma summarizes restrictions imposed by households' and firms' optimality conditions and the resource constraints.

Lemma 1 (Implementability) *The allocation $\{C_{it}, N_{it}, L_{it}, X_{it}, B^h_{it}\}$ and prices $\{\pi_{it}, \pi_{iit}, \pi^*_{it}, \mathcal{E}_{it}, \mathcal{Q}^h_t\}$ constitute part of the equilibrium if and only if equations* (3) – (11) *hold.*

While the lemma describes the set of all implementable allocations, to solve for the optimal *non-cooperative* policy, one needs to specify further the strategic interactions between policymakers.

Definition *We are looking for a SPNE in the following game: (i) each country chooses a state-contingent plan of PPI inflation $\{\pi_{iit}\}$, (ii) the U.S. moves first and other economies move simultaneously after that, (iii) all countries have full commitment.*

Thus, given the special role of the dollar in the global economy, we assume that the U.S. is a Stackelberg leader, which internalizes the effect of its decisions on other economies when choosing the optimal monetary policy. In contrast, each non-U.S. economy takes the policy of the U.S. and other countries as given. We assume that strategies are formed in terms of inflation, i.e. for each history of exogenous shocks and inflation rates in other countries, a planner chooses its best response. While there are several restrictions imposed by this definition, the next lemma shows that the same allocation arises as an equilibrium outcome in a much larger class of games.

Lemma 2 (Equilibrium) *The equilibrium remains the same in each of the following cases:*

1. *instead of π_{iit}, countries choose $C_{it}, N_{it}, L_{it}, X_{it}, \pi_{it}, \pi^*_{it}$, or any their combination,*
2. *all countries simultaneously choose their strategies in terms of π_{iit},*
3. *non-U.S. economies lack commitment and choose the optimal discretionary policy.*

The lemma effectively relaxes all three assumptions embodied in our benchmark definition of equilibrium. First, the equilibrium remains the same independently of whether countries form their strategies in terms of CPI/PPI inflation, aggregate consumption, output gap, etc.[4] Intuitively, this is the case because non-U.S. countries are small and take *all* foreign variables as given. Second, the baseline sequential game can be reformulated as a particular simultaneous-move game without any first-mover advantage on the U.S. side. This result follows directly from Proposition 1 below, which shows that the optimal non-U.S. policy can be formulated in terms of a simple rule $\pi_{iit} = 0$. Given this best response of other economies, the problem of the U.S. becomes invariant to the timing assumption. Finally, we also show below that the optimal non-U.S. policy is time consistent and therefore, the equilibrium would not change if countries other than the U.S. lacked commitment.

In sum, while the equilibrium allocation does depend on assumptions about strategic interactions across countries, Lemma 2 shows that there is a rich class of games, which — largely due to the particular form of the optimal policy in our model — result in the same equilibrium outcome, supporting the robustness of our analysis.

2.5 Efficient allocation

As emphasized above, our setup is fairly general in many dimensions and can be generalized even further without altering the results. In contrast, the next two assumptions are crucial for the analysis: combined together, they ensure that the monetary policy is not used as a second-best instrument to eliminate distortions in the economy other than the ones associated with nominal rigidities and allow us to focus exclusively on the constraints imposed by the structure of the international price system.

A 1 *Time-invariant production subsidies $\tau_i = \frac{\varepsilon-1}{\varepsilon}$, $\tau^*_i = 1$ eliminate domestic monopolistic distortions and the terms-of-trade externality. There are no exogenous shocks to markups.*

[4]The only important restriction that we impose on the set of strategies is that policies are formulated in terms of domestic objectives. In particular, to avoid pathological cases, countries are not allowed to set policies that they cannot directly implement, e.g. in terms of foreign output. While we exclude bilateral exchange rates from Lemma 2 for the same reason, the equilibrium would not change if non-U.S. economies were choosing $\mathcal{E}_{it}$ and the U.S. used a domestic objective.

As is well known, the standard New-Keynesian open economy models have two types of distortions on top of sticky prices: monopolistic markups and the terms-of-trade externality (see Corsetti and Pesenti 2001, Benigno and Benigno 2003, Faia and Monacelli 2008). The former increases domestic prices, lowers the real wage and labor supply, and results in suboptimal production and consumption. The latter implies that the country can exploit its market power abroad by setting the export price above domestic ones. A destination-specific production subsidy A1 eliminates both distortions, offsetting the markups in domestic markets and, at the same time, maintaining the monopolistic markups abroad.[5] While most of the sticky-price normative papers employ a time-invariant production subsidy, this instrument is usually applied uniformly across *all* firms, which does not allow one to simultaneously eliminate both externalities. Relative to this standard approach, assuming more sophisticated instrument(s) comes at little cost in terms of realism, but significantly improves the tractability of the analysis. In particular, it allows us to disentangle new motives of monetary policy associated with DCP from the standard and well-understood inflationary bias and terms-of-trade externality. For the same reason, we also abstract from markup shocks: while arguably important in practice, the markup shocks have been widely studied in the previous normative literature (see e.g. Clarida, Gali, and Gertler 1999).

A2 *Expressed in foreign currency, the payoffs of internationally traded assets D_t^h, $h \in H_t$ and wealth shocks ψ_{it} are independent from the monetary policies of individual countries.*

This assumption aims to exclude the additional motive for the monetary policy that arises under incomplete markets: by manipulating the payoffs, the policy can "complete" — at least partially — the span of the assets, improving the risk sharing between countries.[6] In particular, condition A2 is trivially satisfied under the "original sin" when countries can only borrow and save in *foreign currency* debt, while it does not hold for the *local currency* debt as monetary policy would inflate it away in bad states of the world (see e.g. Engel and Park 2018, Ottonello and Perez 2019). On top of that, changing the real returns of the assets also allows the monetary authorities to manipulate the risk premium charged by foreign investors (Itskhoki and Mukhin 2019b). Both channels have been studied recently from the perspective of the optimal policy (see Fanelli 2017, Hassan, Mertens, and Zhang 2019, Fornaro 2019) and are not the focus of this paper. Assumption A2 excludes such motives for monetary interventions, while still allowing for a rich set of asset market structures.[7]

In what follows, we assume that conditions A1-A2 are satisfied. The next lemma describes the case of flexible prices, which provides an important benchmark for our analysis.

[5]There are, of course, several alternative instruments that can sustain the same allocation, e.g. a uniform production subsidy to all firms combined with an export tax. In fact, two instruments would be necessary if demand elasticities and hence, optimal markups were different across domestic and foreign markets. Notice that one can also use import tariffs instead of export taxes as the Lerner (1936) symmetry holds for the time-invariant fiscal instruments in the model.

[6]A similar motive also arises in a closed economy under distortionary taxation (see Chari, Christiano, and Kehoe 1991).

[7]In particular, although we abstract from borrowing constraints to keep the analysis more tractable, the results remain unchanged if one allows for constraints that are not associated with pecuniary externalities.

Lemma 3 (Efficient allocation) *The flexible-price equilibrium $\varphi = 0$ is efficient from the perspective of an individual economy and can be implemented under PCP by the monetary policy targeting $\pi_{iit} = 0$.*

The former result is based on two properties of the model. First, due to assumption A2, the monetary policy can affect equilibrium allocation only through nominal rigidities and hence, becomes neutral when prices are flexible. Second, assumption A1 eliminates all distortions in the flexible-price economy, which implies that a decentralized equilibrium is efficient, and a non-cooperative social planner who is subject to resource and budget constraints cannot improve upon it. Note, however, that this allocation is neither globally efficient (due to export markups) nor the first best (due to incomplete risk sharing).

Importantly, even when prices are sticky, this efficient allocation can still be implemented as long as firms use producer currency pricing.[8] Intuitively, because there is only one sticky price in each country, the monetary policy can simultaneously implement the optimal domestic and foreign demand. For example, the easing of the monetary stance in response to a positive productivity shock raises local demand, but also depreciates the nominal exchange rate, which makes exported goods cheaper in the currency of destination and increases foreign demand. This open-economy version of the "divine coincidence" underlies the famous argument of Friedman (1953) in favor of free floating exchange rates and provides an important benchmark for our analysis.

3 Non-U.S. Policy

This section describes the optimal policy in a representative non-U.S. economy. We start with the central result that the monetary authorities should target domestic prices, which implies a peg to the dollar and leads to a global monetary cycle. We then consider complementary policy instruments and discuss the redundancy of capital controls and the optimal trade tariffs.

3.1 Optimal policy

The planner's problem in a representative non-U.S. economy i is

$$\max_{\{C_{it}, N_{it}, L_{it}, X_{it}, B^h_{it}, P_{it}, P_{iit}, P^*_{it}, \mathcal{E}_{it}\}_t} \mathbb{E} \sum_{t=0}^{\infty} \beta^t U\big(C_{it}, N_{it}, \xi_{it}\big)$$
$$\text{s.t.} \qquad (3) - (10).$$

where all foreign variables are taken as given and independent from domestic policy. Solving this problem, we establish the following result (see Appendix A.2.2 for the proof):

[8] Despite the first-mover advantage of the U.S., the allocation is symmetric across countries in this case.

Proposition 1 (Non-U.S. policy) *The optimal monetary policy in a non-U.S. economy stabilizes prices of domestic producers:*

$$\pi_{iit} = 0. \tag{12}$$

The resulting allocation is generically not efficient.

Before discussing the intuition behind it, it is worth emphasizing first the key features of this result. First, the proposition shows that the optimal monetary policy — no matter how complicated the interest rate rule that implements it is ï£¡ can be summarized in terms of a simple policy target, namely, zero domestic inflation. This "sufficient statistic" does not depend on any parameters of the model, including the openness of the economy or the inter-/intra-temporal elasticities of substitution, and is potentially directly measurable in the data. Second, and closely related, it is precisely this simple form of the optimal policy that allows us to solve the planner's problem in a full non-linear stochastic environment without using the second-order approximations, which are usually employed in such cases. Third, while formally the policy-relevant price index P_{iit} corresponds to the PPI rather than to the CPI, the interpretation becomes more ambiguous once we take into account that most imported consumer goods can be considered as intermediate inputs of wholesalers and retailers. Indeed, more generally, the optimal policy stabilizes the average price of *all goods that are sticky in domestic currency* including the retail prices of foreign products (see Section 4 for details).

Finally, note that while the optimal policy target — the stabilization of domestic prices — is exactly the same as in a closed economy or in an open economy with PCP (Lemma 3), the intuition behind this result under DCP is fundamentally different. Indeed, in contrast to the former two cases, the monetary policy can no longer implement the efficient allocation when import and export prices are sticky in dollars and exchange rate devaluations have only a limited effect on the terms of trade. In other words, the "divine coincidence" does not hold and targeting inflation does not close the output gap. In fact, the monetary policy loses any stabilizing effect as the economy becomes fully open $\gamma \to 1$ and there are no prices left that are sticky in domestic currency. A standard second-best logic suggests in this case that the planner should allow for (distortionary) domestic inflation to alleviate the terms-of-trade distortion and diminish the output gap. Proposition 1 shows, however, that this is not optimal. To understand the intuition behind this surprising result, we start with two special cases considered in the previous literature and then contrast them with the general case.

Special case #1: Following the early literature on the optimal monetary policy under DCP (Corsetti and Pesenti 2007, Devereux, Shi, and Xu 2007, Goldberg and Tille 2009), consider the case with prices fully sticky in the currency of invoicing $\varphi \to \infty$. The key property of the planner's problem in this limiting case is that a country's terms of trade, exports, and imports are all independent from monetary policy. Indeed, because export prices P^*_{it} are constant in dollars, a depreciation of the local exchange rate does not change their value in the currency of destination and generates no expenditure switching towards exported goods. With both export prices and quantities exogenous to monetary policy, the trade balance implies that the dollar value of imports is also independent of the planner's policy.[9] But import prices P_t^* are constant in dollars as well, so the quantity imported — the value divided by price — is exogenous too. Notice this is true despite expenditure switching between domestic and foreign goods generated by exchange rate depreciation: in equilibrium, such policy only changes production and consumption of local goods, but leaves the volume of imports unchanged.

Being unable to affect foreign demand for home goods, the planner focuses exclusively on the domestic margin. As in a closed economy, the "divine coincidence" implies, in turn, that implementing the optimal level of domestic demand is equivalent to stabilizing the prices of local producers. Intuitively, the monetary policy takes producer price index P_{iit} that is sticky in local currency as a numeraire and adjusts all non-rigid prices to replicate the relative prices from the flexible-price equilibrium.

Special case #2: Consider next a setup from Casas, Díez, Gopinath, and Gourinchas (2017) with gradually adjusting prices, which in addition assumes complete asset markets, log-linear preferences $U = \log C - L$, $\theta = 1$, and no intermediate goods in production $F_X = 0$.[10] The key insight of the paper is that combined together, these assumptions ensure that a local monetary shock increases the nominal wage W_{it} and depreciates the nominal exchange rate $\mathcal{E}_{it}$ by the same amount, leaving the dollar value of marginal costs $\frac{MC_{it}}{\mathcal{E}_{it}} = \frac{W_{it}}{A_{it}\mathcal{E}_{it}}$ unchanged.[11] This, in turn, implies that dollar export prices and the terms of trade are exogenous from the planner's perspective and therefore, similarly to the case of fully sticky prices, the optimal policy focuses exclusively on domestic demand and targets local prices $\pi_{iit} = 0$.

General case: Though insightful, the two cases discussed above are clearly quite special — the monetary policy stabilizes domestic prices because it *cannot* affect the terms of trade. In general, away from these two knife-edge cases, the terms of trade are endogenous to monetary policy and yet, according to Proposition 1, the planner *finds it optimal* to target exclusively domestic prices. To see the intuition, consider again the case of PCP: a depreciation of the exchange rate — without any adjustment of export prices in the currency of invoicing — automatically makes exported goods cheaper in the currency of destination and generates expenditure switching towards them. This is no longer the case under DCP, as the local monetary policy cannot affect the exchange rate between the currency of invoicing (dollar) and the currency of destination. The only way to affect export prices for the planner is to change the dollar value of marginal costs and make firms adjust their prices, which is costly under sticky prices. Thus, there is a non-trivial trade-off facing the policymaker: in which direction should it distort domes-

[9]Because of the Cole and Obstfeld (1991) parametrization of these models with logarithmic preferences, no intermediates, no wealth or preference shocks, the risk sharing is independent from completeness of asset markets and country's trade is balanced in every state of the world.

[10]While they also assume a unit elasticity of substitution between goods from different countries and the Calvo price setting, the results remain the same in our setup with an optimal export tax and the Rotemberg pricing.

[11]Indeed, under log-linear preferences, the labor supply condition implies $W_{it} = P_{it}C_{it}$, while the complete risk-sharing translates into $\mathcal{E}_{it} = \frac{P_{it}C_{it}}{P_{0t}C_{0t}}$, where U.S. nominal spending $P_{0t}C_{0t}$ is exogenous to country i.

tic margin to improve the external one? For example, if there is a positive productivity shock, should it respond aggressively and lower export prices at the expense of higher price-adjustment costs or should it instead respond mildly to avoid these costs at the expense of more suboptimal terms of trade? It turns out that neither is optimal, and the policy should not sacrifice domestic margin at all.

More formally, one can strengthen Proposition 1 by showing that under the optimal monetary policy that targets $\pi_{iit} = 0$, the planner who is allowed to choose export prices π^*_{it} subject to the same adjustment costs as private firms, would set them at the same level as in a decentralized equilibrium. In other words, conditional on their marginal costs, exporters set prices in a socially optimal way. By stabilizing the marginal costs of local producers, the optimal monetary policy ensures the optimality of domestic prices, which combined with flexible nominal wages and dollar-invoiced import prices imply that the marginal costs of exporters are also optimal, and therefore, the economy achieves a *constrained efficient* allocation. The robustness of the intuition can be clearly seen from the extensions of the model. As shown in Appendix A.5, the same logic applies under Calvo friction despite no explicit costs of price adjustment in this case. The optimal price is trivially the same from private and social perspectives for firms that cannot change their prices in a given period, while all adjusting firms set constrained efficient prices conditional on production costs. Section 4.1 below extends the result to the setup with variable markups and heterogeneous firms.

Counterexample To see the limits of this result, consider a counterexample: a Calvo model with domestic varieties aggregated into a separate CES bundle with elasticity $\eta \neq \varepsilon$ before being exported to other countries. In this case, adjusting exporters set suboptimal prices from the social perspective as they do not internalize the *externality* on non-adjusting firms. Indeed, if all firms experience a positive productivity shock, demand for non-adjusting products is inefficiently low and one can stimulate it by lowering less the adjusting prices. Even when the planner cannot directly choose the prices of exporters, this demand-redistribution motive remains relevant and affects the optimal monetary policy, which therefore, deviates from stabilizing $\pi_{iit} = 0$ and violates Proposition 1.

While useful to build intuition, this counterexample is, however, unlikely to be important from a practical point of view because the externalities between exporters from a given country in global markets are small. Moreover, the recent empirical evidence shows that even in the context of a closed economy, the monetary policy has no significant effect on price dispersion within product categories and hence, can hardly redistribute demand between products with adjusting and non-adjusting prices (see Nakamura, Steinsson, Sun, and Villar 2018). Thus, quantitatively, Proposition 1 is likely to provide a good approximation to the optimal policy even when export prices are not fully constrained efficient.

Time consistency The optimal policy is time consistent, i.e. if the planner could revise its policy in the future, it would optimally choose not to do so. This result stems from assumptions A1-A2. For example, with some foreign debt denominated in local currency, the government has incentives to inflate it away. Similarly, there is a well-known inflationary bias when output is inefficiently low due to monopolistic markups. Finally, the price-level stabilization and inflation targeting are no longer equivalent in the presence of markup shocks, which break the "divine coincidence".

Corollary 1.1 (Time consistency) *The optimal non-U.S. monetary policy is time consistent.*

As emphasized in Lemma 2, the important implication of this result is that the global equilibrium is independent of whether we assume that non-U.S. economies choose their policies under commitment or under discretion.

3.2 Global monetary cycle

While the previous section shows that the optimal policy targets domestic inflation, this does not necessarily mean that the policy is purely "inward-looking" and does not respond to foreign shocks. Indeed, it seems intuitive that given the effect of U.S. policy on foreign trade, consumption, and output, it might be optimal for other countries to use their monetary policy to "lean against the wind" and counteract these negative spillovers. To check this hypothesis, we next discuss the *implementation* of the optimal policy, i.e. the dynamics of the nominal interest rates set by the central bank that support the equilibrium allocation.[12]

Although the generality of our baseline model makes it impossible to characterize the equilibrium allocation and interest rates in closed form, there are important robust predictions that can be inferred from a subset of equilibrium conditions. In particular, consider the marginal costs of domestic producers that are a function of nominal wages and prices of intermediates and are stabilized by the optimal policy:

$$MC_{it} = \frac{m(W_{it}, P_{it})}{A_{it}} = \frac{m(\frac{-U_{Nit}}{U_{Cit}/P_{it}}, P_{it})}{A_{it}} = \text{const},$$

where $m(\cdot)$ is an increasing function homogeneous of degree one and the second equality follows from the labor supply condition. The term U_{Cit}/P_{it} is part of the nominal stochastic discount factor and summarizes the response of future nominal interest rates to local and foreign shocks (see Appendix A.2.4 for details). It follows immediately from this expression that despite the fact that the optimal policy targets domestic inflation π_{iit}, it is *generically outward-looking*. In particular, there are two channels through which foreign shocks affect domestic monetary policy. First, consumer price index (4) increases in $\mathcal{E}_{it}$. Therefore, as long as imported goods are used as intermediates in production or have to go through the wholesale and retail sectors before reaching consumers and the prices of final goods are sticky in local currency, the stabilization of domestic prices requires that the policy partially offsets the fluctuations

[12] We focus in this section on the *equilibrium* behavior of interest rates. As in other New-Keynesian models, one needs to go beyond that and specify the policies *off equilibrium* to ensure this equilibrium is a *unique* possible outcome, which might also require the use of complementary monetary instruments such as money supply (see Atkeson, Chari, and Kehoe 2010).

in the nominal exchange rate and hence, reacts to foreign shocks. Second, movements in exchange rates affect country's exports and domestic production, changing the demand for labor and requiring the response of monetary policy to stabilize nominal wages. Both channels are, however, absent in the previous normative literature that focused on a knife-edge case with no intermediates $F_X = 0$ and infinite Frisch elasticity $U_N =$ const, resulting in a purely inward-looking policy (see Corsetti and Pesenti 2007, Goldberg and Tille 2009, Casas, Díez, Gopinath, and Gourinchas 2017).

While both channels are also at work under PCP, the response of monetary policy is much more asymmetric under DCP. When international prices are sticky in dollars, it is the U.S. exchange rate that changes the prices of imported intermediates and the foreign demand for exported goods and through both channels described above makes other economies "import" the monetary stance of the U.S. in spite of the U.S.'s trivial share in global production, consumption and trade. It follows that even if exogenous shocks are completely uncorrelated across countries, the synchronized response of individual economies to U.S. shocks gives rise to a *"Global Monetary Cycle"*. The country-specific loadings on this global factor, in turn, increase in the share of DCP in imports and exports and the openness of the economy. Importantly, this normative prediction of the model is strongly supported by the recent empirical evidence by Zhang (2018), which shows that the pass-through of U.S. monetary shocks into foreign policy rates is systematically related to the cross-country variation in DCP. This logic, however, applies not only to the Fed's shocks, but also more generally to any U.S. and local shocks that move the bilateral exchange rate.

What are the implications of the optimal policy under DCP for the exchange rate regime? Is it more or less volatile than under PCP? While in general the answer is ambiguous, two assumptions allow us to make much progress. First, assume a linear disutility from labor (see Hansen 1985, Rogerson 1988), so that the import channel dominates the export one, i.e. monetary response to foreign shocks is only due to the presence of imported intermediates.[13] Second, assume that other things equal, the exchange rate appreciates in response to a positive interest rate shock, as is the case under complete markets in our model, but is not necessarily true more generally because of the valuation effects.[14]

Combined together, these assumptions imply that it is optimal for non-U.S. economies to have *a partial peg to the dollar*. Indeed, given an exogenous shock that depreciates the domestic currency relative to the dollar, the local monetary policy raises interest rates, which partially offsets the depreciation of the local currency and smooths movements in the bilateral exchange rate against the dollar. This mechanism is consistent with the fact that central banks often mention volatile import prices as a rationale for pegging exchange rates and suggests that the wide use of the dollar in international trade contributes to the fact that the U.S. retains its dominant position in the global monetary system even in the aftermath of the Bretton Woods System. In particular, the model can rationalize the fact that

[13]The latter would also be true if flexible-price commodities accounted for most of country's exports.

[14]While this assumption is in line with most empirical evidence (see e.g. Eichenbaum and Evans 1995), there can be important exceptions, e.g. the effect of U.S. forward guidance during the Great Recession (Stavrakeva and Tang 2019).

most countries in the world experience a "fear of floating" and use the dollar as an anchor currency in their monetary policy (Calvo and Reinhart 2002). It also helps to explain why emerging economies like Argentina, Brazil, and Turkey, with a high ratio of imports-to-GDP and almost all imported goods invoiced in dollars, are more sensitive to U.S. shocks and have a stronger peg to the dollar (Ilzetzki, Reinhart, and Rogoff 2018).

Corollary 1.2 (Global monetary cycle) *The optimal monetary policy in non-U.S. economies (i) is generically outward-looking, (ii) responds to U.S. monetary shocks giving rise to a global monetary cycle, and (iii) if* $U_N = const$ *and* $\frac{\partial \mathcal{E}_{it}}{\partial R_{it}} < 0$*, implements a partial peg to the dollar.*

Thus, the implications of DCP are similar and highly complementary to the international *financial* spillovers of U.S. monetary policy, which are the focus of a growing "Global Financial Cycle" literature (see e.g. Rey 2013, Aoki, Benigno, and Kiyotaki 2016, Giovanni, Kalemli-Ozcan, Ulu, and Baskaya 2017). Just as in our model, the trade-off facing the policymakers in non-U.S. economies is worsened by U.S. spillovers: the free floating exchange rate does not fully insulate from negative foreign shocks and does not allow countries to achieve the efficient allocation, transforming the Trilemma into a "Dilemma". At the same time, both in our setup and in the case of financial spillovers, this does not mean that the planner should give up on exchange rates altogether, as they still constitute an important margin of the stabilization mechanism: in general, it might be optimal to have more volatile exchange rates relative to the PCP case if the export channel dominates (*cf.* Gourinchas 2018, Kalemli-Ozcan 2019), and even when the import channel is more important, it is still optimal to fully adjust exchange rates in response to local productivity shocks. In contrast to the literature on the global financial cycle, however, our results are *not* driven by frictions in the international asset markets — as they remain true even when the markets are complete — and instead are solely due to the dominance of the dollar as the currency of invoicing in global trade.

To take stock, we show that the optimal policy actively responds to U.S. shocks and partially pegs the exchange rate to the dollar. These results hold both conditional on U.S. monetary shocks, but also unconditionally, i.e. for most local and U.S. shocks. The described optimal policy under DCP is much closer to the one observed in the data, rather than the normative predictions of standard open-economy models.

3.3 Capital controls

The inability of exchange rates to shield economies from negative U.S. spillovers under DCP and the failure of the Trilemma raise a question about whether additional restrictions on capital mobility can be used to improve the allocation and increase a countries' welfare. Indeed, a modern consensus among both policymakers and scholars is that "[the use of capital controls by emerging economies] allows advanced economies to use monetary policy to increase domestic demand, while shielding emerging economies of the undesirable exchange rate effects" (Blanchard 2017). In other words, the U.S. can focus on its domestic objectives when setting monetary policy, while other countries can insulate their economies from the arising spillovers by augmenting the optimal monetary policy with appropriate capital controls. Though often made in a context of international financial spillovers, this argument might be equally important for the spillovers from DCP in global trade.

To address this question, we allow the planner to choose a state-specific subsidy τ^c_{it} on foreign investment, so that household portfolio choice is described by the no-arbitrage conditions[15]

$$\mathbb{E}_t \Theta_{it,t+1} \frac{1-\tau^c_{it}}{1-\tau^c_{it+1}} \frac{\mathcal{E}_{it+1}}{\mathcal{E}_{it}} \frac{\mathcal{Q}^h_{t+1}+D^h_{t+1}}{\mathcal{Q}^h_t} = 1.$$

Following the primal approach, we can exclude these equilibrium conditions from the planner's problem and allow the government to choose directly the international asset positions $\{B^h_{it}\}$ of the country, in addition to the monetary policy:

$$\max_{\{C_{it},N_{it},L_{it},X_{it},B^h_{it},P_{it},P_{iit},P^*_{it},\mathcal{E}_{it}\}_t} \quad \mathbb{E}\sum_{t=0}^{\infty}\beta^t U\big(C_{it},N_{it},\xi_{it}\big)$$
$$\text{s.t.} \qquad (4)-(10).$$

Solving for the optimal allocation and then using household no-arbitrage conditions to back out the optimal taxes on capital flows, we obtain the following result:

Proposition 2 (Capital controls) *Given the optimal monetary policy, capital controls do not insulate other economies from U.S. spillovers and are not used by the planner, i.e.* $\tau^c_{it}=0$ *under the optimal policy.*

Thus, relaxing the planner's problem by augmenting the monetary instrument with the macroprudential tools does not change the optimal policy or the equilibrium allocation in our model as the planner optimally chooses not to intervene into asset markets.[16] At first glance, this may look like a surprising result: after all, the general principle of the second-best policy is that it is usually optimal to mitigate distortions in one market by distorting other margins in the economy. In our model, this would correspond to distorting the intertemporal portfolio decisions of households to improve the allocation in static goods markets. Indeed, as shown by Farhi and Werning (2016), the laissez-faire risk sharing is *generically* inefficient when monetary policy cannot implement the first-best allocation. Yet, Proposition 2 shows that despite suboptimal allocation under the optimal monetary policy, capital controls are redundant and are not used by the planner in our setting independently from the completeness of asset markets. The international spillovers arising from DCP are therefore very different from the ones arising from financial frictions and cannot be eliminated with macroprudential policy.

[15]Without loss of generality, it is convenient to normalize the value of capital controls in period 0 to $\frac{U_{Ci0}}{P_{i0}}\frac{\mathcal{E}_{i0}}{1-\tau^c_{i0}}=1$.

[16]In contrast to the "approximate efficiency" of the risk sharing in Fanelli (2017), the laissez-faire portfolio choice is *exactly* optimal in our setting.

To understand the intuition, we map our result into the general setup of Farhi and Werning (2016). For the sake of clarity, assume no intermediate goods in production and complete asset markets. There are three types of goods produced in each country — local consumption goods, exported goods, and labor. Adopting notation from the paper, let $C_{iit}(I_{it}, \{P_t\})$ and $C^*_{it}(I_{it}, \{P_t\})$ denote household demand for local and exported goods as functions of income I_{it} and a vector of prices $\{P_t\}$, and define wedges for two goods against labor as

$$1 - \bar{\tau}_{iit} \equiv \frac{W_{it}}{P_{iit}} \frac{1}{A_{it}}, \qquad 1 - \bar{\tau}^*_{it} \equiv \frac{\varepsilon}{\varepsilon - 1} \frac{W_{it}}{\mathcal{E}_{it} P^*_{it}} \frac{1}{A_{it}}.$$

Clearly, $\bar{\tau}_{iit} = \bar{\tau}^*_{it} = 0$ at the first-best allocation, while positive (negative) values of wedges correspond to a deficit (excess) of demand for the respective good. Note that τ^c_{it} can be interpreted as a subsidy on the Arrow-Debreu security that pays one unit conditional on a given history and zero otherwise. Its optimal value is then shown to be

$$\tau^c_{it} = P_{iit} C_{I,iit} \bar{\tau}_{iit} + \mathcal{E}_{it} P^*_{it} C^*_{I,it} \bar{\tau}^*_{it},$$

where $C_I \equiv \frac{\partial C}{\partial I}$ is the marginal propensity to consume for a given product. Intuitively, the reason the risk sharing is generically inefficient under sticky prices is the aggregate demand externality: when making portfolio decisions, individual agents do not internalize the fact that a wealth transfer changes the aggregate demand C_I for depressed/overheated goods and mitigates wedges $\bar{\tau}$ in goods markets. Therefore, the planner subsidizes the purchase of assets $\tau^c_{it} > 0$ that pay in the states of the world with inefficiently low demand.

In our setting, however, the optimal monetary policy ensures that local demand is optimal and fully eliminates the wedge for domestic goods by stabilizing the marginal costs of local producers, i.e. $\bar{\tau}_{iit} = 0$. In contrast, because of DCP, foreign demand is in general inefficient under the optimal policy, so that the wedge for export goods remains open $\bar{\tau}^*_{it} \neq 0$, and the equilibrium allocation is not the first best. This inefficiency, however, cannot be eliminated with domestic capital controls, which in the case of a small open economy have no effect on foreign demand and can only change local demand. But domestic households do not consume export goods $C^*_{I,it} = 0$ and hence, a redistribution of wealth across states has no effect on the export output gap. Crucially, a zero demand for export goods is not a knife-edge case that is fragile to an arbitrary small perturbation of preferences, but is rather true *by definition*, which explains why the main theorem from Farhi and Werning (2016) that taxes τ^c_{it} are generically non-zero does not apply in our model. Thus, even though the optimal monetary policy cannot implement the first-best allocation under dollar pricing — similarly to the settings with the zero lower bound or fixed nominal exchange rates (see e.g. Farhi and Werning 2017) — it does eliminate the local aggregate demand externality and closes the gap between private and social value of insurance.

To be clear, our result should not be interpreted as an argument against the macroprudential policy, which might be important to offset financial spillovers and local pecuniary externalities (see e.g. Bianchi 2011, Jeanne and Korinek 2010). Instead, it shows that capital controls are not a panacea and cannot be used unilaterally to insulate a country from spillovers arising under DCP. We also show in Section 5.2 below that it is optimal to use macroprudential instruments under the *cooperative* solution.

3.4 Trade policy

Given the limited efficiency of monetary policy and capital controls, a natural question is: what other tools can be used to restore the first-best allocation? Indeed, as argued in the recent literature, a sufficiently rich set of fiscal instruments can always replicate the effects of a given monetary policy (Adao, Correia, and Teles 2009) and implement the optimal allocation (Correia, Nicolini, and Teles 2008). Most closely related to our setting, Chen, Devereux, Xu, and Shi (2018) show how state-contingent taxes can be used to restore efficiency in global economy when international prices are sticky in the currency of destination (LCP). To address the question in the context of DCP, we augment the planner with two additional *state-contingent* fiscal instruments: a production subsidy to exporters τ^*_{it} and an export tax levied *on top* of firms' prices at the dock τ^E_{it}. While the two instruments would be largely isomorphic under flexible prices, their effects are quite different in the presence of nominal rigidities: e.g. in the limit of fully sticky prices, an unexpected change in τ^*_{it} has no effect on export prices in the currency of destination and only changes the profits of exporters, while τ^E_{it} allows the planner to have direct control over export prices and affects foreign demand.

Proposition 3 (Trade policy) *The non-cooperative efficient allocation can be implemented by (i) monetary policy stabilizing domestic prices $\pi_{iit} = 0$, (ii) production subsidy to exporters τ^*_{it} stabilizing their dollar prices $\pi^*_{it} = 0$, and (iii) export tax stabilizing destination prices in domestic currency $\tau^E_{it}\mathcal{E}_{it} = 1$.*

Thus, a mix of monetary policy with two fiscal instruments is sufficient to replicate the flexible price equilibrium, which according to Lemma 3, is efficient. Intuitively, this is because sticky prices generate three types of distortions. The first one comes from the suboptimal production and consumption of domestic goods and, as explained above, is fully eliminated by the monetary policy. The second distortion is due to the price-adjustment costs of exporters. The only way to avoid these costs for the economy is to ensure that the marginal costs of exporters are constant in dollars and hence, there is no need for firms to change their prices. This goal can be achieved by using a time-varying production subsidy to exporters τ^*_{it}. Having export prices constant in dollars, however, implies suboptimal terms of trade. To eliminate this last distortion, the planner uses export tax τ^E_{it} to target the prices faced by consumers in foreign markets of destination. In particular, since domestic prices are stable in local currency and export prices are constant in dollars, this last instrument is necessary to eliminate deviations from the law of one price.

While it is hardly surprising that the efficient allocation can be restored using a sufficient number of fiscal instruments, it is interesting that the optimal policy is actually quite simple and can be characterized in terms of three directly observable targets: domestic prices and two measures of export prices. No other details of the models, including the values of any parameters, are relevant for the policymaker given these three sufficient statistics, although the resulting allocation and the particular values of taxes and interest rates implementing this allocation are sensitive to the details of the model. Thus, similarly to the fiscal devaluations in Farhi, Gopinath, and Itskhoki (2014), our policies are also "robust", but in terms of objectives rather than implementation.

Clearly, this is just one set of policies that are sufficient to implement the optimal allocation: as usual, there exist alternative instruments that can achieve the same goal. For example, one can use a production subsidy to local firms τ_{it} to stabilize their marginal costs and allow the monetary policy to target instead other distortions. That said, the export tax τ_{it}^{E} is crucial for the implementation of the efficient allocation and cannot be substituted with other instruments. In particular, the Lerner symmetry does not hold under DCP, as pointed out by Barbiero, Farhi, Gopinath, and Itskhoki (2019), and hence, one cannot replace the export tax with import tariffs.

We conclude this section by emphasizing that in spite of the benefits and relative simplicity of the optimal fiscal policy, it is still uncommon in the modern world to have state-dependent production subsidies and export taxes. The relative flexibility of monetary policy, on the other hand, makes it a primary instrument to be used against exogenous shocks. Interestingly, our results show that the optimal monetary policy is the same regardless if it is the only policy tool or if it is complemented with export taxes and subsidies.

4 Extensions

To clarify the generality and limitations of the main results, this section extends the baseline model in three important dimensions. Motivated by empirical evidence, we first allow for pricing-to-market and heterogeneity between domestic and exporting firms and show robustness of the optimal policy. The second extension introduces *endogenous* currency choice and proves that our normative results withstand the Lucas (1976) critique. Finally, we consider the case when both exporters and some domestic firms set their prices in dollars and show how the arising aggregate demand externality affects the optimal policy.

4.1 Robustness

To keep equilibrium conditions more transparent and directly comparable to the previous literature, the baseline model assumes that all firms and consumers in an economy share the same simple CES aggregator of home and foreign products. This section relaxes this assumption to show the robustness of our main findings in an arguably more realistic environment and to clarify whether the policy should stabilize the costs of local firms or exporters.

In particular, we generalize the baseline model in three directions. First, the home bias is allowed to be different for consumers, local producers and exporters:

$$C_{it} = \left[(1-\gamma_c)^{\frac{1}{\theta}} C_{iit}^{\frac{\theta-1}{\theta}} + \gamma_c^{\frac{1}{\theta}} C_{it}^{*\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},$$
$$X_{it}^d = \left[(1-\gamma_d)^{\frac{1}{\theta}} X_{iit}^{d\frac{\theta-1}{\theta}} + \gamma_d^{\frac{1}{\theta}} X_{it}^{d*\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},$$
$$X_{it}^e = \left[(1-\gamma_e)^{\frac{1}{\theta}} X_{iit}^{e\frac{\theta-1}{\theta}} + \gamma_e^{\frac{1}{\theta}} X_{it}^{e*\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},$$

where X_{it}^d and X_{it}^e are, respectively, the bundles of intermediates for domestic firms and exporters. Heterogeneous γ's allow the model to capture, among other things, the fact that exporting firms are also the largest importers and that consumers might not have direct access to foreign goods, but rather have to buy them from local retailers.

Second, we allow for heterogeneous production functions used by local firms and exporters

$$Y_{iit} = A_{iit} F(L_{iit}, X_{it}^d) \quad \text{and} \quad Y_{it}^* = A_{it}^* G(L_{it}^*, X_{it}^e),$$

where $F(\cdot)$ and $G(\cdot)$ are both constant returns to scale. Thus, the labor intensity and productivity shocks might differ across two types of firms.

Finally, instead of the CES bundle, we assume that both local and foreign varieties are combined via the Kimball (1995) aggregator, e.g. consumption bundles C_{iit} and C_{it}^* are defined by

$$\int \Upsilon\left(\frac{C_{iit}(\omega)}{C_{iit}}\right) \mathrm{d}\omega = 1, \quad \text{and} \quad \int\int \Upsilon\left(\frac{C_{jit}(\omega)}{C_{it}^*}\right) \mathrm{d}\omega \mathrm{d}j = 1,$$

where $\Upsilon(1) = \Upsilon'(1) = 1$, $\Upsilon'(\cdot) > 0$ and $\Upsilon''(\cdot) < 0$, and the bundles of intermediates X_{iit} and X_{it}^* are defined symmetrically. The key difference from the baseline model introduced by the Kimball aggregator is that the prices of individual firms now depend not only on their marginal costs, but also on the prices of competitors. This, in turn, implies that the model can reproduce the pricing-to-market and the incomplete pass-through that have been extensively documented in the data even for the long horizons when the prices adjust fully (for a survey of the literature see Burstein and Gopinath 2012).

The extended version of the model is significantly more complex than the baseline setup, yet our main results still hold. Intuitively, assumptions A1-A2 still ensure that the flexible-price equilibrium is efficient and that exporters set prices in a constrained efficient way when prices are sticky.[17] The latter result is due to the fact that individual producers and the planner face the same foreign demand for each exported variety, and there are no externalities not internalized by exporters. Therefore, as in the baseline model, the planner targets $\pi_{iit} = 0$, giving rise to a global monetary cycle, and finds it

[17] More generally, one needs firm-specific subsidies to eliminate heterogeneous markups when firms are not symmetric.

suboptimal to use capital controls.[18]

Lemma 4 (Robustness) *All results from Propositions 1-2 hold in this general setup.*

Importantly, the lemma shows not only the robustness of our results, but also clarifies the key feature of the optimal policy: it is the marginal costs of *local* sellers that the monetary authorities should stabilize. On the one hand, that might include the prices of most imported goods if they have to go through the wholesale and retail sectors before reaching consumers. On the other hand, this target does not include foreign intermediates used by local exporters and does not directly depend on the productivity shocks in the export sector. Thus, the important corollary of Lemma 4 is that the optimal policy can vary across countries not only because of different openness and the share of DCP (as discussed in Section 3.2), but also because of differences in input-output linkages. In particular, economies with a higher share of consumption goods in imports sold by local retailers respond more strongly to U.S. shocks and have a tighter peg to the dollar than countries in which exporters account for most of imported goods.

It is worth emphasizing that the important assumption that underlies Lemma 4 and, in fact, our next results about the currency choice is that the Kimball aggregator applies *separately* to local and foreign goods. If the two were combined into a single nest, exporters would set different prices in each market of destination depending on the competitors' price index P_{it}. In this case, import prices depend on prices of domestic firms and are no longer exogenous from the planner's perspective. The monetary policy and other tools would then deviate from price targeting to manipulate import prices. To the best of our knowledge, this trade-off has not yet been studied in the literature, and we leave it for future research. At the same time, it remains an open empirical question as to what level of aggregation provides a better approximation to the real world: one of very few existing pieces of evidence by Cavallo, Neiman, and Rigobon (2014, 2015) actually shows that the law of one price holds quite well across markets with the same currency, which supports our approach.

4.2 Currency choice

So far, we have treated the currency of invoicing as a primitive of the model following most of the previous normative literature. One might, however, be concerned that our results are subject to the Lucas critique and might change drastically once the firms are allowed to make *optimally* the invoicing decisions. In what follows, we argue that this is not the case and that both DCP and the optimal policy described above can be sustained as an equilibrium outcome. To this end, we augment the generalized version of the model with the endogenous currency choice from Mukhin (2018), which in turn, builds on the seminal contributions of Engel (2006) and Gopinath, Itskhoki, and Rigobon (2010).[19]

[18] At the same time, in contrast to the baseline model, the efficient allocation cannot be implemented under PCP as the optimal pass-through into export prices is no longer complete. As a result, although the planner can still implement the efficient allocation using the trade instruments from Proposition 3, the optimal target for export tax τ_{it}^{E} is more involved.

[19] See also closely related models by Corsetti and Pesenti (2002), Bacchetta and van Wincoop (2005), Goldberg and Tille (2008), Cravino (2014). The two appealing features of these models are that they do not require any additional frictions on

Assume in particular, that in the initial period, domestic firms and exporters can choose any currency, in which to set prices. The currency choice is discrete, i.e. firms cannot set their prices in terms of a basket of currencies (see Appendix A.3.2 for details). Because of nominal rigidities, future prices deviate from the desired level that maximizes firms' profits state by state. In this environment, the currency choice aims to minimize such deviations and replicate the desired prices as close as possible. To give a simple example, if the desired price is $\$100$ in all states of the world, setting the price in dollars is clearly optimal as no ex-post adjustments are required in this case, while setting the price in pounds is suboptimal as the ex-post price would deviate from the optimal level due to movements in exchange rates. More generally, a firm chooses the currency, in which its desired price is most stable: e.g. if the latter is $\$70 + £30$, it is still optimal to set the price in dollars.

The "weights" of currencies in the desired price, in turn, depend on the marginal costs of production and the prices of competitors. As before, the former includes wages and the prices of local and foreign intermediates, while the latter is due to the Kimball demand, which in contrast to the CES demand, generates strategic complementarities in price setting. Given that the currency choice depends on the properties of nominal wages and exchange rates, there is a two-way interaction between the monetary policy and firms' invoicing decisions.

Proposition 4 (Endogenous currency choice) *Assume strong enough price linkages across exporters. Then (i) under the optimal policy there is an equilibrium with PCP in local markets and DCP in foreign markets, (ii) the optimal policy that internalizes its effect on firms' currency choice is the same as in Lemma 4.*

The first result speaks to the case when invoicing is taken as given by the monetary authorities and we are looking for a Nash equilibrium in a game where planner and firms move simultaneously. Since the optimal monetary policy fully stabilizes domestic prices in local currency $\pi_{iit} = 0$, firms unambiguously choose PCP in the home market. In contrast, exporters face a non-trivial trade-off: the wages and prices of local intermediates are more stable in producer currency, while the prices of imported inputs and competing products are more stable in foreign currencies, in which exporters from other countries set their prices. When the price linkages between exporters, which arise from the use of foreign intermediates and complementarities in price setting, are strong enough, the latter effect dominates, and it is optimal for exporters to coordinate on a single currency, allowing to sustain a DCP equilibrium.[20]

Perhaps more surprisingly, Proposition 4 shows that the optimal policy does not change even when the planner acts as a Stackelberg leader and commits to monetary policy before firms make their invoicing decisions. On the one hand, PCP in local markets increases the efficiency of the monetary policy to stabilize the economy, so the planner has no incentives to distort this margin. On the other hand, exporters' invoicing decisions are constrained efficient, i.e. even if the planner were allowed to choose

top of sticky prices and are supported by empirical studies.

[20] As usual, strong strategic complementarities can potentially lead to multiple equilibria with different currencies used as a dominant one. The dollar is more likely to play this role when the U.S. is larger and more stable than other economies.

both monetary policy and the currency of invoicing, it would not change it relative to the decentralized equilibrium. This result strengthens the intuition behind Proposition 1: conditional on monetary policy that stabilizes domestic prices, both prices and currency choice of exporters are socially optimal, which — combined with the fact that targeting π_{iit} is the optimal response of monetary policy under DCP — implies that the planner has no incentives to deviate from this equilibrium.

Finally, the currency choice of importers is almost exogenous to the planner as exporters from other countries choose the same currency for all foreign markets. The only way, a planner can affect the invoicing decisions of importers is by manipulating the properties of its exchange rate, e.g. by stabilizing the average desired prices of foreign firms in its currency. This motive of monetary policy is, however, weak for two reasons. First, it might prove to be too costly as it requires giving up on stabilizing domestic demand. Second, if complementarities in currency choice are strong enough, it might be impossible to make foreign firms switch to a new currency once they coordinate on DCP.[21]

4.3 Domestic price dollarization

In addition to its dominant status in international trade, the dollar is also frequently used in emerging economies as a currency of invoicing in *domestic* markets (see e.g. Drenik and Perez 2019). Such "dollarization" can potentially significantly lower effectiveness of local monetary policy and result in additional policy trade-offs. To address these issues, this section extends our baseline model allowing for DCP in domestic markets and describes the optimal monetary and fiscal policy.

To this end, consider a non-U.S. economy and assume that consumption bundle C_{it} as well as the bundle of intermediate goods X_{it} include three types of goods — domestic products invoiced in local currency C_{iit}, domestic products invoiced in dollars C^*_{iit}, and foreign goods C^*_{it}:

$$C_{it} = \left[(1-\gamma^*-\gamma)^{\frac{1}{\theta}} C_{iit}^{\frac{\theta-1}{\theta}} + \gamma^{*\frac{1}{\theta}} C_{iit}^{*\,\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} C_{it}^{*\,\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

and similarly for X_{it}, where as before, each category is a CES aggregator of individual products with elasticity of substitution ε.[22] For simplicity, all firms have the same round-about production function independently from invoicing or market of destination, and the currency choice is exogenous. We allow producers to set different dollar prices in local and foreign markets. Extending Assumption A1, all firms selling in domestic markets get a time-invariant production subsidy $\tau_i = \frac{\varepsilon-1}{\varepsilon}$ that eliminates monopolistic distortion.[23] The planner is free to choose a state-dependent monetary policy R_{it}, capi-

[21] A similar argument applies to the optimal cooperative policy: the costs of promoting PCP with monetary policy can outweigh the benefits from enhanced expenditure switching (see Mukhin 2018 for details).

[22] The fact that dollarized products enter consumption as a separate bundle with elasticity of substitution θ with other local and imported goods means that most variation in currency choice is across sectors. This assumption simplifies the notation, but is not crucial for our results.

[23] We also allow for a time-invariant tax on dollar-invoicing firms that subsidizes their price-adjustment costs $\tau_{ii}^{*R} = \frac{\varepsilon-1}{\theta}$. Intuitively, it eliminates both markups and externalities in price-adjustment decisions of private firms (see Appendix A.3.3).

tal controls τ_{it}^c (as defined in Section 3.3), and production subsidies to exporters τ_{it}^*, but falls short of using export taxes τ_{it}^E to restore efficient terms of trade. We allow for this rich set of instruments to disentangle different policy motives with the understanding that once some of the tools (e.g. fiscal) are excluded, the remaining ones (monetary) pick up their functions. Appendix A.3.3 provides further details about the equilibrium conditions and the planner's problem.

Proposition 5 (Domestic dollarization) *The optimal policy in dollarized economies*

- *stabilizes domestic prices* $\pi_{iit} = 0$,
- *taxes capital flows*

$$\frac{\tau_{it}^c}{1-\tau_{it}^c} = \frac{\gamma^*}{\gamma}\left(\frac{P_{iit}^*}{P_t^*}\right)^{-\theta}\frac{\mathcal{E}_{it}P_{iit}^* - P_{iit}}{\mathcal{E}_{it}P_t^*},$$

- *uses production subsidies to exporters to implement*

$$\pi_{it}^*\left(\pi_{it}^*+1\right)W_{it} = -\kappa\left(\frac{\mathcal{E}_{it}P_{it}^*}{1-\tau_{it}^c} - \frac{\varepsilon}{\varepsilon-1}MC_{it}\right)\frac{Y_{it}^*}{\gamma} + \mathbb{E}_t\Theta_{it,t+1}\pi_{it+1}^*\left(\pi_{it+1}^*+1\right)W_{it+1}.$$

The first thing to notice is that given additional instruments, the optimal monetary stance remains exactly the same as in the baseline model stabilizing prices of domestic producers. Thus, the monetary policy is used exclusively to achieve the optimal level of aggregate demand for domestic goods invoiced in local currency and does not aim to improve demand for dollar-invoiced goods, which is generically inefficient due to deviations from the law of one price. Combined together, Propositions 1 and 5 show that whether a product is included in the price index targeted by the monetary policy depends more on its *currency* of invoicing rather than its *country* of origin: as discussed above, π_{iit} may include retail prices of imported goods, but excludes domestic products with prices set in dollars. Moreover, the important implication of this result is that a high level of dollarization should not be interpreted as a prima facie evidence in favor of pegging the exchange rate to the dollar.

In contrast to the baseline case, it is no longer optimal to have zero capital controls when some local prices are sticky in dollars. The latter implies that monetary policy can no longer achieve efficient demand for all domestically produced goods, leaving room for macroprudential tools to further improve the allocation. In line with the analysis of Farhi and Werning (2017), the planner internalizes the effect of international transfers on local demand and, given the optimal monetary policy that closes the output gap for goods invoiced in local currency, aims to stabilize demand for dollar-invoiced goods. Indeed, as can be seen from the formula above, the optimal subsidy τ_{it}^c is positive for assets that pay more in states of the world, in which prices of dollarized products are inefficiently high $\mathcal{E}_{it}P_{iit}^* > P_{iit}$. The absolute size of the optimal capital controls, in turn, depends on the share of additional income spent on local DCP goods relative to the fraction spent on imported goods. Consistent with the results above, no capital controls are needed when the share of dollarized goods converges to zero $\gamma^* \to 0$ or when local demand is fully efficient $\mathcal{E}_{it}P_{iit}^* = P_{iit}$.

Perhaps more novel to the literature, we find that the same argument applies to export revenues. Because individual exporters do not internalize the increase in demand for local goods associated with higher income earned abroad, their price setting is not constrained efficient. Remarkably, the comparison of the optimal NKPC from the proposition with the private one (7) shows that the externality can be fixed by applying the same state-dependent tax on foreign revenues as the one imposed on capital flows. In particular, the planner subsidizes exporters $\tau^c_{it} > 0$ to boost their revenues in states of the world with insufficient demand for dollarized goods $\mathcal{E}_{it} P^*_{iit} > P_{iit}$. This shows that the aggregate demand externality is the common source of inefficiencies in risk sharing and price setting. This extends the main insight of Farhi and Werning (2016) and shows that — in addition to private portfolio decisions — the price setting of exporters is also generically constrained inefficient when monetary policy cannot achieve the optimal local demand.

5 U.S. Policy

The previous sections describe the optimal policy in non-U.S. economies for an arbitrary monetary policy of the U.S. To characterize the subgame-perfect equilibrium, we next apply the backward induction and use the best responses of other countries to solve the U.S. planner's problem.[24] We characterize the motives of U.S. optimal policy, contrast them with the optimal cooperative policy, and discuss the welfare implications.

5.1 Optimal policy

As a Stackelberg leader, the U.S. planner maximizes the welfare over all prices and quantities in the world economy, taking as given the optimal policy in other countries:

$$\max_{\{C_{jt}, N_{jt}, L_{jt}, X_{jt}, B^h_{jt}, P_{jt}, P_{jjt}, P^*_{jt}, \mathcal{E}_{jt}, \mathcal{Q}^h_t\}_{j,t}} \quad \mathbb{E} \sum_{t=0}^{\infty} \beta^t U\big(C_{it}, N_{it}, \xi_{it}\big)$$
$$\text{s.t.} \qquad (3) - (12).$$

Here and for the rest of this section we implicitly assume $i = 0$. This problem is fundamentally different from the problem of other economies discussed in Section 3. Non-U.S. countries take global prices and quantities as given and their monetary policy can only affect local margins. In contrast, because of the dollar invoicing in international trade, U.S. monetary policy can affect the relative prices outside of its economy and can potentially implement allocations that are unattainable under flexible prices. This can perhaps be seen best in the extreme case with no home bias $\gamma \to 1$ when all prices in the global

[24]Notice, however, that none of our results below are driven by the timing assumption as the same equilibrium would arise in a simultaneous-move game with all countries choosing π_{iit} (Lemma 2). Indeed, in both sequential and simultaneous games, the U.S. takes the strategies of other economies $\pi_{iit} = 0$ as given and hence, chooses the same optimal policy.

economy are sticky in dollars and hence, non-U.S. monetary policy becomes completely ineffective, while the U.S. policy determines allocations in all countries.

Because of these global effects, the optimal U.S. policy deviates from inflation targeting and pursues other objectives as well. As a result, it cannot be summarized with a simple targeting rule and depends on the details of the setup. To make progress in understanding the motives of U.S. policy, we adopt a different approach than in Section 3 and impose an additional restriction:

A3 *Domestic and export prices are fully rigid $\varphi \to \infty$, and the asset markets are complete.*

While in the general case, the optimal policy is implicitly determined by a large system of dynamic equations, assumption A3 provides a lot of tractability by allowing for a state-by-state analysis. It is worth emphasizing, however, that the same motives that arise in this "static" case are equally applicable in the general dynamic setup. Combining the optimality conditions and taking the first-order approximation around the symmetric steady state, one can decompose the optimal policy in a given state into three separate motives.[25] Intuitively, the U.S. problem can be partitioned into three blocks — the static market clearing conditions, the intertemporal risk-sharing, and the ex-ante price-setting — each of which generate a separate motive for the optimal policy. The next proposition expresses the policy rule in terms of simple sufficient statistics, which even if not directly observable in the data, provide important intuition for the motives of U.S. policy.

Proposition 6 (U.S. policy) *Under assumptions A1-A3, the optimal U.S. policy rule balances three motives and to the first-order of approximation is given by*

$$\Gamma \cdot p_{iit} + \gamma\epsilon \cdot nx_{it} + \gamma\Xi \cdot \int p^*_{jt} \mathrm{d}j = 0, \tag{13}$$

where p_{iit}, p^*_{it} and nx_{it} are the deviation from the steady-state values of $\frac{P_{iit}}{MC_{it}}$, $\frac{\mathcal{E}_{it}P^*_{it}}{MC_{it}}$ and net exports, ϵ is the steady-state elasticity of global SDF with respect to U.S. monetary policy, and Γ, Ξ are steady-state constants — all defined in Appendix A.4.2.

Motive #1: The first term in (13) corresponds to local demand and constitutes the *price-stabilization motive* of the monetary policy. Notice that this motive dominates when home bias is strong $\gamma \approx 0$ and all other terms drop out. While it is the same motive pursued by non-U.S. economies, its implications are largely different in the U.S. than in other countries. First, in contrast to other economies, in which inflation targeting stabilizes only local demand, but leaves exports at the suboptimal level, the monetary policy of the U.S. simultaneously affects domestic and foreign demand for its products.

[25] An alternative and widely used way to characterize the motives of the optimal policy is to focus on the terms in a quadratic approximation of the objective function (see e.g. Engel 2011, Corsetti, Dedola, and Leduc 2010). At the same time, the presence of a "gap" in the loss function does not necessarily imply that the optimal policy should target it (see Casas, Díez, Gopinath, and Gourinchas 2017 for a counterexample).

Indeed, a depreciation of the dollar decreases export prices in the currency of destination and boosts the country's exports.[26] Interestingly, this stimulative effect holds even when other countries peg their exchange rates to the dollar. Intuitively, to prevent the appreciation of their currencies in response to monetary easing in the U.S., other economies follow a similar stimulative policy, raising demand for both local and imported goods, including ones exported from the U.S.

The second key difference between the policy in the U.S. and in other countries is that its implementation leads to an asymmetric response to foreign shocks. According to Corollary 1.2, movements of the U.S. exchange rate affect the prices of imported intermediates in all other countries and require the intervention of local monetary policy to keep domestic inflation at zero. In contrast, because of dollar pricing, U.S. import prices are less sensitive to movements in foreign exchange rates, making U.S. monetary policy inward-looking and responding mostly to local shocks when $\gamma \approx 0$.

Motive #2: The fact that U.S. monetary policy affects production and consumption in other countries implies that it also shapes global stochastic discount factor and hence, the prices of Arrow-Debreu securities. This gives rise to the second term in the policy rule (13), which corresponds to the *dynamic terms-of-trade manipulation motive* as defined by Costinot, Lorenzoni, and Werning (2014). As they show in the context of a flexible-price model with one internationally traded good, a large economy faces a downward sloping demand for funds from the rest of the world: the larger is the promised transfer to foreigners in a given state of the world, the higher is the consumption in other economies and the lower is their willingness to pay for a marginal unit. Therefore, a large economy can act as a monopolist and extract additional rents by altering its current account relative to the laissez-faire equilibrium. Our analysis shows that the same motive is also relevant for the monetary policy of an infinitely *small* economy if it is the issuer of the dominant currency.

Consider, for example, the case when stimulative monetary policy in the U.S. boosts world consumption and decreases global SDF, i.e. $\epsilon < 0$. The optimal policy then overstimulates the economy — i.e. generates inflationary pressure by setting marginal costs above the price level — in the states of the world, in which the U.S. borrows from the rest of the world $nx_{it} < 0$. This lowers the SDF and allows the country to borrow more cheaply. Symmetrically, the U.S. has incentives to contract the economy and raise the global SDF in order to enjoy higher interest rates in the states of the world when the country runs a positive current account $nx_{it} > 0$.[27] Thus, the effect of the Fed's policy on international asset markets under DCP can potentially contribute to the "exorbitant privilege" of the U.S. (Gourinchas and Rey 2007). It also implies that as long as U.S. trade with other countries depends on foreign shocks, the optimal policy will respond to them and is to some extent outward-looking.

Note that this policy motive is new to the literature because previous models focused on log prefer-

[26]As a result, the price-stabilization motive remains relevant even in the limit with no home bias $\gamma \to 1$.

[27]Our analysis focuses on the approximation around the symmetric steady-state with zero net foreign asset positions. If instead one assumes that the U.S. runs a negative trade balance in the steady-state, the optimal policy rule has an additional term with the time-varying stochastic discount factor.

ences, which make trade balanced $nx_{it} = 0$ in every state of the world (see Corsetti and Pesenti 2007, Goldberg and Tille 2009). The dynamic terms-of-trade motive also drops out under financial autarky when there are no internationally traded assets, but is relevant under any other form of (in)complete markets and does not rely on the simplifying assumption A3.

Motive #3: The final term in the optimal policy rule (13) comes from the price-setting constraint of exporters in other economies. Intuitively, in contrast to other countries, the U.S. can affect its terms of trade not only through export prices, but also by changing the prices of foreign exporters. As a result, the optimal markups of U.S. exporters do not fully eliminate the *terms-of-trade motive* of the monetary policy. Indeed, if import prices were preset at an exogenously given level, the U.S. could inflate them away and buy foreign goods at an arbitrary low price. Of course, this does not happen under rational expectations when foreign prices cannot deviate too much from the average marginal costs, imposing an important constraint on the global real effects of U.S. monetary policy. The optimal policy, however, aims to relax this constraint by partially stabilizing the dollar prices of global exporters.[28] This shifts U.S. policy in the direction of international cooperation (see below) and makes it respond to global shocks outside the U.S.

To what extent should the Fed be concerned about the global spillovers of its policy? Proposition 6 provides new insights to this classical question. On the one hand, the optimal *ex-post* policy in a given state of the world focuses exclusively on the first motive — price stability. Independently from the sign and size of the spillovers on other economies, such a policy allows the U.S. to achieve the optimal level of both local and foreign demand. In this sense, the Fed does not need to worry about the global effects of its policy and can focus exclusively on domestic objectives, consistent with the view of several U.S. policymakers (see e.g. Bernanke 2017). On the other hand, as the two additional motives in the policy rule (13) make clear, ignoring the spillovers is suboptimal from the *ex-ante* perspective as they can backfire through both international financial and goods markets making U.S. borrowing and imports more expensive. It is therefore in the Fed's interest to pay attention to the global effects of its monetary policy, even though the relative weight of these motives is proportional to the openness of the U.S. economy γ and is likely to be small in practice. The ex-ante motives also imply that the optimal policy is not time consistent for the U.S.

Gains from DCP The analysis above shows that both the international transmission of shocks and the optimal monetary policy are markedly different for the U.S. than for other economies. Do these asymmetries translate into differences in countries' welfare? Does the U.S. actually benefit from the dominance of the dollar in world trade? As it turns out, the answers to these questions are ambiguous

[28]This motive was first described in a context of the LCP model by Devereux and Engel (1998) and is also present in DCP models of Devereux, Shi, and Xu (2007), Corsetti and Pesenti (2007), Goldberg and Tille (2009).

and depend on the details of the model.

Intuitively, there are three sources of difference in welfare between countries, which roughly correspond to the three motives of U.S. monetary policy discussed above. First, consider the terms-of-trade motive and notice that it works against the U.S. Indeed, in contrast to other economies that stabilize local demand, U.S. optimal rule (13) deviates from this objective to partially stabilize global export prices. Such policy increases the absolute welfare of the U.S., but also decreases its welfare relative to other countries: lower international prices benefit all economies, but they come at the expense of domestic price stabilization for the U.S. One can show, however, that the differences in the optimal policy have only a second-order effect on countries' welfare when economies are close enough to the autarky limit $\gamma \to 0$.

Second, there is the dynamic terms-of-trade effect: depending on the correlation between the global stochastic discount factor and the U.S. trade balance induced by the monetary policy, the U.S. can win or lose relative to other countries. Empirically, U.S. net exports have a tendency to go up in recessions when the global SDF is high, consistent with the "exorbitant privilege/duty" paradigm (see Gourinchas and Rey 2007), but to what extent this pattern is due to DCP remains an open question. Finally, as discussed above, in contrast to other economies that face a trade-off between stabilizing local and foreign demand, U.S. monetary policy can simultaneously close the domestic output gap and generate the optimal expenditure switching towards its goods abroad. The U.S. unambiguously benefits from this form of the divine coincidence, which is arguably the most important welfare implication of DCP. More formally, the next corollary shows that the U.S. gains from DCP in the special case that eliminates the other motives.[29]

Corollary 6.1 (Welfare) *Depending on parameter values, the welfare of the U.S. can be higher or lower than the welfare of other economies. In the special case of A3, $U = \frac{C^{1-\sigma}}{1-\sigma} - N$, $\sigma\theta = 1$, $F_X = 0$, $\gamma \to 0$, and symmetric shocks, the welfare is higher for the U.S.*

Finally, note that while we focus on the relative welfare of the U.S. vs. non-U.S. economies, one can consider an alternative counterfactual of the PCP world with symmetric use of currencies. Again, the U.S. can gain or lose from DCP relative to this benchmark depending on the strength of the three motives: while dollar pricing allows the U.S. to exploit the dynamic terms-of-trade externality, it also results in higher import prices. Although instructive, this comparison with PCP should be interpreted with caution as firms' currency choices are endogenous and it might be not possible to sustain a PCP equilibrium given today's fundamentals, such as input-output linkages and complementarities in price setting. This implies that if price complementarities across firms are strong enough — as they arguably

[29]While clearly very special, this parametrization is fairly standard and provides an important counterexample to the conclusions of the previous literature that emphasized the losses of the U.S. from DCP (see e.g. Devereux, Shi, and Xu 2007). This discrepancy arises from the fact that in our model, firms use DCP in trade between third countries rather than just in bilateral trade with the U.S.

are in the modern globalized world — making exporters coordinate on one currency and the price-stabilization motive dominates, a competition among countries can emerge for the status of the issuer of the dominant currency.

5.2 International cooperation

So far, we assumed that a policy is chosen independently by each individual country in order to maximize its own welfare. This section contrasts the Nash equilibrium with the optimal cooperative solution for both monetary and fiscal policies, which in particular, might be of special interest to the International Monetary Fund. We also discuss potential welfare gains from international coordination and whether the latter is in mutual interests.

Consider a global planner who can simultaneously use three instruments in each economy — monetary policy, capital controls, and production subsidies — to maximize the world welfare. This set of tools is rich enough to disentangle different policy motives and to make sure that each instrument is not used as a second-best instrument against other distortions. Notice, however, that we do not allow for export taxes τ_{it}^E that could implement the optimal terms of trade and the first-best allocation. Finally, we relax the assumption of symmetric trade flows between countries and allow for heterogeneous demand shifters γ_{ji} of country i for goods from country j. As we discuss below, this generalization has important implications for optimal capital controls. The global planner's problem is then

$$\max_{\{C_{it},N_{it},L_{it},X_{it},B^h_{it},P_{it},P_{iit},P^*_{it},\mathcal{E}_{it},\mathcal{Q}^h_t\}_{it}} \quad \mathbb{E} \int \sum_{t=0}^{\infty} \beta^t U\big(C_{it}, N_{it}, \xi_{it}\big) \mathrm{d}i$$
$$\text{s.t.} \qquad (4)-(6), (8)-(11).$$

The next proposition shows that the optimal policy is substantially different from the non-cooperative case, indicating there are potentially large gains from coordination.

Proposition 7 (Cooperative policy) *Under the optimal cooperative policy, capital controls τ_{it}^c and production subsidies to exporters τ_{it}^* are generically non-zero, non-U.S. monetary policy stabilizes domestic prices $\pi_{iit} = 0$, and U.S. monetary policy stabilizes average international dollar prices.*

We next discuss separately each individual policy instrument starting with monetary policy. Interestingly, the optimal rule for non-U.S. economies remains exactly the same as before: monetary policy stabilizes demand for locally produced goods by targeting domestic prices $\pi_{iit} = 0$. In contrast, the optimal U.S. policy changes dramatically relative to the non-cooperative case: U.S. welfare has an infinitely small weight in the objective function of the global planner, but its monetary policy affects all international prices. It follows that the optimal cooperative solution is to use U.S. monetary tools to stabilize the global demand for dollar-invoiced goods responding to global shocks, rather than to idiosyncratic shocks in the U.S.[30] For concreteness, consider the case of complete markets and symmetric trade flows between economies. The optimal U.S. policy rule can then be written as

$$\int \varpi_{kt}\Psi_{kt}\mathrm{d}k = 0,$$

where $\varpi_{kt} \equiv \left(\frac{P^*_{kt}}{P^*_t}\right)^{1-\varepsilon}$ is the import share of goods from country k and $\Psi_{kt} \equiv \frac{P_{kkt}}{\mathcal{E}_{kt}P^*_{kt}}$ is the law-of-one-price deviation. Thus, under cooperation, the U.S. stabilizes average markups of world exporters Ψ_{kt} weighted by their sale shares ϖ_{kt}. This policy ensures the optimal world demand for the basket of internationally-traded goods, though it falls short of closing the output gap for each individual product as one instrument is not sufficient to target all country-specific deviations from the law of one price.

The important corollary of Proposition 7 is the conflict of interests between countries. Indeed, the optimal cooperative solution requires the U.S. to sacrifice domestic objectives to stabilize global demand, while leaving the monetary policy of other economies unchanged.[31] Therefore, unless the U.S. is compensated via other instruments, it might not be interested in cooperation with other countries under DCP. This discrepancy between local and global incentives is especially pronounced when countries are at different phases of the business cycle and there is a tension between the U.S. responding to domestic vs. world shocks. On the other hand, countries' interests are perfectly aligned in response to global shocks as price stabilization in all economies including the U.S. achieves the first-best allocation. This prediction of the model is consistent with the high level of cooperation between central bankers around the world during the global financial crisis of 2008–2009.

Moving next to the capital controls, Proposition 7 shows that from a global perspective, private risk sharing is generically constrained inefficient. On the one hand, the incomplete asset markets result in a pecuniary externality with agents from different economies not internalizing the effect of their portfolio decisions on the static terms of trade (Geanakoplos and Polemarchakis 1986). On the other hand, even if asset markets are complete, the risk sharing is still inefficient due to the aggregate demand externality (Farhi and Werning 2016). In the notation of Section 3.3, domestic production wedges $\bar{\tau}_{iit}$ are closed by local monetary policy, but the export wedges $\bar{\tau}^*_{it}$ remain generically open given the inability of U.S. policy to close all of them simultaneously. Since private agents take these gaps as given and do not internalize the effect of transfers on aggregate demand, the optimal capital controls are non-zero.

More formally, consider the case of complete markets, so that the aggregate demand externality is the only reason for the macroprudential policy. The optimal tax $1-\tau^c_{it}$ on an Arrow-Debreu security is then determined by the following system of equations for each country i:

$$1-\tau^c_{it} = \int \varpi_{kit}(1-\tau^c_{kt})\Psi_{kt}\mathrm{d}k,$$

[30]This is, of course, an extreme result that is due to the small size of the U.S. Under a more realistic assumption that the U.S. accounts for a significant fraction of global GDP, the policy would target a weighted average of local and global shocks.

[31]This result generalizes the insight from simple models by Corsetti and Pesenti (2007) and Goldberg and Tille (2009).

where $\varpi_{kit} \equiv \frac{\gamma_{ki} P_{kt}^{*1-\varepsilon}}{\int \gamma_{ji} P_{jt}^{*1-\varepsilon} \mathrm{d}j}$ are the import shares. Notice first that the capital controls are zero in the absence of law-of-one-price deviations $\Psi_{kt} = 1$, i.e. when demand for foreign goods is at the optimal level. Similarly, the taxes are uniform across countries and can be set to zero when import shares are the same across countries $\varpi_{kit} = \varpi_{kt}$ for all i because the redistribution of wealth across countries does not change global demand for individual imported products. The capital controls are, however, non-zero away from these two knife-edge cases. To see the intuition behind the optimality condition, consider a special case when all imports of country i come from some other economy j, i.e. $\varpi_{jit} = 1$. If $\Psi_{jt} < 1$, country j has dollar prices that are too high and hence, its export is inefficiently low. The global planner can, therefore, improve the allocation by redistributing demand from country j to country i imposing relatively lower capital taxes in the latter $\frac{1-\tau_{it}^c}{1-\tau_{jt}^c} < 1$. More generally, the optimal capital controls in country i relative to other economies depend on all foreign law-of-one-price deviations weighted by their market shares. Thus, an important policy implication of this analysis is that the macroprudential tools should target *foreign consumers* of depressed goods rather than their *producers*.

Finally, Proposition 7 also implies that the laissez-faire price setting of exporters is not constrained efficient from a global perspective. In particular, assuming symmetric trade flows γ_{ji}, the planner uses production subsidies to implement the optimal export prices:

$$\pi_{it}^* \left(\pi_{it}^* + 1\right) W_{it} = \bar{\kappa} \mathcal{E}_{it} P_{it}^* \left[\frac{MC_{it}}{\mathcal{E}_{it} P_{it}^*} - 1 - \frac{1}{\varepsilon} \int \vartheta_{ikt} \Lambda_{kit} \mathrm{d}k \right] \frac{Y_{it}^*}{\gamma} + \mathbb{E}_t \Theta_{it,t+1} \pi_{it+1}^* \left(\pi_{it+1}^* + 1\right) W_{it+1},$$

where $\bar{\kappa} \equiv \frac{\varepsilon}{\varphi}$ reflects the costs of price adjustment, $\vartheta_{ikt} \equiv \frac{Y_{ikt}}{Y_{it}^*}$ is the share of country k in total exports of country i, and $\Lambda_{kit} \equiv \frac{U_{Ckt}}{U_{Cit}} \frac{\mathcal{E}_{kt} P_{it}}{\mathcal{E}_{it} P_{kt}} - 1$ is the deviation from the full risk sharing. Comparison of this optimality condition with the private NKPC (7) reveals two important differences. First, the planner eliminates the standard terms-of-trade externality by offsetting the markups of exporters with a time-invariant subsidy $\tau_i^* = \frac{\varepsilon - 1}{\varepsilon}$ (see Corsetti and Pesenti 2001). In particular, if prices are flexible $\bar{\kappa} \to \infty$ and the risk sharing is perfect $\Lambda_{kit} = 0$, this ensures that the export prices are equal to the marginal costs of production $P_{it}^* = MC_{it}/\mathcal{E}_{it}$. On the other hand, when prices are sticky, the planner also applies the same subsidy to costs of price adjustment, which results in a more steep Phillips curve relative to the private one, $\bar{\kappa} > \kappa$.

Second and perhaps more surprisingly, there is an additional externality associated with the deviations from the full risk sharing Λ_{kit}. Intuitively, away from the first best, the law of one price is not necessarily optimal, as the global planner aims to equalize the marginal utility from a given good across all economies. To see this, consider for simplicity the case of flexible prices $\bar{\kappa} \to \infty$ and country i exporting only to country j, $\vartheta_{ijt} = 1$. If $\Lambda_{jit} > 1$, the marginal utility of having one additional unit of good i is higher in country j and hence, the planner lowers the export price relative to the domestic one $P_{it}^* < P_{iit}/\mathcal{E}_{it}$. More generally, the optimal export price depends on the average gap between domestic and foreign marginal utilities weighted by export shares ϑ_{ikt}. Importantly, this optimality condition holds for an arbitrary structure of financial markets and the deviations from the full risk sharing Λ_{kit}

can be due to an incomplete span of assets and/or the optimal capital controls described above.

We conclude this section by emphasizing that although the active use of capital controls and export subsidies by the global planner resembles the optimal policy in a dollarized economy from Section 4.3, the underlying motives are quite different in each case. A local planner uses both instruments to fight *local* demand externality due to the suboptimal production of dollarized goods, while the optimal cooperative policy uses capital controls and export subsidies to redistribute demand to *foreign* economies with inefficient imports.

Currency union The fact that international cooperation might benefit non-U.S. economies, but not be in U.S. interests raises the question of whether *local* forms of coordination can be sustained and help to internalize international spillovers. In particular, motivated by the recent experience of the Eurozone, we revise the theory of the optimal currency area in the presence of DCP (Mundell 1961).

To this end, consider a currency area formed by a continuum of non-U.S. economies. To make the analysis interesting, we assume that the trade flows between these countries have a positive mass in their bundles of imported goods C_{it}^* and X_{it}^*, but to simplify the analysis, keep the assumption that the currency union accounts for a trivial fraction of the global economy (see Appendix A.4.5 for details). As usual, joining the currency union means losing an independent monetary policy, which unambiguously decreases the country's welfare in a standard model with PCP. While the same costs are still present under DCP, there are also additional benefits of having a coordinated monetary policy. The latter motive dominates if the shocks are sufficiently correlated across countries and hence, the losses from having common monetary policy are small.

Corollary 7.1 (Currency union) *If shocks are sufficiently positively correlated across countries, then forming a currency union increases the welfare of its members.*

Intuitively, in contrast to the non-cooperative case, the planner internalizes the suboptimal demand for dollar-invoiced goods traded between the members of the union. In the absence of other instruments, the optimal monetary policy deviates from stabilizing local prices to improve demand for goods priced in dollars. More generally, consistent with our results for dollarized economies from Section 4.3, the optimal capital controls and production subsidies are also non-zero. Independently from the set of available instruments, however, forming a currency union increases countries' welfare if their shocks are sufficiently correlated. Of course, the benefits are even higher if having a large currency area helps to promote the status of its currency and makes local trade flows switch from DCP to home currency.[32]

[32]Indeed, this is consistent with empirical evidence from Cavallo, Neiman, and Rigobon (2014) and with the theoretical predictions of the currency choice model from Mukhin (2018).

6 Conclusion

This paper characterizes the optimal monetary policy of the U.S. and other economies in a world with international prices sticky in dollars. We show that although targeting domestic inflation remains robustly optimal for non-U.S. economies, this policy cannot implement the efficient allocation, generates a partial peg to the dollar and gives rise to the global monetary cycle. Individual countries cannot unilaterally improve the allocation using capital controls, while a sufficiently rich set of trade instruments can restore the first-best allocation. The optimal U.S. policy, on the other hand, deviates from price stabilization to extract rents in the goods and asset markets. International coordination can improve global welfare by internalizing the aggregate demand externality and making transfers to countries importing depressed goods. While it might be hard to sustain global cooperation, which is not necessarily in the self-interest of the U.S., countries can still benefit from local forms of coordination, such as forming a currency union.

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Appendix — for online publication

A.1 Proofs for Section 2

A.1.1 Proof of Lemma 2

To prove the first part, note that all non-U.S. countries are small open economies and therefore, take all foreign variables as given. Hence, it does not matter for them in which variables foreign strategies are formulated. The U.S. moves first and takes as given the best responses of other economies. At the second stage, non-U.S. countries take all global variables, including U.S. actions, as given. Proposition 1 states that in this case the optimal non-U.S. policy is to set $\pi_{iit} = 0$. This condition (12) along with conditions (3)–(10) is enough to pin down all local non-U.S. variables $\left\{C_{it}, N_{it}, L_{it}, X_{it}, B^h_{it}, \pi_{it}, \pi_{iit}, \pi^*_{it}, \mathcal{E}_{it}\right\}$ as functions of global variables.[33] Thus, the best response functions are uniquely determined by conditions (3)–(10) and (12) regardless of which variable is used by non-U.S. countries to formulate their strategies.

To prove the second part, note that under the optimal policy from Proposition 1, non-U.S. PPI inflation does not depend on the U.S. actions, $\pi_{iit} = 0$. Therefore, the optimal policy condition (12) in a sequential game can be viewed instead as a fixed non-U.S. strategy in a simultaneous game, while the rest of the non-U.S. variables are still determined by conditions (3)–(10).

The third part of the lemma follows from Corollary 1.1, which states that the optimal policy condition (12) stays the same under discretion.

A.1.2 Proof of Lemma 3

Efficient allocation To solve for efficient allocation in a given economy i, allow the planner to choose directly all quantities in this country. At the same time, the planner takes as given international prices and is subject to the country's resource and budget constraints. Thus, the social planner's problem can be written as

$$\max_{C_{it}, N_{it}, X_{it}, P^*_{it}, \{C_{jit}, X_{jit}\}_j, \left\{B^h_{it+1}\right\}_h} \mathbb{E} \sum_{t=0}^{\infty} \beta^t U\left(C_{it}, N_{it}, \xi_{it}\right)$$

$$A_{it} F\left(N_{it}, X_{it}\right) = C_{iit} + X_{iit} + \gamma \left(\frac{P^*_{it}}{P^*_t}\right)^{-\varepsilon} \int \left(\frac{\mathcal{E}_{jt} P^*_t}{P_{jt}}\right)^{-\theta} \left(C_{jt} + X_{jt}\right) \mathrm{d}j$$

$$C_{it} = \left[(1-\gamma)^{\frac{1}{\theta}} C_{iit}^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} \left(\int C_{jit}^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}j\right)^{\frac{\varepsilon}{\varepsilon-1}\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$$

$$X_{it} = \left[(1-\gamma)^{\frac{1}{\theta}} X_{iit}^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} \left(\int X_{jit}^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}j\right)^{\frac{\varepsilon}{\varepsilon-1}\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$$

[33] CPI inflation $\pi_{it} \equiv P_{it}/P_{it-1} - 1$ can be found from the price index condition (4).

$$\sum_{h\in H_t} \mathcal{Q}_t^h B_{it+1}^h - \sum_{h\in H_{t-1}} (\mathcal{Q}_t^h + D_t^h) B_{it}^h$$
$$= \gamma P_{it}^* \left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} \int \left(\frac{\mathcal{E}_{jt} P_t^*}{P_{jt}}\right)^{-\theta} (C_{jt} + X_{jt})\,\mathrm{d}j - \int P_{jt}^* (C_{jit} + X_{jit})\,\mathrm{d}j + \psi_{it}$$

Here the planner can choose any export price in dollars P_{it}^*, but all import dollar prices, P_t^* and P_{jt}^*, are taken as given. Substitute out C_{it} and X_{it} and denote the Lagrange multipliers for the market clearing condition with ν_{it} and for the budget constraint with ρ_{it}. Then the FOCs are

$$U_C(C_{it}, N_{it}, \xi_{it}) \frac{\mathrm{d}C_{it}}{\mathrm{d}C_{iit}} - \nu_{it} = 0$$
$$U_C(C_{it}, N_{it}, \xi_{it}) \frac{\mathrm{d}C_{it}}{\mathrm{d}C_{jit}} - \rho_{it} P_{jt}^* = 0$$
$$U_N(C_{it}, N_{it}, \xi_{it}) + \nu_{it} A_{it} F_L(N_{it}, X_{it}) = 0$$
$$\nu_{it} A_{it} F_X(N_{it}, X_{it}) \frac{\mathrm{d}X_{it}}{\mathrm{d}X_{iit}} - \nu_{it} = 0$$
$$\nu_{it} A_{it} F_X(N_{it}, X_{it}) \frac{\mathrm{d}X_{it}}{\mathrm{d}X_{jit}} - \rho_{it} P_{jt}^* = 0$$
$$\nu_{it} \gamma \varepsilon P_{it}^{*-\varepsilon-1} P_t^{*\varepsilon} + \rho_{it} \gamma (1-\varepsilon) \left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} = 0$$
$$-\rho_{it} \mathcal{Q}_t^h + \beta \mathbb{E}_t \rho_{it+1} \left(\mathcal{Q}_{t+1}^h + D_{t+1}^h\right) = 0$$

Use the FOC with respect to N_{it} to substitute out ν_{it} in all other conditions,

$$\nu_{it} = \frac{-U_N(C_{it}, N_{it}, \xi_{it})}{A_{it} F_L(N_{it}, X_{it})}.$$

Similarly, use the FOC with respect to P_{it}^* to substitute out ρ_{it},

$$\rho_{it} = \frac{-U_N(C_{it}, N_{it}, \xi_{it})}{A_{it} F_L(N_{it}, X_{it})} \frac{\varepsilon}{\varepsilon - 1} P_{it}^{*-1}.$$

Divide the first FOC by the second (and the fourth by the fifth) to find the relative demand for domestic and foreign varieties,

$$\frac{C_{iit}}{C_{it}^*} = \frac{X_{iit}}{X_{it}^*} = \frac{1-\gamma}{\gamma} \left(\frac{\varepsilon}{\varepsilon - 1} \frac{P_t^*}{P_{it}^*}\right)^{\theta}, \tag{A1}$$

and similarly derive the relative demand for different foreign varieties,

$$\frac{C_{jit}}{C_{kit}} = \frac{X_{jit}}{X_{kit}} = \left(\frac{P_{kt}^*}{P_{jt}^*}\right)^{\varepsilon}. \tag{A2}$$

Next, combine the first two FOCs and use the consumption aggregator to derive

$$\left(\frac{U_C(C_{it}, N_{it}, \xi_{it})}{-U_N(C_{it}, N_{it}, \xi_{it})} A_{it} F_L(N_{it}, X_{it})\right)^{1-\theta} = (1-\gamma) + \gamma \left(\frac{\varepsilon}{\varepsilon - 1} \frac{P_t^*}{P_{it}^*}\right)^{1-\theta}.$$

Similarly, combine the the fourth and the fifth FOCs and use the intermediates aggregator to get

$$\left(A_{it}F_X\left(N_{it},X_{it}\right)\right)^{1-\theta}=(1-\gamma)+\gamma\left(\frac{\varepsilon}{\varepsilon-1}\frac{P_t^*}{P_{it}^*}\right)^{1-\theta}, \tag{A3}$$

and therefore the two conditions together imply

$$\frac{-U_N\left(C_{it},N_{it},\xi_{it}\right)}{U_C\left(C_{it},N_{it},\xi_{it}\right)}=\frac{F_L\left(N_{it},X_{it}\right)}{F_X\left(N_{it},X_{it}\right)}, \tag{A4}$$

which determines the relative demand for inputs.

Finally, the portfolio allocation is determined by the last FOC, which becomes

$$\frac{U_{Nit}}{A_{it}F_{Lit}}=\beta\mathbb{E}_t\frac{U_{Nit+1}}{A_{it+1}F_{Lit+1}}\frac{P_{it}^*}{P_{it+1}^*}\frac{\mathcal{Q}_{t+1}^h+D_{t+1}^h}{\mathcal{Q}_t^h}. \tag{A5}$$

Flexible-price allocation Next, consider the flexible price allocation. First, note that the planner's condition (A4) is exactly the same as the private sector equilibrium condition (5) combined with the labor supply condition (1). Second, under flexible prices, the price setting conditions (6) and (7) collapse to

$$P_{iit}=MC_{it},\quad \mathcal{E}_{it}P_{it}^*=\frac{\varepsilon}{\varepsilon-1}MC_{it}.$$

It follows the law of one price holds up to the export markup, $\mathcal{E}_{it}P_{it}^*=\frac{\varepsilon}{\varepsilon-1}P_{iit}$. With the CES demand we get

$$\frac{C_{iit}}{C_{it}^*}=\frac{X_{iit}}{X_{it}^*}=\frac{1-\gamma}{\gamma}\left(\frac{\mathcal{E}_{it}P_t^*}{P_{iit}}\right)^{\theta},\quad \frac{C_{jit}}{C_{kit}}=\frac{X_{jit}}{X_{kit}}=\left(\frac{P_{kt}^*}{P_{jt}^*}\right)^{\varepsilon}.$$

The relative demand then coincides with the planner's conditions (A1) and (A2).

The domestic price setting condition (6), in turn, could be rewritten as

$$P_{iit}=\frac{P_{it}h\left(\frac{-U_{Nit}}{U_{Cit}}\right)}{A_{it}}=\frac{P_{it}}{A_{it}F_X\left(1,g\left(\frac{-U_{Nit}}{U_{Cit}}\right)\right)}=\frac{P_{it}}{A_{it}F_X\left(L_{it},X_{it}\right)}.$$

Combining it with the price index (4) and the law of one price, $\mathcal{E}_{it}P_{it}^*=\frac{\varepsilon}{\varepsilon-1}P_{iit}$, yields the planner's condition (A3), which turns out to be equivalent to the domestic marginal costs stabilization.

Finally, the portfolio allocation is determined by condition (3), which together with the price setting condition above becomes

$$\mathbb{E}_t\beta\frac{U_{Cit+1}}{U_{Cit}}\frac{A_{it}F_{Xit}P_{iit}}{A_{it+1}F_{Xit+1}P_{iit+1}}\frac{\mathcal{E}_{it+1}}{\mathcal{E}_{it}}\frac{\mathcal{Q}_{t+1}^h+D_{t+1}^h}{\mathcal{Q}_t^h}=1.$$

Plug in the law of one price and the relative demand for inputs (A4), and arrive at the last planner's optimality condition, (A5). Thus, the flexible-price equilibrium conditions coincide with the planner's optimality conditions.

Equilibrium under producer currency pricing Assume that both domestic and export prices are sticky in producer currency and that the monetary policy sets $\pi_{iit}=0$. Then the export Phillips curve (7) turns to

$$\pi^*_{it}\left(\pi^*_{it}+1\right)W_{it}=-\kappa\left(\tilde{P}^*_{it}-\frac{\varepsilon}{\varepsilon-1}MC_{it}\right)\frac{Y^*_{it}}{\gamma}+\mathbb{E}_t\Theta_{it,t+1}\pi^*_{it+1}\left(\pi^*_{it+1}+1\right)W_{it+1},$$

where $\tilde{P}^*_{it}$ is expressed in currency i. Domestic price stabilization $\pi_{iit}=0$ implies constant MC_{it}, and thus constant export prices $\tilde{P}^*_{it}$ satisfy the export Phillips curve. The Rotemberg costs are not incurred in equilibrium, $\pi^*_{it}=0$, and the law of one price holds up to the export markup, $\tilde{P}^*_{it}=\frac{\varepsilon}{\varepsilon-1}P_{iit}$. It is then straightforward to verify that all conditions from the flexible-price allocation are satisfied, given that the export prices expressed in dollars move one-to-one with the nominal exchange rate against the dollar, $P^*_{it}=\tilde{P}^*_{it}/\mathcal{E}_{it}$.

A.2 Proofs for Section 3

To economize space, we describe the equilibrium conditions and prove Lemma 4 for the general setup described in Section 4. Propositions 1-2 and Corollaries 1.1-1.2 then follow as special cases.

A.2.1 Equilibrium conditions

The heterogeneity in home bias implies different price indices for consumers P^c_{it}, domestic producers P^d_{it}, and exporters P^e_{it}. In particular, the CPI enters labor supply condition (1) and the definition of the nominal stochastic discount factor (2):

$$\frac{-U_{Nit}}{U_{Cit}}=\frac{W_{it}}{P^c_{it}},\qquad \Theta_{it,t+\tau}\equiv\beta^\tau\frac{U_{Cit+\tau}}{U_{Cit}}\frac{P^c_{it}}{P^c_{it+\tau}}.$$

Under this definition of $\Theta_{it,t+\tau}$, the no-arbitrage condition (3) does not change. The equilibrium price index (4) is replaced with 3 corresponding conditions, where the left hand side is replaced by P^c_{it}, P^d_{it}, or P^e_{it}, while the right hand side differs only by the value of the home bias parameter: γ_c, γ_d, or γ_e.

On the firms' side, the relative demand for inputs (5) is replaced with similar conditions for domestic producers and for exporters,

$$\frac{X^d_{it}}{L_{iit}}=g^d\left(\frac{-U_{Nit}}{U_{Cit}}\frac{P^c_{it}}{P^d_{it}}\right),\qquad \frac{X^e_{it}}{L^*_{it}}=g^e\left(\frac{-U_{Nit}}{U_{Cit}}\frac{P^c_{it}}{P^e_{it}}\right),$$

where as before functions g^d and g^e are implicitly defined by $\frac{F_L\left(1,g^d(z)\right)}{F_X\left(1,g^d(z)\right)}\equiv z$ and $\frac{G_L(1,g^e(z))}{G_X(1,g^e(z))}\equiv z$. Similarly, the resulting marginal costs of production are

$$\frac{MC^d_{it}}{P^d_{it}}=\frac{h^d\left(W_{it}/P^d_{it}\right)}{A_{iit}},\qquad \frac{MC^e_{it}}{P^e_{it}}=\frac{h^e\left(W_{it}/P^e_{it}\right)}{A^*_{it}},$$

where $h^d(z)\equiv 1/F_X\left(1,g^d(z)\right)$ and $h^e(z)\equiv 1/G_X\left(1,g^e(z)\right)$.

Demand for an individual domestic variety solves the following expenditure minimization problem:

$$\min_{\{C_{iit}(\omega)\}}\int P_{iit}(\omega)\,C_{iit}(\omega)\,\mathrm{d}\omega$$

$$\text{s.t.} \int \Upsilon\left(\frac{C_{iit}(\omega)}{C_{iit}}\right) \mathrm{d}\omega = 1.$$

The first-order conditions lead to the demand function

$$C_{iit}(\omega) = d\left(\frac{P_{iit}(\omega)}{\mathcal{P}_{iit}}\right) C_{iit},$$

where $d(z) \equiv \Upsilon'^{-1}(z)$ and the price index $\mathcal{P}_{iit}$ is implicitly defined by

$$\int \Upsilon\left(d\left(\frac{P_{iit}(\omega)}{\mathcal{P}_{iit}}\right)\right) \mathrm{d}\omega = 1.$$

Define also another price index P_{iit} to express total expenses as $P_{iit}C_{iit} \equiv \int P_{iit}(\omega) C_{iit}(\omega) \mathrm{d}\omega$. This price index is then given by

$$P_{iit} \equiv \int P_{iit}(\omega) d\left(\frac{P_{iit}(\omega)}{\mathcal{P}_{iit}}\right) \mathrm{d}\omega.$$

Note, however, that in equilibrium all domestic producers are going to be symmetric and hence, for any ω, $P_{iit}(\omega) = P_{iit} = \mathcal{P}_{iit}$.[34]

The problem of a domestic firm can then be written as

$$\max_{\{P_t\}} \mathbb{E} \sum_{t=0}^{\infty} \Theta_{i0,t} \left[\left(P_t - \tau_i MC_{it}^d\right) d\left(\frac{P_t}{\mathcal{P}_{iit}}\right) Y_{iit} - (1-\gamma) \frac{\varphi}{2} \left(\frac{P_t}{P_{t-1}} - 1\right)^2 W_{it} \right],$$

where demand shifter Y_{iit} combines demand of consumers, domestic producers, and exporters, $Y_{iit} = C_{iit} + X_{iit}^d + X_{iit}^e$. The first-order conditions together with equilibrium relationships lead to the following Phillips curve

$$\pi_{iit}(\pi_{iit}+1)(-U_{Nit}) = -\tilde{\kappa}\left(P_{iit} - MC_{it}^d\right) \frac{U_{Cit}}{P_{it}^c} \frac{Y_{iit}}{1-\gamma} + \beta \mathbb{E}_t \pi_{iit+1}(\pi_{iit+1}+1)(-U_{Nit+1}),$$

where $\tilde{\kappa} \equiv -\frac{1+d'(1)}{\varphi}$ and the production subsidy corrects for the time-invariant markup, $\frac{d'(1)}{1+d'(1)}\tau_i = 1$. Using the labor supply condition and the CES demand, one gets the baseline NKPC (6) as a special case.

In addition, by allowing firms to freely choose prices after the announcement of policy, we ensure that there could be no inflation in the initial period "on average". Formally, this first-order condition with respect to initial price level leads to an additional "ex-ante" price-setting condition, $\mathbb{E}\pi_{ii0}(\pi_{ii0}+1)(-U_{Ni0}) = 0$.

The expenditure minimization problem for imported varieties is similar to the one for domestic varieties considered above, and leads to demand function

$$C_{jit}(\omega) = d\left(\frac{P_{jt}^*(\omega)}{\mathcal{P}_t^*}\right) C_{it}^*,$$

[34]The CES demand is a special case with $\Upsilon(x) = 1 + \frac{\varepsilon}{\varepsilon-1}\left(x^{\frac{\varepsilon-1}{\varepsilon}} - 1\right)$, where it could be verified that the two price indices, P_{iit} and $\mathcal{P}_{iit}$, always coincide.

where the two price indices $\mathcal{P}_t^*$ and P_t^* are defined by

$$\int\int d\left(\frac{P_{jt}^*(\omega)}{\mathcal{P}_t^*}\right)\mathrm{d}\omega\mathrm{d}j = 1, \qquad P_t^* \equiv \int\int P_{jt}^*(\omega)\, d\left(\frac{P_{jt}^*(\omega)}{\mathcal{P}_t^*}\right)\mathrm{d}\omega\mathrm{d}j.$$

In contrast to the case of domestic prices, these price indices do not coincide because of the cross-country differences. However, both of them are taken as given by a small open economy.

The problem of an exporter is

$$\max_{\{P_t\}} \mathbb{E}\sum_{t=0}^{\infty}\Theta_{i0,t}\left[\left(\mathcal{E}_{it}P_t - MC_{it}^e\right) d\left(\frac{P_t}{\mathcal{P}_t^*}\right)Y_t^* - \gamma\frac{\varphi}{2}\left(\frac{P_t}{P_{t-1}}-1\right)^2 W_{it}\right],$$

where the foreign demand shifter combines demand from three types of agents, $Y_t^* \equiv \int\left(C_{jt}^* + X_{jt}^{d*} + X_{jt}^{e*}\right)\mathrm{d}j$, and we assume that there is no production subsidy, $\tau_i^* = 1$. As before, the first-order conditions together with equilibrium relationships lead to the export Phillips curve:

$$\pi_{it}^*\left(\pi_{it}^*+1\right)\left(-U_{Nit}\right) = \beta\mathbb{E}_t\pi_{it+1}^*\left(\pi_{it+1}^*+1\right)\left(-U_{Nit+1}\right) + \frac{d\left(\frac{P_{it}^*}{\mathcal{P}_t^*}\right) + \frac{P_{it}^*}{\mathcal{P}_t^*}d'\left(\frac{P_{it}^*}{\mathcal{P}_t^*}\right)}{\varphi}\left(\mathcal{E}_{it}P_{it}^* - \frac{\frac{P_{it}^*}{\mathcal{P}_t^*}d'\left(\frac{P_{it}^*}{\mathcal{P}_t^*}\right)}{d\left(\frac{P_{it}^*}{\mathcal{P}_t^*}\right) + \frac{P_{it}^*}{\mathcal{P}_t^*}d'\left(\frac{P_{it}^*}{\mathcal{P}_t^*}\right)}MC_{it}^e\right)\frac{U_{Cit}}{P_{it}^c}\frac{Y_t^*}{\gamma}.$$

The key difference of pricing in the export market compared to the domestic market is that the optimal markup is time-varying. The reason is that the optimal markup depends on the prices of competitors. In the domestic market, all firms are symmetric and thus the relevant relative price, $P_{iit}(\omega)/\mathcal{P}_{iit}$, is always 1. In the export market, only exporters from one country are symmetric, $P_{it}^*(\omega) = P_{it}^*$, but they compete with exporters from all over the world, and thus the relevant relative price, $P_{it}^*/\mathcal{P}_t^*$, is time-varying. As before, the baseline export NKPC (7) is the special case and the free choice of prices in the initial period result in the "ex-ante" price setting condition $\mathbb{E}\pi_{i0}^*\left(\pi_{i0}^*+1\right)\left(-U_{Ni0}\right) = 0$.

Finally, the goods market clearing condition (8) splits into one condition for domestic goods

$$A_{iit}F\left(L_{iit}, X_{it}^d\right) = (1-\gamma_c)\left(\frac{P_{iit}}{P_{it}^c}\right)^{-\theta}C_{it} + (1-\gamma_d)\left(\frac{P_{iit}}{P_{it}^d}\right)^{-\theta}X_{it}^d + (1-\gamma_e)\left(\frac{P_{iit}}{P_{it}^e}\right)^{-\theta}X_{it}^e,$$

and one condition for exported goods

$$A_{it}^*G\left(L_{it}^*, X_{it}^e\right) = d\left(\frac{P_{it}^*}{\mathcal{P}_t^*}\right)\int\left(C_{jt}^* + X_{jt}^{d*} + X_{jt}^{e*}\right)\mathrm{d}j.$$

A.2.2 Proof of Proposition 1

The full policy problem is

$$\max_{\left\{C_{it},X_{it}^d,X_{it}^e,L_{iit},L_{it}^*,N_{it},P_{iit},P_{it}^c,P_{it}^d,P_{it}^e,P_{it}^*,\mathcal{E}_{it},\pi_{iit},\pi_{it}^*,\left\{B_{it+1}^h\right\}_h\right\}_t} \mathbb{E}\sum_{t=0}^{\infty}\beta^t U\left(C_{it}, N_{it}, \xi_{it}\right)$$

$$\text{s.t. } A_{iit}F\left(L_{iit}, X_{it}^d\right) = (1-\gamma_c)\left(\frac{P_{iit}}{P_{it}^c}\right)^{-\theta} C_{it} + (1-\gamma_d)\left(\frac{P_{iit}}{P_{it}^d}\right)^{-\theta} X_{it}^d + (1-\gamma_e)\left(\frac{P_{iit}}{P_{it}^e}\right)^{-\theta} X_{it}^e,$$

$$A_{it}^* G\left(L_{it}^*, X_{it}^e\right) = d\left(\frac{P_{it}^*}{\mathcal{P}_t^*}\right) Y_t^*,$$

$$\frac{X_{it}^d}{L_{iit}} = g^d\left(\frac{-U_{Nit}}{U_{Cit}}\frac{P_{it}^c}{P_{it}^d}\right), \quad \frac{X_{it}^e}{L_{it}^*} = g^e\left(\frac{-U_{Nit}}{U_{Cit}}\frac{P_{it}^c}{P_{it}^e}\right),$$

$$\sum_{h\in H_t} \mathcal{Q}_t^h B_{it+1}^h - \sum_{h\in H_{t-1}} (\mathcal{Q}_t^h + D_t^h) B_{it}^h$$

$$= P_{it}^* d\left(\frac{P_{it}^*}{\mathcal{P}_t^*}\right) Y_t^* - P_t^*\gamma_c\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}^c}\right)^{-\theta} C_{it} - P_t^*\gamma_d\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}^d}\right)^{-\theta} X_{it}^d - P_t^*\gamma_e\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}^e}\right)^{-\theta} X_{it}^e + \psi_{it},$$

$$L_{iit} + L_{it}^* = N_{it} - \frac{\varphi}{2}(1-\gamma)\pi_{iit}^2 - \frac{\varphi}{2}\gamma\pi_{it}^{*2},$$

$$\beta\mathbb{E}_t \frac{U_{Cit+1}}{U_{Cit}}\frac{\mathcal{E}_{it+1}}{P_{it+1}^c}\frac{P_{it}^c}{\mathcal{E}_{it}}\frac{\mathcal{Q}_{t+1}^h + D_{t+1}^h}{\mathcal{Q}_t^h} = 1,$$

$$(1-\gamma_c)\left(\frac{P_{iit}}{P_{it}^c}\right)^{1-\theta} + \gamma_c\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}^c}\right)^{1-\theta} = 1,$$

$$(1-\gamma_d)\left(\frac{P_{iit}}{P_{it}^d}\right)^{1-\theta} + \gamma_d\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}^d}\right)^{1-\theta} = 1,$$

$$(1-\gamma_e)\left(\frac{P_{iit}}{P_{it}^e}\right)^{1-\theta} + \gamma_e\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}^e}\right)^{1-\theta} = 1,$$

$$\pi_{iit} = \frac{P_{iit}}{P_{iit-1}} - 1, \quad \pi_{it}^* = \frac{P_{it}^*}{P_{it-1}^*} - 1,$$

$$\pi_{iit}\left(\pi_{iit}+1\right)\left(-U_{Nit}\right) = \beta\mathbb{E}_t\pi_{iit+1}\left(\pi_{iit+1}+1\right)\left(-U_{Nit+1}\right)$$

$$-\tilde{\kappa}\left(\frac{\mathcal{P}_{iit}}{P_{it}^c} - \frac{P_{it}^d}{P_{it}^c}\frac{h^d\left(\frac{-U_{Nit}}{U_{Cit}}\frac{P_{it}^c}{P_{it}^d}\right)}{A_{it}}\right) U_{Cit}P_{iit}^{\theta}\frac{(1-\gamma_c)\,(P_{it}^c)^{\theta}C_{it} + (1-\gamma_d)\,(P_{it}^d)^{\theta}X_{it}^d + (1-\gamma_e)\,(P_{it}^e)^{\theta}X_{it}^e}{1-\gamma},$$

$$\pi_{it}^*\left(\pi_{it}^*+1\right)\left(-U_{Nit}\right) = \beta\mathbb{E}_t\pi_{it+1}^*\left(\pi_{it+1}^*+1\right)\left(-U_{Nit+1}\right)$$

$$+\frac{d\left(\frac{P_{it}^*}{\mathcal{P}_t^*}\right) + \frac{P_{it}^*}{\mathcal{P}_t^*}d'\left(\frac{P_{it}^*}{\mathcal{P}_t^*}\right)}{\varphi}\left(\frac{\mathcal{E}_{it}P_{it}^*}{P_{it}^c} - \frac{\frac{P_{it}^*}{\mathcal{P}_t^*}d'\left(\frac{P_{it}^*}{\mathcal{P}_t^*}\right)}{d\left(\frac{P_{it}^*}{\mathcal{P}_t^*}\right) + \frac{P_{it}^*}{\mathcal{P}_t^*}d'\left(\frac{P_{it}^*}{\mathcal{P}_t^*}\right)}\frac{P_{it}^e}{P_{it}^c}\frac{h^e\left(\frac{-U_{Nit}}{U_{Cit}}\frac{P_{it}^c}{P_{it}^e}\right)}{A_{it}^*}\right) U_{Cit}\frac{Y_t^*}{\gamma},$$

$$\mathbb{E}\pi_{ii0}\left(\pi_{ii0}+1\right)\left(-U_{Ni0}\right) = 0, \quad \mathbb{E}\pi_{i0}^*\left(\pi_{i0}^*+1\right)\left(-U_{Ni0}\right) = 0.$$

Denote the Lagrange multipliers on these constraints respectively with ν_t^d, ν_t^e, η_t^d, η_t^e, ρ_t, ϑ_t, χ_t, λ_t^c, λ_t^d, λ_t^e, ζ_t, ζ_t^*, μ_t, μ_t^*, μ, μ^*. We guess and verify later that some of the constraints are not binding: $\eta_t^d = \eta_t^e = \chi_t = \zeta_t = \mu_t = \mu_t^* = \mu = \mu^* = 0$. With this in mind, take the first-order conditions:

- wrt C_{it}:

$$U_{Cit} - \nu_t^d(1-\gamma_c)\left(\frac{P_{iit}}{P_{it}^c}\right)^{-\theta} + \rho_t P_t^*\gamma_c\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}^c}\right)^{-\theta} = 0,$$

- wrt X_{it}^{d}:

$$\nu_t^d A_{iit} F_X \left(L_{iit}, X_{it}^d\right) - \nu_t^d \left(1-\gamma_d\right) \left(\frac{P_{iit}}{P_{it}^d}\right)^{-\theta} + \rho_t P_t^* \gamma_d \left(\frac{\mathcal{E}_{it} P_t^*}{P_{it}^d}\right)^{-\theta} = 0,$$

- wrt X_{it}^{e}:

$$\nu_t^e A_{it}^* G_X \left(L_{it}^*, X_{it}^e\right) - \nu_t^d \left(1-\gamma_e\right) \left(\frac{P_{iit}}{P_{it}^e}\right)^{-\theta} + \rho_t P_t^* \gamma_e \left(\frac{\mathcal{E}_{it} P_t^*}{P_{it}^e}\right)^{-\theta} = 0,$$

- wrt L_{iit}:

$$\nu_t^d A_{iit} F_L \left(L_{iit}, X_{it}^d\right) + \vartheta_t = 0,$$

- wrt L_{it}^{*}:

$$\nu_t^e A_{it}^* G_L \left(L_{it}^*, X_{it}^e\right) + \vartheta_t = 0,$$

- wrt N_{it}:

$$U_{Nit} - \vartheta_t = 0,$$

- wrt P_{iit}:

$$\nu_t^d \theta P_{iit}^{-\theta} \left(\left(1-\gamma_c\right) \left(P_{it}^c\right)^{\theta} C_{it} + \left(1-\gamma_d\right) \left(P_{it}^d\right)^{\theta} X_{it}^d + \left(1-\gamma_e\right) \left(P_{it}^e\right)^{\theta} X_{it}^e \right)$$
$$- \left(1-\theta\right) \left[\lambda_t^c \left(1-\gamma_c\right) \left(\frac{P_{iit}}{P_{it}^c}\right)^{1-\theta} + \lambda_t^d \left(1-\gamma_d\right) \left(\frac{P_{iit}}{P_{it}^d}\right)^{1-\theta} + \lambda_t^e \left(1-\gamma_e\right) \left(\frac{P_{iit}}{P_{it}^e}\right)^{1-\theta} \right] = 0,$$

- wrt P_{it}^{c}:

$$-\nu_t^d \left(1-\gamma_c\right) \theta \left(\frac{P_{iit}}{P_{it}^c}\right)^{-\theta} C_{it} + \rho_t P_t^* \gamma_c \theta \left(\frac{\mathcal{E}_{it} P_t^*}{P_{it}^c}\right)^{-\theta} C_{it}$$
$$-\lambda_t^c \left(1-\gamma_c\right) \left(\theta-1\right) \left(\frac{P_{iit}}{P_{it}^c}\right)^{1-\theta} - \lambda_t^c \gamma_c \left(\theta-1\right) \left(\frac{\mathcal{E}_{it} P_t^*}{P_{it}^c}\right)^{1-\theta} = 0,$$

- wrt P_{it}^{d}:

$$-\nu_t^d \left(1-\gamma_d\right) \theta \left(\frac{P_{iit}}{P_{it}^d}\right)^{-\theta} X_{it}^d + \rho_t P_t^* \gamma_d \theta \left(\frac{\mathcal{E}_{it} P_t^*}{P_{it}^d}\right)^{-\theta} X_{it}^d$$
$$-\lambda_t^d \left(1-\gamma_d\right) \left(\theta-1\right) \left(\frac{P_{iit}}{P_{it}^d}\right)^{1-\theta} - \lambda_t^d \gamma_d \left(\theta-1\right) \left(\frac{\mathcal{E}_{it} P_t^*}{P_{it}^d}\right)^{1-\theta} = 0,$$

- wrt P_{it}^{e}:

$$-\nu_t^d \left(1-\gamma_e\right) \theta \left(\frac{P_{iit}}{P_{it}^e}\right)^{-\theta} X_{it}^e + \rho_t P_t^* \gamma_e \theta \left(\frac{\mathcal{E}_{it} P_t^*}{P_{it}^e}\right)^{-\theta} X_{it}^e$$
$$-\lambda_t^e \left(1-\gamma_e\right) \left(\theta-1\right) \left(\frac{P_{iit}}{P_{it}^e}\right)^{1-\theta} - \lambda_t^e \gamma_e \left(\theta-1\right) \left(\frac{\mathcal{E}_{it} P_t^*}{P_{it}^e}\right)^{1-\theta} = 0,$$

- wrt P^*_{it}:

$$-\nu_t^e d'\left(\frac{P^*_{it}}{\mathcal{P}^*_t}\right)\frac{1}{\mathcal{P}^*_t}Y^*_t - \rho_t d\left(\frac{P^*_{it}}{\mathcal{P}^*_t}\right)Y^*_t - \rho_t\frac{P^*_{it}}{\mathcal{P}^*_t}d'\left(\frac{P^*_{it}}{\mathcal{P}^*_t}\right)Y^*_t - \zeta^*_t\frac{1}{P^*_{it-1}} + \beta\mathbb{E}_t\zeta^*_{t+1}\frac{P^*_{it+1}}{P^{*2}_{it}} = 0,$$

- wrt $\mathcal{E}_{it}$:

$$-\theta\rho_t P^*_t\left[\gamma_c\left(\frac{\mathcal{E}_{it}P^*_t}{P^c_{it}}\right)^{-\theta}C_{it} + \gamma_d\left(\frac{\mathcal{E}_{it}P^*_t}{P^d_{it}}\right)^{-\theta}X^d_{it} + \gamma_e\left(\frac{\mathcal{E}_{it}P^*_t}{P^e_{it}}\right)^{-\theta}X^e_{it}\right]$$
$$-(1-\theta)\left[\lambda^c_t\gamma_c\left(\frac{\mathcal{E}_{it}P^*_t}{P^c_{it}}\right)^{1-\theta} + \lambda^d_t\gamma_d\left(\frac{\mathcal{E}_{it}P^*_t}{P^d_{it}}\right)^{1-\theta} + \lambda^e_t\gamma_e\left(\frac{\mathcal{E}_{it}P^*_t}{P^e_{it}}\right)^{1-\theta}\right] = 0,$$

- wrt B^h_{it+1}:

$$\rho_t\mathcal{Q}^h_t - \beta\mathbb{E}_t\rho_{t+1}\left(\mathcal{Q}^h_{t+1} + D^h_{t+1}\right) = 0,$$

- wrt π_{iit}:

$$\vartheta_t\varphi(1-\gamma)\pi_{iit} = 0,$$

- wrt π^*_{it}:

$$\vartheta_t\varphi\gamma\pi^*_{it} + \zeta^*_t = 0.$$

Conjecture that the optimal policy targets $\pi_{iit} = 0$, which along with the domestic Phillips curve implies

$$A_{iit}\frac{P_{iit}}{P^d_{it}} = h^d\left(\frac{-U_{Nit}}{U_{Cit}}\frac{P^c_{it}}{P^d_{it}}\right) = \frac{1}{F_X\left(L_{iit}, X^d_{it}\right)} = \frac{-U_{Nit}}{U_{Cit}}\frac{P^c_{it}}{P^d_{it}}\frac{1}{F_L\left(L_{iit}, X^d_{it}\right)}, \tag{A6}$$

where the second equality follows from the definition of $h^d(\cdot)$, and the last equality uses demand for inputs.

Now let's verify our guesses by showing that all first-order conditions are satisfied. The FOC wrt N_{it} is just $\vartheta_t = U_{Nit}$, so that the FOC wrt L_{iit} becomes $\nu^d_t = -U_{Nit}/\left(A_{iit}F_L\left(L_{iit}, X^d_{it}\right)\right)$, which under the optimal policy implies $\nu^d_t = P_{iit}U_{Cit}/P^c_{it}$. Similarly, the FOC wrt L^*_{it} results in $\nu^e_t = -U_{Nit}/\left(A^*_{it}G_L\left(L^*_{it}, X^e_{it}\right)\right)$. Guess further that $\rho_t = -U_{Cit}\mathcal{E}_{it}/P^c_{it}$. Then the FOC wrt B^h_{it+1} coincides with the no-arbitrage condition, and thus it is satisfied. The FOC wrt P^c_{it} implies $\lambda^c_t = -\frac{\theta}{\theta-1}U_{Cit}C_{it}$. Similarly, the FOCs wrt P^d_{it} and wrt P^e_{it} become

$$\lambda^d_t = -\frac{P^d_{it}}{P^c_{it}}\frac{\theta}{\theta-1}U_{Cit}X^d_{it}, \quad \lambda^e_t = -\frac{P^e_{it}}{P^c_{it}}\frac{\theta}{\theta-1}U_{Cit}X^e_{it}.$$

It is straightforward then to show that the FOCs wrt $\mathcal{E}_{it}$, P_{iit}, C_{it}, X^d_{it}, X^e_{it} are satisfied. Finally, the FOC wrt π^*_{it} implies $\zeta^*_t = -\vartheta_t\varphi\gamma\pi^*_{it} = \varphi\gamma\pi^*_{it}\left(-U_{Nit}\right)$. By plugging this condition into the FOC wrt P^*_{it}, it can be shown that it ultimately leads to the same condition as the export Phillips curve, and thus it is also satisfied.

In sum, we have shown that there exists a set of values of Lagrange multipliers such that all optimality conditions are satisfied under our policy, $\pi_{iit} = 0$. Since it is feasible, i.e. all constraints of the policy problem are satisfied, this policy solves the planner's problem.

A.2.3 Proof of Corollary 1.1

Consider the system of constraints in the optimal policy problem in Section A.2.2. Note that the private agents' expectations enter only the no-arbitrage condition (3) and the last four constraints, the price setting conditions. The proof of Proposition 1, shows that all these constraints do not bind under the optimal policy. Thus, the policymaker under commitment does not use policy to influence private agents' expectations. Moreover, the optimal policy stays the same regardless of how (and whether) the policy can affect these expectations. Therefore, the optimal policy under commitment coincides with the optimal policy under discretion and is time-consistent.

A.2.4 Proof of Corollary 1.2

Iterate forward the Euler equation (2) for local bonds to back out the nominal interest rates:

$$\frac{U_{Cit}}{P^c_{it}} = \beta R_{it} \mathbb{E}_t \frac{U_{Cit+1}}{P^c_{it+1}} = \lim_{T\to\infty} \beta^T \mathbb{E}_t \left(\prod_{\tau=0}^{T-1} R_{it+\tau} \right) \frac{U_{Cit+T}}{P^c_{it+T}}.$$

Assume stationarity, so that the long-run values of all real variables are constant.[35] This, in turn, implies that the long-run values of the relative prices are constant as well, and because the monetary policy stabilizes P_{iit}, the long-run CPI P^c_{it} is also independent of shocks. It follows that $\lim_{T\to\infty} \frac{U_{Cit+T}}{P^c_{it+T}} = \text{const}$ and $\frac{U_{Cit}}{P^c_{it}}$ is equal to the expected present value of future interest rates — the characteristic of the monetary policy we focus on henceforth.

Under the optimal monetary policy, the nominal marginal costs of local firms are constant, i.e.

$$MC^d_{it} = \frac{m\left(W_{it}, P^d_{it}\right)}{A_{iit}} = \frac{m\left(\frac{-U_{Nit}}{U_{Cit}/P^c_{it}}, P^d_{it}\right)}{A_{iit}} = \text{const}.$$

It follows that the monetary policy has to react to foreign shocks: U_{Nit} fluctuates with foreign demand for domestic products and the price index of intermediates P^d_{it} depends on import prices

$$\left(P^d_{it}\right)^{1-\theta} = (1-\gamma_d)\, P^{1-\theta}_{iit} + \gamma_d \left(\mathcal{E}_{it} P^*_t\right)^{1-\theta}.$$

Moreover, because both import and export prices are sticky in dollars, the dollar exchange rate has a disproportionately large effect on local monetary policy through both channels.

Lastly, assume $U_N = \text{const}$ and that the exchange rate appreciates in response to a positive interest rate shock, and consider an exogenous shock that depreciates domestic currency relative to the dollar. Since domestic prices P_{iit} and dollar import prices P^*_t are sticky, a higher nominal exchange rate $\mathcal{E}_{it}$ increases price index for intermediates P^d_{it}. This, in turn, puts an upward pressure on domestic marginal costs, MC^d_{it}. To stabilize them, the monetary policy raises its monetary instrument U_{Cit}/P^c_{it}, which leads to an appreciation of domestic currency. Thus, the optimal policy smooths movements in $\mathcal{E}_{it}$, which results in a partial peg to the dollar.

[35] While the stationarity is in general not guaranteed under incomplete markets, one can ensure it by adding infinitely small portfolio adjustment costs (see Schmitt-Grohé and Uribe 2003, Itskhoki and Mukhin 2019a).

A.2.5 Proof of Proposition 2

Augment the policy problem in Section A.2.2 with a set of state-contingent taxes $\{\tau_{it}^c\}$ that enter the no-arbitrage condition (3). The solution to the problem stays the same since the no-arbitrage condition (3) was not binding even in the absence of these instruments, that is $\chi_t = 0$ in Section A.2.2. Moreover, after substituting out the equilibrium value of the Lagrange multiplier ρ_t, the FOC wrt B_{it+1}^h coincides with the no-arbitrage condition (3). This implies that the optimal allocation can be decentralized with zero taxes, $\tau_{it}^c = 0$.

A.2.6 Proof of Proposition 3

Assume that monetary policy stabilizes domestic prices, $\pi_{iit} = 0$. The production subsidy to exporters stabilizes dollar prices, $\pi_{it}^* = 0$, which implies that all price-setting conditions are satisfied with zero inflation, there are no output losses due to Rotemberg costs, and $L_{it} = N_{it}$. Next, we choose the export tax τ_{it}^E so that the law of one price holds, $\mathcal{E}_{it}\tau_{it}^E P_{it}^* = \frac{\varepsilon}{\varepsilon-1}P_{iit}$. Since both domestic prices P_{iit} and pre-tax export prices P_{it}^* are constant, this means that the export tax follows the nominal exchange rate movements, $\mathcal{E}_{it}\tau_{it}^E = 1$. Then it is straightforward to verify that all conditions from the flexible-price allocation (Section A.1.2) are satisfied since the after-tax export prices $\tau_{it}^E P_{it}^*$ replicate the path of the flexible dollar prices. From Lemma 3, the resulting allocation is efficient.

A.3 Proofs for Section 4

A.3.1 Proof of Lemma 4

See Sections A.2.2 and A.2.5.

A.3.2 Proof of Proposition 4

Private currency choice The problem of a representative firm is now to choose not only the path of export prices, but also the currency, in which the prices are set:

$$\max_{\{P_t^k\},k} \mathbb{E}\sum_{t=0}^{\infty}\Theta_{i0,t}\left[\left(\mathcal{E}_{it}P_t^k/\mathcal{E}_{kt} - MC_{it}^e\right)d\left(\frac{P_t^k/\mathcal{E}_{kt}}{\mathcal{P}_t^*}\right)Y_t^* - \gamma\frac{\varphi}{2}\left(\frac{P_t^k}{P_{t-1}^k}-1\right)^2 W_{it}\right],$$

where P_t^k is a price sticky in currency k. The optimality conditions with respect to the choice of prices lead to the following export Phillips curve

$$\begin{aligned}\pi_{it}^{*k}\left(\pi_{it}^{*k}+1\right)(-U_{Nit}) = \beta\mathbb{E}_t\pi_{it+1}^{*k}\left(\pi_{it+1}^{*k}+1\right)(-U_{Nit+1}) + \frac{d\left(\frac{P_{it}^{*k}/\mathcal{E}_{kt}}{\mathcal{P}_t^*}\right)+\frac{P_{it}^{*k}/\mathcal{E}_{kt}}{\mathcal{P}_t^*}d'\left(\frac{P_{it}^{*k}/\mathcal{E}_{kt}}{\mathcal{P}_t^*}\right)}{\varphi}\times \\ \times\left(\frac{\mathcal{E}_{it}P_{it}^{*k}/\mathcal{E}_{kt}}{P_{it}^c} - \frac{\frac{P_{it}^{*k}/\mathcal{E}_{kt}}{\mathcal{P}_t^*}d'\left(\frac{P_{it}^{*k}/\mathcal{E}_{kt}}{\mathcal{P}_t^*}\right)}{d\left(\frac{P_{it}^{*k}/\mathcal{E}_{kt}}{\mathcal{P}_t^*}\right)+\frac{P_{it}^{*k}/\mathcal{E}_{kt}}{\mathcal{P}_t^*}d'\left(\frac{P_{it}^{*k}/\mathcal{E}_{kt}}{\mathcal{P}_t^*}\right)}\frac{P_{it}^e}{P_{it}^c}\frac{h^e\left(\frac{-U_{Nit}}{U_{Cit}}\frac{P_{it}^c}{P_{it}^e}\right)}{A_{it}^*}\right)U_{Cit}\frac{Y_t^*}{\gamma}.\end{aligned} \tag{A7}$$

The result that under sufficiently strong price complementarities, domestic firms choose PCP and exporters choose DCP follows directly from the analysis of Mukhin (2018) and is suppressed here for brevity.

Optimal currency choice We show next that the optimal policy does not change when the currency choice is endogenous. To this end, we augment the policy problem from Section A.2.2 with endogenous currency choice:[36]

$$\max_{\left\{C_{it},X^d_{it},X^e_{it},L_{iit},L^*_{it},N_{it},P_{iit},P^c_{it},P^d_{it},P^e_{it},P^{*k}_{it},\mathcal{E}_{it},\pi_{iit},\pi^{*k}_{it},\left\{B^h_{it+1}\right\}_h\right\}_t,k} \mathbb{E}\sum_{t=0}^{\infty}\beta^t U\left(C_{it},N_{it},\xi_{it}\right)$$

$$\text{s.t. } A_{iit}F\left(L_{iit},X^d_{it}\right)=\left(1-\gamma_c\right)\left(\frac{P_{iit}}{P^c_{it}}\right)^{-\theta}C_{it}+\left(1-\gamma_d\right)\left(\frac{P_{iit}}{P^d_{it}}\right)^{-\theta}X^d_{it}+\left(1-\gamma_e\right)\left(\frac{P_{iit}}{P^e_{it}}\right)^{-\theta}X^e_{it},$$

$$A^*_{it}G\left(L^*_{it},X^e_{it}\right)=d\left(\frac{P^{*k}_{it}/\mathcal{E}_{kt}}{\mathcal{P}^*_t}\right)Y^*_t,$$

$$\sum_{h\in H_t}\mathcal{Q}^h_t B^h_{it+1}-\sum_{h\in H_{t-1}}\left(\mathcal{Q}^h_t+D^h_t\right)B^h_{it}-P^{*k}_{it}/\mathcal{E}_{kt}d\left(\frac{P^{*k}_{it}/\mathcal{E}_{kt}}{\mathcal{P}^*_t}\right)Y^*_t$$
$$=-P^*_t\gamma_c\left(\frac{\mathcal{E}_{it}P^*_t}{P^c_{it}}\right)^{-\theta}C_{it}-P^*_t\gamma_d\left(\frac{\mathcal{E}_{it}P^*_t}{P^d_{it}}\right)^{-\theta}X^d_{it}-P^*_t\gamma_e\left(\frac{\mathcal{E}_{it}P^*_t}{P^e_{it}}\right)^{-\theta}X^e_{it}+\psi_{it},$$

$$\left(1-\gamma_c\right)\left(\frac{P_{iit}}{P^c_{it}}\right)^{1-\theta}+\gamma_c\left(\frac{\mathcal{E}_{it}P^*_t}{P^c_{it}}\right)^{1-\theta}=1,$$

$$\left(1-\gamma_d\right)\left(\frac{P_{iit}}{P^d_{it}}\right)^{1-\theta}+\gamma_d\left(\frac{\mathcal{E}_{it}P^*_t}{P^d_{it}}\right)^{1-\theta}=1,$$

$$\left(1-\gamma_e\right)\left(\frac{P_{iit}}{P^e_{it}}\right)^{1-\theta}+\gamma_e\left(\frac{\mathcal{E}_{it}P^*_t}{P^e_{it}}\right)^{1-\theta}=1,$$

$$L_{iit}+L^*_{it}=N_{it}-\frac{\varphi}{2}\left(1-\gamma\right)\pi^2_{iit}-\frac{\varphi}{2}\gamma\left(\pi^{*k}_{it}\right)^2,\quad \pi^{*k}_{it}=\frac{P^{*k}_{it}}{P^{*k}_{it-1}}-1.$$

For the sake of brevity, we omit from the set of constraints the relative demand for inputs (5), the no-arbitrage condition (3), the definition of domestic inflation, and all the price setting conditions (6)-(7) because it can be shown that none of them bind in equilibrium (similar to the proof of Proposition 1 in Section A.2.2).

We proceed in two steps. First, for given currency choice k, we take first order conditions with respect to all other variables and find their optimal values. Second, we plug these optimal values — for all variables except for k — back into the policy problem and show that the social currency choice problem is equivalent to the private one. For the first step, denote the Lagrange multipliers as in Section A.2.2, ν^d_t, ν^e_t, ρ_t, λ^c_t, λ^d_t, λ^e_t, ϑ_t, ζ^*_t. It can then be shown that the system of first order conditions is satisfied under the optimal monetary policy that stabilizes domestic prices $\pi_{iit}=0$. The optimal dynamics of export prices coincides with the export Phillips curve (A7), and the values of Lagrange multipliers are the same as in Section A.2.2, including $\nu^e_t=-U_{Nit}/\left(A^*_{it}G_L\left(L^*_{it},X^e_{it}\right)\right)$, $\rho_t=-U_{Cit}\mathcal{E}_{it}/P^c_{it}$, $\vartheta_t=U_{Nit}$, and $\zeta^*_t=\varphi\gamma\pi^{*k}_{it}\left(-U_{Nit}\right)$.

For the second step, we explicitly formulate the Lagrangian including, for brevity, only those terms that

[36] We focus primarily on exporters: as explained in Section 4.2, domestic invoicing is optimal anyway, while import prices are largely exogenous to a small open economy.

depend on currency k:

$$\mathcal{L} \equiv \max_k \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[-\nu_t^e d \left(\frac{P_{it}^{*k}/\mathcal{E}_{kt}}{\mathcal{P}_t^*} \right) Y_t^* - \rho_t P_{it}^{*k}/\mathcal{E}_{kt} d \left(\frac{P_{it}^{*k}/\mathcal{E}_{kt}}{\mathcal{P}_t^*} \right) Y_t^* \right.$$
$$\left. + \vartheta_t \frac{\varphi}{2} \gamma \left(\pi_{it}^{*k} \right)^2 + \zeta_t^* \left(\pi_{it}^{*k} - \frac{P_{it}^{*k}}{P_{it-1}^{*k}} + 1 \right) + ... \right].$$

Plug in the values of Lagrange multipliers and rewrite it as

$$\mathcal{L} = \max_k \mathbb{E} \sum_{t=0}^{\infty} \beta^t \frac{U_{Cit}}{P_{it}^c} \left[- \frac{-U_{Nit} P_{it}^c}{A_{it}^* U_{Cit} G_L \left(L_{it}^*, X_{it}^e \right)} d \left(\frac{P_{it}^{*k}/\mathcal{E}_{kt}}{\mathcal{P}_t^*} \right) Y_t^* \right.$$
$$\left. + \mathcal{E}_{it} P_{it}^{*k}/\mathcal{E}_{kt} d \left(\frac{P_{it}^{*k}/\mathcal{E}_{kt}}{\mathcal{P}_t^*} \right) Y_t^* - \frac{-U_{Nit}}{U_{Cit}} P_{it}^c \frac{\varphi}{2} \gamma \left(\pi_{it}^{*k} \right)^2 + ... \right].$$

Recall the labor supply condition (1), the definition of the nominal stochastic discount factor (2), and the definition of the marginal costs

$$MC_{it}^e = P_{it}^e \frac{h^e \left(\frac{-U_{Nit}}{U_{Cit}} \frac{P_{it}^c}{P_{it}^e} \right)}{A_{it}^*} = \frac{-U_{Nit}}{U_{Cit}} \frac{P_{it}^c}{A_{it}^*} \frac{1}{G_L \left(L_{it}^*, X_{it}^e \right)},$$

and use them to rewrite the Lagrangian as follows:

$$\mathcal{L} = \max_k \mathbb{E} \sum_{t=0}^{\infty} \Theta_{i0,t} \left[\left(\mathcal{E}_{it} P_{it}^{*k}/\mathcal{E}_{kt} - MC_{it}^e \right) d \left(\frac{P_{it}^{*k}/\mathcal{E}_{kt}}{\mathcal{P}_t^*} \right) Y_t^* - \gamma \frac{\varphi}{2} \left(\pi_{it}^{*k} \right)^2 W_{it} + ... \right].$$

This expression is the same as exporters' profits. Moreover, as we have shown in the first step, both the planner and exporters choose the same dynamics of export prices conditional on currency k. Thus, the two currency choice problems are equivalent and have the same solution.[37]

A.3.3 Proof of Proposition 5

We start with the price-setting condition for domestic dollar prices. The firm's problem is

$$\max_{\{P_t\}} \mathbb{E} \sum_{t=0}^{\infty} \Theta_{i0,t} \left[\left(\mathcal{E}_{it} P_t - \tau_{ii}^* MC_{it} \right) \gamma^* \left(\frac{P_t}{P_{iit}^*} \right)^{-\varepsilon} \left(\frac{\mathcal{E}_{it} P_{iit}^*}{P_{it}} \right)^{-\theta} \left(C_{it} + X_{it} \right) - \tau_{ii}^{*R} \gamma^* \frac{\varphi}{2} \left(\frac{P_t}{P_{t-1}} - 1 \right)^2 W_{it} \right],$$

where we normalize the Rotemberg costs by the demand parameter γ^* and allow for both production subsidy τ_{ii}^* and price-adjustment subsidy τ_{ii}^{*R}. The first-order conditions lead to the following Phillips curve:

$$\pi_{iit}^* \left(\pi_{iit}^* + 1 \right) \left(-U_{Nit} \right) = \beta \mathbb{E}_t \pi_{iit+1}^* \left(\pi_{iit+1}^* + 1 \right) \left(-U_{Nit+1} \right)$$
$$- \frac{\varepsilon - 1}{\tau_{ii}^{*R} \varphi} \left(\mathcal{E}_{it} P_{iit}^* - MC_{it} \right) \frac{U_{Cit}}{P_{it}} \left(\frac{\mathcal{E}_{it} P_{iit}^*}{P_{it}} \right)^{-\theta} \left(C_{it} + X_{it} \right), \tag{A8}$$

[37] Note that our proof is effectively an application of the envelope theorem: when solving the optimal invoicing problem, it is sufficient to focus on the direct effect of currency choice because all indirect ones are equal to zero according to the first order conditions.

where we assumed that the production subsidy eliminates monopolistic markups, $\frac{\varepsilon}{\varepsilon-1}\tau^*_{ii} = 1$.

The planner's problem is

$$\max_{\left\{C_{it},X_{it},L_{it},N_{it},P_{iit},P_{it},P^*_{it},P^*_{iit},\mathcal{E}_{it},\pi_{iit},\pi^*_{it},\pi^*_{iit},\left\{B^h_{it+1}\right\}_h\right\}_t} \mathbb{E}\sum_{t=0}^{\infty}\beta^t U\left(C_{it},N_{it},\xi_{it}\right)$$

$$\text{s.t. } A_{it}F\left(L_{it},X_{it}\right) = \left(1-\gamma^*-\gamma\right)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right)+\gamma^*\left(\frac{\mathcal{E}_{it}P^*_{iit}}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right)+\gamma\left(\frac{P^*_{it}}{P^*_t}\right)^{-\varepsilon}Y^*_t,$$

$$\sum_{h\in H_t}\mathcal{Q}^h_t B^h_{it+1}-\sum_{h\in H_{t-1}}\left(\mathcal{Q}^h_t+D^h_t\right)B^h_{it} = \gamma P^*_{it}\left(\frac{P^*_{it}}{P^*_t}\right)^{-\varepsilon}Y^*_t-\gamma P^*_t\left(\frac{\mathcal{E}_{it}P^*_t}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right)+\psi_{it},$$

$$L_{it} = N_{it}-\frac{\varphi}{2}\left(1-\gamma^*-\gamma\right)\pi^2_{iit}-\frac{\varphi}{2}\gamma^*\pi^{*2}_{iit}-\frac{\varphi}{2}\gamma\pi^{*2}_{it},$$

$$1 = \left(1-\gamma^*-\gamma\right)\left(\frac{P_{iit}}{P_{it}}\right)^{1-\theta}+\gamma^*\left(\frac{\mathcal{E}_{it}P^*_{iit}}{P_{it}}\right)^{1-\theta}+\gamma\left(\frac{\mathcal{E}_{it}P^*_t}{P_{it}}\right)^{1-\theta},$$

$$\pi^*_{iit} = \frac{P^*_{iit}}{P^*_{iit-1}}-1, \quad \pi^*_{it} = \frac{P^*_{it}}{P^*_{it-1}}-1.$$

Following the primal approach, we have excluded the no-arbitrage condition and the export price-setting because the planner is free to choose capital controls and production subsidies to exporters. Further, we guess (and verify below) that the relative input demand, the definition of domestic inflation, and all price-setting conditions do not bind, and thus can be excluded from the planner's problem. Denote the Lagrange multipliers for the remaining constraints with ν_{it}, ρ_{it}, ϑ_{it}, λ_{it}, ζ^*_{iit}, ζ^*_{it} and take the first-order conditions:

- wrt C_{it}:

$$U_{Cit}-\nu_{it}\left(1-\gamma^*-\gamma\right)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}-\nu_{it}\gamma^*\left(\frac{\mathcal{E}_{it}P^*_{iit}}{P_{it}}\right)^{-\theta}+\rho_{it}\gamma P^*_t\left(\frac{\mathcal{E}_{it}P^*_t}{P_{it}}\right)^{-\theta} = 0,$$

- wrt X_{it}:

$$\nu_{it}A_{it}F_X\left(L_{it},X_{it}\right)-\nu_{it}\left(1-\gamma^*-\gamma\right)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}-\nu_{it}\gamma^*\left(\frac{\mathcal{E}_{it}P^*_{iit}}{P_{it}}\right)^{-\theta}+\rho_{it}\gamma P^*_t\left(\frac{\mathcal{E}_{it}P^*_t}{P_{it}}\right)^{-\theta} = 0,$$

- wrt L_{it}:

$$\nu_{it}A_{it}F_L\left(L_{it},X_{it}\right)+\vartheta_{it} = 0,$$

- wrt N_{it}:

$$U_{Nit} = \vartheta_{it},$$

- wrt P_{iit}:

$$\nu_{it}\theta P^{-1}_{iit}\left(1-\gamma^*-\gamma\right)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right) = \lambda_{it}\left(1-\gamma^*-\gamma\right)\left(1-\theta\right)P^{-1}_{iit}\left(\frac{P_{iit}}{P_{it}}\right)^{1-\theta},$$

- wrt P_{it}:

$$-\nu_{it}\left(1-\gamma^{*}-\gamma\right)\theta P_{it}^{-1}\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right)-\nu_{it}\gamma^{*}\theta P_{it}^{-1}\left(\frac{\mathcal{E}_{it}P_{iit}^{*}}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right)$$
$$+\rho_{it}\gamma P_{t}^{*}\theta P_{it}^{-1}\left(\frac{\mathcal{E}_{it}P_{t}^{*}}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right)-\lambda_{it}\left(\theta-1\right)P_{it}^{-1}=0,$$

- wrt P_{it}^{*}:

$$\nu_{it}\varepsilon P_{it}^{*-1}\gamma\left(\frac{P_{it}^{*}}{P_{t}^{*}}\right)^{-\varepsilon}Y_{t}^{*}-\rho_{it}\left(1-\varepsilon\right)\gamma\left(\frac{P_{it}^{*}}{P_{t}^{*}}\right)^{-\varepsilon}Y_{t}^{*}-\zeta_{it}^{*}\frac{1}{P_{it-1}^{*}}+\beta\mathbb{E}_{t}\zeta_{it+1}^{*}\frac{P_{it+1}^{*}}{P_{it}^{*2}}=0,$$

- wrt P_{iit}^{*}:

$$\nu_{it}\theta P_{iit}^{*-1}\gamma^{*}\left(\frac{\mathcal{E}_{it}P_{iit}^{*}}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right)-\lambda_{it}\gamma^{*}\left(1-\theta\right)P_{iit}^{*-1}\left(\frac{\mathcal{E}_{it}P_{iit}^{*}}{P_{it}}\right)^{1-\theta}$$
$$-\zeta_{iit}^{*}\frac{1}{P_{iit-1}^{*}}+\beta\mathbb{E}_{t}\zeta_{iit+1}^{*}\frac{P_{iit+1}^{*}}{P_{iit}^{*2}}=0,$$

- wrt $\mathcal{E}_{it}$:

$$\nu_{it}\theta\mathcal{E}_{it}^{-1}\gamma^{*}\left(\frac{\mathcal{E}_{it}P_{iit}^{*}}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right)-\rho_{it}\theta\mathcal{E}_{it}^{-1}\gamma P_{t}^{*}\left(\frac{\mathcal{E}_{it}P_{t}^{*}}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right)$$
$$-\lambda_{it}\gamma^{*}\left(1-\theta\right)\mathcal{E}_{it}^{-1}\left(\frac{\mathcal{E}_{it}P_{iit}^{*}}{P_{it}}\right)^{1-\theta}-\lambda_{it}\gamma\left(1-\theta\right)\mathcal{E}_{it}^{-1}\left(\frac{\mathcal{E}_{it}P_{t}^{*}}{P_{it}}\right)^{1-\theta}=0,$$

- wrt B_{it+1}^{h}:

$$\rho_{it}\mathcal{Q}_{t}^{h}=\beta\mathbb{E}_{t}\rho_{it+1}\left(\mathcal{Q}_{t+1}^{h}+D_{t+1}^{h}\right),$$

- wrt π_{iit}:

$$\vartheta_{it}\varphi\left(1-\gamma^{*}-\gamma\right)\pi_{iit}=0,$$

- wrt π_{iit}^{*}:

$$\zeta_{iit}^{*}=-\vartheta_{it}\varphi\gamma^{*}\pi_{iit}^{*},$$

- wrt π_{it}^{*}:

$$\zeta_{it}^{*}=-\vartheta_{it}\varphi\gamma\pi_{it}^{*}.$$

From our guesses, we immediately get the domestic price stabilization, $\pi_{iit}=0$. Combine the FOCs wrt C_{it} and X_{it} to get $\nu_{it}=U_{Cit}/\left(A_{it}F_{X}\left(L_{it},X_{it}\right)\right)$. Together with the FOC wrt L_{it}, it implies the relative demand for inputs (5) and confirms our guess that this condition does not bind.

Next, combine the FOCs wrt P_{it} and $\mathcal{E}_{it}$ to eliminate ρ_{it}. We get $\lambda_{it}=\nu_{it}\frac{\theta}{1-\theta}\frac{P_{it}}{P_{iit}}\left(C_{it}+X_{it}\right)$ and plug it back into one of these FOCs to obtain

$$\nu_{it}\gamma^*\left(\frac{\mathcal{E}_{it}P^*_{iit}}{P_{it}}\right)^{-\theta}-\nu_{it}\frac{P_{it}}{P_{iit}}\left[1-(1-\gamma^*-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{1-\theta}\right]=\rho_{it}\gamma P^*_t\left(\frac{\mathcal{E}_{it}P^*_t}{P_{it}}\right)^{-\theta}.$$

Together with the FOC wrt C_{it}, this equation results in $\nu_{it}=P_{iit}U_{Cit}/P_{it}$, which combined with our previous conditions implies domestic price stabilization. Then it is straightforward to check that the FOC wrt P_{iit} is also satisfied, confirming our earlier guess that the domestic price-setting condition does not bind.

From the same set of FOCs we can back out ρ_{it}:

$$\rho_{it}=-U_{Cit}\left[\frac{\gamma^*}{\gamma}\frac{\mathcal{E}_{it}P^*_{iit}-P_{iit}}{P_{it}P^*_t}\left(\frac{P^*_{iit}}{P^*_t}\right)^{-\theta}+\frac{\mathcal{E}_{it}}{P_{it}}\right].$$

Then the optimal portfolio allocation can be found from the FOC wrt B^h_{it+1},

$$\mathbb{E}_t\Theta_{it,t+1}\frac{\frac{\gamma^*}{\gamma}\frac{\mathcal{E}_{it+1}P^*_{iit+1}-P_{iit+1}}{\mathcal{E}_{it+1}P^*_{t+1}}\left(\frac{P^*_{iit+1}}{P^*_{t+1}}\right)^{-\theta}+1}{\frac{\gamma^*}{\gamma}\frac{\mathcal{E}_{it}P^*_{iit}-P_{iit}}{\mathcal{E}_{it}P^*_t}\left(\frac{P^*_{iit}}{P^*_t}\right)^{-\theta}+1}\frac{\mathcal{E}_{it+1}}{\mathcal{E}_{it}}\frac{\mathcal{Q}^h_{t+1}+D^h_{t+1}}{\mathcal{Q}^h_t}=1.$$

Comparing this condition with private risk sharing allows us to back out the values of capital controls τ^c_{it}.

The optimal dynamics of domestic dollar prices can be found from the FOCs wrt P^*_{iit} and π^*_{iit}:

$$\begin{aligned}\pi^*_{iit}\left(\pi^*_{iit}+1\right)\left(-U_{Nit}\right)) &= \beta\mathbb{E}_t\pi^*_{iit+1}\left(\pi^*_{iit+1}+1\right)\left(-U_{Nit+1}\right)\\ &\quad-\frac{\theta}{\varphi}\left(\mathcal{E}_{it}P^*_{iit}-MC_{it}\right)\frac{U_{Cit}}{P_{it}}\left(\frac{\mathcal{E}_{it}P^*_{iit}}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right).\end{aligned}$$

This coincides with the private sector price-setting condition (A8) under $\tau^{*R}_{ii}=(\varepsilon-1)/\theta$. Similarly, the optimal dynamics of export prices follows from the FOCs wrt P^*_{it} and π^*_{it}:

$$\begin{aligned}\pi^*_{it}\left(\pi^*_{it}+1\right)\left(-U_{Nit}\right) &= \beta\mathbb{E}_t\pi^*_{it+1}\left(\pi^*_{it+1}+1\right)\left(-U_{Nit+1}\right)\\ &\quad-\kappa\left(\mathcal{E}_{it}P^*_{it}-\frac{\varepsilon}{\varepsilon-1}MC_{it}-\frac{\gamma^*}{\gamma}\frac{P_{iit}-\mathcal{E}_{it}P^*_{iit}}{P^*_t}\left(\frac{P^*_{iit}}{P^*_t}\right)^{-\theta}P^*_{it}\right)\frac{U_{Cit}}{P_{it}}\frac{Y^*_{it}}{\gamma}.\end{aligned}$$

Using the value of capital controls τ^c_{it}, one can see this condition is equivalent to subsidizing export revenues in the same way as revenues from financial assets.

To conclude, we have shown that there exists a set of Lagrange multipliers such that all first-order conditions and constraints of the policy problem are satisfied under the proposed policy.

A.4 Proofs for Section 5

A.4.1 Complete markets

For the case of complete markets, we consider the full set of Arrow-Debreu securities and formulate the single consumer's budget constraint (instead of a recursive formulation),

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t Z_t \mathcal{E}_{it}^{-1}\left[W_{it}N_{it}+\Pi_{it}^f+T_{it}-P_{it}C_{it}\right]=0,$$

where with a slight abuse of the notation, Z_t denotes the period-zero price of an Arrow-Debreu security that pays one dollar in period t for a given history of shocks. Then the no-arbitrage condition (3) is replaced with

$$\frac{U_{Cit}}{U_{Ci0}}\frac{P_{i0}}{P_{it}}\frac{\mathcal{E}_{it}}{\mathcal{E}_{i0}}\frac{1}{Z_t}=1,$$

which can be also rewritten as

$$\frac{U_{Cit}}{P_{it}}\mathcal{E}_{it}=\Lambda_i Z_t, \tag{A9}$$

where Λ_i is the country i's consumers' Lagrange multiplier on the budget constraint. Intuitively, the constant Λ_i determines the wealth of country i relative to the rest of the world. In equilibrium, all non-US countries are ex-ante symmetric, and therefore $\Lambda_i=1$ for $i\neq 0$. The country's budget constraint (10) is then

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t Z_t\gamma\left[P_{it}^*\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}Y_t^*-P_t^*\left(\frac{P_t^*}{P_{it}}\right)^{-\theta}(C_{it}+X_{it})\right]=0.$$

A.4.2 Proof of Proposition 6

Because of Assumption A3, the problem is effectively static with agents setting prices and sharing the risk before the realization of shocks. To simplify notation, we therefore, focus on a one-period version of the model. To prove the proposition, we first formulate the policy problem and take the non-linear first-order conditions. We then find the non-stochastic steady state, and linearize optimality conditions around it. The solution of this system describes the optimal (linear) policy.

Policy problem and optimality conditions The U.S. policy problem has two blocks of constraints: the local and the global. The local block is exactly the same as for any other country. The global block includes equilibrium conditions for all other countries, as well as their optimal policy.[38]

To be more precise, for each country j, the global block consists of the export price setting, the relative demand for inputs, the domestic price stabilization, the domestic price index, the risk-sharing condition, and the market clearing. On top of it, we add the definition of the global aggregate Y_t^*, so that the global block becomes

$$\mathbb{E}\left(\frac{\mathcal{E}_{jt}P_t^*}{P_{jt}}-\frac{\varepsilon}{\varepsilon-1}h\left(\frac{-U_{Njt}}{U_{Cjt}}\right)/A_{jt}\right)U_{Cjt}Y_t^*=0,$$

[38] It can be shown that all global balances follow from these constraints.

$$\frac{X_{jt}}{N_{jt}} = g\left(\frac{-U_{Njt}}{U_{Cjt}}\right), \qquad \frac{P_{jjt}}{P_{jt}} = h\left(\frac{-U_{Njt}}{U_{Cjt}}\right)/A_{jt},$$

$$(1-\gamma)\left(\frac{P_{jjt}}{P_{jt}}\right)^{1-\theta} + \gamma\left(\frac{\mathcal{E}_{jt}P_t^*}{P_{jt}}\right)^{1-\theta} = 1,$$

$$\frac{\mathcal{E}_{jt}U_{Cjt}}{P_{jt}} = Z_t,$$

$$A_{jt}F\left(N_{jt}, X_{jt}\right) = (1-\gamma)\left(\frac{P_{jjt}}{P_{jt}}\right)^{-\theta}\left(C_{jt}+X_{jt}\right) + \gamma Y_t^*,$$

$$Y_t^* = \int\left(\frac{\mathcal{E}_{jt}P_t^*}{P_{jt}}\right)^{-\theta}\left(C_{jt}+X_{jt}\right)\mathrm{d}j,$$

where we have already used the ex-ante symmetry of all non-U.S. countries (leading to $P_{it}^* = P_t^*$ and $\Lambda_j = 1$). Each country j has six local variables in this block: $\mathcal{E}_{jt}$,P_{jt}, N_{jt}, C_{jt}, X_{jt}. P_{jjt}. Without loss of generality, normalize stabilized domestic prices to $P_{jjt} = 1$. The remaining five constraints can then be used to solve for the five local variables as functions of the remaining global variables: P_t^*, Y_t^*, Z_t. Similarly, the last condition defines the global demand as a function of the other global variables, $Y_t^* = Y_t^*\left(P_t^*, Z_t\right)$. Thus, the global block reduces to this condition and the export price setting, which we denote as $\mathbb{E}\,\Omega\left(P_t^*, Z_t\right) = 0$.

The full U.S. policy problem can then be written as

$$\max_{C_{it},X_{it},L_{it},N_{it},P_{iit},P_{it},P_{it}^*,Z_t,P_t^*} \mathbb{E}\,U\left(C_{it}, N_{it}, \xi_{it}\right)$$

$$\text{s.t. } A_{it}F\left(L_{it}, X_{it}\right) = (1-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right) + \gamma\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} Y_t^*\left(P_t^*, Z_t\right),$$

$$\frac{X_{it}}{L_{it}} = g\left(\frac{-U_{Nit}}{U_{Cit}}\right), \qquad L_{it} = N_{it},$$

$$\mathbb{E}\gamma Z_t\left[P_{it}^*\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} Y_t^*\left(P_t^*, Z_t\right) - P_t^*\left(\frac{P_t^*}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right)\right] = 0,$$

$$(1-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{1-\theta} + \gamma\left(\frac{P_t^*}{P_{it}}\right)^{1-\theta} = 1, \qquad \frac{U_{Cit}}{P_{it}} = \Lambda_i Z_t,$$

$$\mathbb{E}\left(\frac{P_{iit}}{P_{it}} - h\left(\frac{-U_{Nit}}{U_{Cit}}\right)/A_{it}\right)U_{Cit}\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right) = 0,$$

$$\mathbb{E}\left(\frac{P_{it}^*}{P_{it}} - \frac{\varepsilon}{\varepsilon-1}h\left(\frac{-U_{Nit}}{U_{Cit}}\right)/A_{it}\right)U_{Cit}\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} Y_t^*\left(P_t^*, Z_t\right) = 0,$$

$$\mathbb{E}\Omega\left(P_t^*, Z_t\right) = 0.$$

This problem is different from the one in non-U.S. economies in several dimensions. First, the U.S. can choose global variables P_t^* and Z_t, and also has one more ex-ante constraint. Second, by construction $\mathcal{E}_{it} = 1$, and the U.S. policy is effectively choosing state-dependent Z_t. Also, because of the ex-ante symmetry of other countries, it is sufficient to focus on the export price setting of just one non-U.S. economy.

Following previous sections, let's denote the Lagrange multipliers as ν_{it}, η_{it}, ϑ_{it}, ρ_i, λ_{it}, χ_{it}, μ_{ii}, μ_i^*, μ_j^*.[39] Then the system of first-order conditions is:

- wrt C_{it}:

$$U_{Cit} - \nu_{it}(1-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta} - \eta_{it}g'\left(\frac{-U_{Nit}}{U_{Cit}}\right)\frac{-U_{NCit}U_{Cit}+U_{Nit}U_{CCit}}{U_{Cit}^2}$$
$$-\rho_i\gamma Z_t P_t^*\left(\frac{P_t^*}{P_{it}}\right)^{-\theta} + \chi_{it}\frac{U_{CCit}}{P_{it}} + \mu_{ii}[...] + \mu_i^*[...] = 0,$$

- wrt X_{it}:

$$\nu_{it}A_{it}F_X(L_{it},X_{it}) - \nu_{it}(1-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta} + \eta_{it}\frac{1}{L_{it}} - \rho_i Z_t\gamma P_t^*\left(\frac{P_t^*}{P_{it}}\right)^{-\theta} + \mu_{ii}[...] + \mu_i^*[...] = 0,$$

- wrt L_{it}:

$$\nu_{it}A_{it}F_L(L_{it},X_{it}) - \eta_{it}\frac{X_{it}}{L_{it}^2} + \vartheta_{it} + \mu_{ii}[...] + \mu_i^*[...] = 0,$$

- wrt N_{it}:

$$U_{Nit} - \eta_{it}g'\left(\frac{-U_{Nit}}{U_{Cit}}\right)\frac{-U_{NNit}U_{Cit}+U_{Nit}U_{CNit}}{U_{Cit}^2} - \vartheta_{it} + \chi_{it}\frac{U_{CNit}}{P_{it}} + \mu_{ii}[...] + \mu_i^*[...] = 0,$$

- wrt (state-invariant) P_{iit}:

$$\mathbb{E}\left[\nu_{it}(1-\gamma)\theta\frac{1}{P_{iit}}\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}(C_{it}+X_{it}) - (1-\gamma)\lambda_{it}(1-\theta)\frac{1}{P_{iit}}\left(\frac{P_{iit}}{P_{it}}\right)^{1-\theta} + \mu_{ii}[...]\right] = 0,$$

- wrt (state-invariant) P_{it}:

$$\mathbb{E}\left[-\nu_{it}(1-\gamma)\theta\frac{1}{P_{it}}\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}(C_{it}+X_{it}) - \rho_i\gamma Z_t P_t^*\theta\frac{1}{P_{it}}\left(\frac{P_t^*}{P_{it}}\right)^{-\theta}(C_{it}+X_{it}) + \mu_{ii}[...]\right.$$
$$\left.-\chi_{it}\frac{U_{Cit}}{P_{it}^2} - \lambda_{it}(1-\gamma)(\theta-1)\frac{1}{P_{it}}\left(\frac{P_{iit}}{P_{it}}\right)^{1-\theta} - \lambda_{it}\gamma(\theta-1)\frac{1}{P_{it}}\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{1-\theta} + \mu_i^*[...]\right] = 0,$$

- wrt (state-invariant) P_{it}^*:

$$\mathbb{E}\left[\nu_{it}\gamma\varepsilon\frac{1}{P_{it}^*}\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}Y_t^* + \rho_i\gamma Z_t(1-\varepsilon)\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}Y_t^* + \mu_i^*[...]\right] = 0,$$

- wrt (state-invariant) Λ_i:

$$\mathbb{E}[\chi_{it}Z_t] = 0,$$

[39] The Lagrange multipliers without time subscripts are state-invariant constants, while Lagrange multipliers with time subscripts vary state-by-state.

- wrt Z_t:

$$-\nu_{it}\gamma\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}\frac{\partial Y_t^*\left(P_t^*,Z_t\right)}{\partial Z_t}+\rho_i\gamma\left[P_{it}^*\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}Y_t^*-P_t^*\left(\frac{P_t^*}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right)\right]$$
$$+\rho_i\gamma Z_tP_{it}^*\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}\frac{\partial Y_t^*\left(P_t^*,Z_t\right)}{\partial Z_t}-\chi_{it}\Lambda_i+\mu_i^*\left[...\right]+\mu_j^*\frac{\partial\Omega\left(P_t^*,Z_t\right)}{\partial Z_t}=0,$$

- wrt (state-invariant) P_t^*:

$$\mathbb{E}\left[-\nu_{it}\gamma\varepsilon P_t^{*-1}\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}Y_t^*-\nu_{it}\gamma\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}\frac{\partial Y_t^*\left(P_t^*,Z_t\right)}{\partial P_t^*}+\rho_i\gamma Z_tP_{it}^*\varepsilon P_t^{*-1}\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}Y_t^*\right.$$
$$-\rho_i\gamma Z_t\left(1-\theta\right)\left(\frac{P_t^*}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right)+\rho_i\gamma Z_tP_{it}^*\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}\frac{\partial Y_t^*\left(P_t^*,Z_t\right)}{\partial P_t^*}$$
$$\left.-\lambda_{it}\gamma\left(1-\theta\right)P_t^{*-1}\left(\frac{P_t^*}{P_{it}}\right)^{1-\theta}+\mu_i^*\left[...\right]+\mu_j^*\frac{\partial\Omega\left(P_t^*,Z_t\right)}{\partial P_t^*}\right]=0,$$

where we omitted terms with μ_{ii} and μ_i^* for the reasons discussed below.

Non-stochastic steady state We drop time subscript to denote the values of variables in a non-stochastic steady-state. Because the price stickiness does not play any role in the non-stochastic steady state, the U.S. is symmetric to all other countries. It follows that $\Lambda_i=1$, $P_i^*=P^*$, the law of one price holds up to a production subsidy, $P_i^*=\frac{\varepsilon}{\varepsilon-1}P_{ii}$, and domestic prices are equal to the marginal costs, $P_{ii}=P_i/\left(A_iF_X\left(L_i,X_i\right)\right)$. Then it is straightforward to verify that the system of FOCs and constraints holds under $\eta_i=\chi_i=\mu_{ii}=\mu_i^*=0$, $\vartheta_i=U_{Ni}$, $\rho_i=1$, $\nu_i=U_{Ci}P_{ii}/P_i$, $\lambda_i=\frac{\theta}{1-\theta}U_{Ci}\left(C_i+X_i\right)$, and two global conditions

$$\frac{U_{Ci}}{P_i}\gamma\left(P_{ii}-P_i^*\right)\frac{\partial Y^*}{\partial Z}=\mu_j^*\frac{\partial\Omega}{\partial Z},\qquad\frac{U_{Ci}}{P_i}\gamma\left(P_{ii}-P_i^*\right)\frac{\partial Y^*}{\partial P^*}=\mu_j^*\frac{\partial\Omega}{\partial P^*}.\tag{A10}$$

First-order approximations Note that the state-invariant Largrange multipliers (ρ_i, μ_{ii}, μ_i^*, μ_j^*) are of the second order and hence, up to the first order, are equal to their steady-state values. In particular, this means that we do not need to take derivatives of the local price-setting conditions since both μ_{ii} and μ_i^* are zero to the first order. The same argument applies to state-invariant variables such as Λ_i, P_{iit}, P_{it}, P_{it}^*, P_t^* and therefore, we do not need to take approximations of the FOCs wrt state-invariant variables.

We denote the first-order deviations of variables with small letters, e.g. c_{it} is a linear approximation of C_{it}. For Greek letters (Lagrange multipliers), we keep the original letters. Then the approximations to the remaining FOCs are:

- wrt C_{it}:

$$U_{CCi}c_{it}+U_{CNi}n_{it}-\eta_{it}g'\left(\frac{-U_{Ni}}{U_{Ci}}\right)\frac{-U_{NCi}U_{Ci}+U_{Ni}U_{CCi}}{U_{Ci}^2}$$
$$-\nu_{it}\left(1-\gamma\right)\left(\frac{P_{ii}}{P_i}\right)^{-\theta}-\gamma z_tP^*\left(\frac{P^*}{P_i}\right)^{-\theta}+\chi_{it}\frac{U_{CCi}}{P_i}=0,$$

- wrt X_{it}:

$$\nu_{it} A_i F_X \left(L_i, X_i\right) + U_{Ci} \frac{P_{ii}}{P_i} a_{it} F_X \left(L_i, X_i\right) + U_{Ci} \frac{P_{ii}}{P_i} A_i F_{XL} \left(L_i, X_i\right) l_{it}$$
$$+U_{Ci} \frac{P_{ii}}{P_i} A_i F_{XX} \left(L_i, X_i\right) x_{it} - \nu_{it} \left(1-\gamma\right) \left(\frac{P_{ii}}{P_i}\right)^{-\theta} + \eta_{it} \frac{1}{L_i} - z_t \gamma P^* \left(\frac{P^*}{P_i}\right)^{-\theta} = 0,$$

- wrt L_{it}:

$$\nu_{it} A_i F_L \left(L_i, X_i\right) + U_{Ci} \frac{P_{ii}}{P_i} a_{it} F_L \left(L_i, X_i\right) + U_{Ci} \frac{P_{ii}}{P_i} A_i F_{LL} \left(L_i, X_i\right) l_{it}$$
$$+U_{Ci} \frac{P_{ii}}{P_i} A_i F_{LX} \left(L_i, X_i\right) x_{it} - \eta_{it} \frac{X_i}{L_i^2} + \vartheta_{it} = 0,$$

- wrt N_{it}:

$$U_{NNi} n_{it} + U_{NCi} c_{it} - \eta_{it} g' \left(\frac{-U_{Ni}}{U_{Ci}}\right) \frac{-U_{NNi} U_{Ci} + U_{Ni} U_{CNi}}{U_{Ci}^2} - \vartheta_{it} + \chi_{it} \frac{U_{CNi}}{P_i} = 0,$$

- wrt Z_t:

$$\gamma \left[P_i^* \left(\frac{P_i^*}{P^*}\right)^{-\varepsilon} c_t^* - P^* \left(\frac{P^*}{P_i}\right)^{-\theta} \left(c_{it} + x_{it}\right)\right] - \chi_{it} + \mu_j^* \frac{\partial^2 \Omega}{\partial Z^2} z_t$$
$$+\gamma \left(z_t P_i^* - \nu_{it}\right) \frac{\partial Y^* \left(P^*, Z\right)}{\partial Z} + \gamma \left(Z P_i^* - \nu_i\right) \frac{\partial^2 Y^* \left(P^*, Z\right)}{\partial Z^2} z_t = 0.$$

The first four FOCs represent the local block of the system. They can be solved for the four Lagrange multipliers: ν_{it}, η_{it}, χ_{it}, ϑ_{it}. We then plug the values of χ_{it} and ν_{it} into the last FOC, which represents the global block, to obtain the optimal policy rule. We proceed to implement these steps.

Local block This linear system of four equations can be solved in terms of four variables: ν_{it}, η_{it}, χ_{it}, ϑ_{it}. The solution becomes simpler if we use two more equilibrium conditions. First, take the first-order approximation to the relative demand for inputs, $U_{Cit} / \left(-U_{Nit}\right) = F_X \left(L_{it}, X_{it}\right) / F_L \left(L_{it}, X_{it}\right)$:

$$U_{Ci} \frac{P_{ii}}{P_i} A_i \left(F_{XL} \left(L_i, X_i\right) - \frac{U_{Ci}}{-U_{Ni}} F_{LL} \left(L_i, X_i\right)\right) l_{it} - \left(U_{CCi} + U_{NCi} \frac{U_{Ci}}{-U_{Ni}}\right) c_{it}$$
$$+U_{Ci} \frac{P_{ii}}{P_i} A_i \left(F_{XX} \left(L_i, X_i\right) - \frac{U_{Ci}}{-U_{Ni}} F_{LX} \left(L_i, X_i\right)\right) x_{it} - \left(\frac{U_{Ci}}{-U_{Ni}} U_{NNi} + U_{CNi}\right) n_{it} = 0.$$

Second, linearize the U.S. risk-sharing condition, $U_{Cit} / P_{it} = Z_t$:

$$U_{CCi} c_{it} + U_{CNi} n_{it} = P_i z_t.$$

Using these conditions, the values of Lagrange multipliers can be shown to be

$$\nu_{it} = z_t P_{ii} - \frac{\Phi P_{ii} U_{Ci}/P_i}{\gamma \left(\frac{P^*}{P_i}\right)^{1-\theta} (\Phi - 1) + 1} p_{iit},$$

$$\eta_{it} = \frac{\gamma \left(\frac{P^*}{P_i}\right)^{1-\theta} - 1}{\gamma \left(\frac{P^*}{P_i}\right)^{1-\theta} (\Phi - 1) + 1} L_i U_{Ci} p_{iit},$$

$$\chi_{it} = \eta_{it} \frac{\Phi}{L_i} \frac{P_i}{U_{CCi}} + \eta_{it} g' \left(\frac{-U_{Ni}}{U_{Ci}}\right) \left(\frac{U_{CCi}}{U_{Ci}} - \frac{U_{CNi}}{U_{Ni}}\right) \frac{U_{Ni}}{U_{Ci}} \frac{P_i}{U_{CCi}},$$

where p_{iit} is defined as a linear deviation of $\frac{P_{iit}}{MC_{it}}$, that is

$$p_{iit} = \frac{P_{ii}}{P_i} a_{it} F_X (L_i, X_i) + \frac{P_{ii}}{P_i} A_i F_{XX} (L_i, X_i) x_{it} + \frac{P_{ii}}{P_i} A_i F_{XL} (L_i, X_i) l_{it},$$

and Φ is a constant,

$$\Phi \equiv \frac{U_{CCi}}{U_{CNi} \frac{U_{Ci}}{-U_{Ni}} + U_{CCi}} \left(g' \left(\frac{-U_{Ni}}{U_{Ci}}\right) \frac{\frac{U_{CNi}^2}{U_{CCi}} - U_{NNi}}{-U_{Ni}} L_i + 1 + \frac{X_i}{L_i} \frac{U_{Ci}}{-U_{Ni}} \right).$$

Global block Take a linear approximation to the net exports:

$$nx_{it} \equiv P_i^* \left(\frac{P_i^*}{P^*}\right)^{-\varepsilon} c_t^* - P^* \left(\frac{P^*}{P_i}\right)^{-\theta} (c_{it} + x_{it}).$$

Then the remaining FOC can be simplified to

$$\gamma nx_{it} - \chi_{it} + \gamma (z_t P_i^* - \nu_{it}) \frac{\partial Y^*}{\partial Z} + \gamma \frac{U_{Ci}}{P_i} (P_i^* - P_{ii}) \frac{\partial^2 Y^*}{\partial Z^2} z_t + \mu_j^* \frac{\partial^2 \Omega}{\partial Z^2} z_t = 0.$$

Now plug in the Lagrange multipliers and obtain

$$\Gamma \cdot p_{iit} + \gamma \epsilon \cdot nx_{it} + \left[\gamma (P_i^* - P_{ii}) + \gamma \frac{U_{Ci}}{P_i} (P_i^* - P_{ii}) \frac{\partial^2 Y^*/\partial Z^2}{\partial Y^*/\partial Z} + \epsilon \mu_j^* \frac{\partial^2 \Omega}{\partial Z^2} \right] z_t = 0.$$

Here we denote the steady-state derivative of the global demand (determined by the U.S. monetary policy) with respect to the global SDF (given by the Arrow-Debreu price) as $1/\epsilon \equiv \partial Y^*/\partial Z$. Γ is a constant defined by

$$\Gamma \equiv (1 - \omega) \times \epsilon \frac{P_i U_{Ni}}{U_{CCi}} \left[\Phi \frac{U_{Ci}}{U_{Ni}} + g' \left(\frac{-U_{Ni}}{U_{Ci}}\right) \left(\frac{U_{CCi}}{U_{Ci}} - \frac{U_{CNi}}{U_{Ni}}\right) L_i \right] + \omega \times \left(\frac{P^*}{P_i}\right)^{\theta - 1} \frac{U_{Ci}}{P_i} P_{ii},$$

where

$$\omega \equiv \frac{\gamma \left(\frac{P^*}{P_i}\right)^{1-\theta} \Phi}{\gamma \left(\frac{P^*}{P_i}\right)^{1-\theta} \Phi + 1 - \gamma \left(\frac{P^*}{P_i}\right)^{1-\theta}}.$$

Intuitively, the first term, weighted by $1-\omega$, represents the domestic price stabilization, while the second term, weighted by ω, reflects the stabilization of export prices. Since both domestic and export prices are sticky in dollars, these policy motives turn out to be perfectly aligned.

Next, use the steady-state relationship (A10) to plug in the Lagrange multiplier μ_j^*:

$$\Gamma \cdot p_{iit} + \gamma\epsilon \cdot nx_{it} + \gamma \left(P_i^* - P_{ii}\right) \left[1 + \frac{U_{Ci}}{P_i}\frac{\partial^2 Y^*/\partial Z^2}{\partial Y^*/\partial Z} - \frac{U_{Ci}}{P_i}\frac{\partial^2 \Omega/\partial Z^2}{\partial \Omega/\partial Z}\right] z_t = 0.$$

Note that the global term is proportional to the openness of the economy γ and the markup earned by the U.S. in foreign markets, $P_i^* - P_{ii} = P_{ii}/\left(\varepsilon - 1\right) > 0$.

Finally, recall that all variables in the global block of constraints can be written as functions of P_t^* and Z_t. State-invariant P_t^* is a constant up to the first order and hence, linear approximations of all global variables can be expressed in terms of z_t alone. In particular, one can implicitly solve for the deviations of $\frac{\mathcal{E}_{jt}P_{jt}^*}{MC_{jt}}$ and find the corresponding constant Ξ.

A.4.3 Proof of Corollary 6.1

First, note that in equilibrium, the welfare is equal to the value of the Lagrangian as all constraints hold with equality. Thus, instead of comparing welfare across countries we can compare the values of the Lagrangians. Next, to eliminate the first-order differences in optimal policy across countries we consider the autarky limit $\gamma \to 0$. However, at the point of $\gamma = 0$, all countries are ex-ante symmetric and achieve the same welfare, or have the same Lagrangians, $\left(\mathcal{L}^{US} - \mathcal{L}^{nUS}\right)|_{\gamma=0} = 0$. Instead, we focus on the limit $\gamma \to 0$, as the welfare across countries starts to differ as soon as we deviate from the autarky point:

$$\lim_{\gamma\to 0}\frac{\mathcal{L}^{US} - \mathcal{L}^{nUS}}{\gamma} = \lim_{\gamma\to 0}\left(\frac{\mathrm{d}\mathcal{L}^{US}}{\mathrm{d}\gamma} - \frac{\mathrm{d}\mathcal{L}^{nUS}}{\mathrm{d}\gamma}\right) = \frac{\mathrm{d}\mathcal{L}^{US}}{\mathrm{d}\gamma}\,|_{\gamma=0} - \frac{\mathrm{d}\mathcal{L}^{nUS}}{\mathrm{d}\gamma}\,|_{\gamma=0}\,.$$

Non-U.S. Recall the policy problem of a non-U.S. economy from Section A.2.2. Write down the Lagrangian for this problem, keeping only the binding constraints:

$$\mathcal{L}^{nUS} \equiv \mathbb{E}\left[\frac{C_{it}^{1-\sigma}}{1-\sigma} - N_{it} + \nu_{it}\left(A_{it}L_{it} - (1-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta} C_{it} - \gamma\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} Y_t^*\right) + \vartheta_{it}\left(L_{it} - N_{it}\right)\right.$$
$$\left. -\rho_i\gamma Z_t\left(P_{it}^*\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} Y_t^* - P_t^*\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{-\theta} C_{it}\right) + \lambda_{it}\left(1 - (1-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{1-\theta} - \gamma\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{1-\theta}\right)\right].$$

We fix all primitives of the model as we change only the openness parameter γ and investigate how it affects the value of $\mathcal{L}^{nUS}$. Parameter γ enters $\mathcal{L}^{nUS}$ both directly and indirectly through the equilibrium values of the global variables (Y_t^*, Z_t, P_t^*) and of the local non-U.S. variables (C_{it}, N_{it}, etc.). From the envelope theorem, the effects of the latter variables are all zero: the optimality conditions for the non-U.S. economy ensure that the derivatives of the Lagrangian with respect to all local variables (including the Lagrange multipliers) are zero. Then we need to consider only the partial derivative wrt γ and the derivatives wrt all global variables:

$$\begin{aligned}\frac{\mathrm{d}\mathcal{L}^{nUS}}{\mathrm{d}\gamma} = \mathbb{E}\Bigg[&\nu_{it}\left(\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta} C_{it} - \left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} Y_t^*\right) + \lambda_{it}\left(\left(\frac{P_{iit}}{P_{it}}\right)^{1-\theta} - \left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{1-\theta}\right) \\ &- \rho_i Z_t\left(P_{it}^*\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} Y_t^* - P_t^*\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{-\theta} C_{it}\right) - \gamma\left(\nu_{it}\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} + \rho_i Z_t P_{it}^*\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}\right)\frac{\mathrm{d}Y_t^*}{\mathrm{d}\gamma} \\ &- \gamma\left(\nu_{it}\varepsilon\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} Y_t^* + \rho_i Z_t\left(P_{it}^*\varepsilon\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} Y_t^* - (1-\theta)\,P_t^*\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{-\theta} C_{it}\right)\right) P_t^{*-1}\frac{\mathrm{d}P_t^*}{\mathrm{d}\gamma} \\ &-\gamma\lambda_{it}\frac{1-\theta}{P_t^*}\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{1-\theta}\frac{\mathrm{d}P_t^*}{\mathrm{d}\gamma} - \rho_i\gamma\left(P_{it}^*\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} Y_t^* - P_t^*\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{-\theta} C_{it}\right)\frac{\mathrm{d}Z_t}{\mathrm{d}\gamma}\Bigg].\end{aligned}$$

We evaluate this derivative in the autarky limit $\gamma = 0$. Note that all terms with the derivatives of the global variables drop out. Moreover, the price index constraint (4) implies $P_{it} = P_{iit}$, and the optimal policy (the marginal cost stabilization) (A6) collapses to $C_{it}^{\sigma} = A_{it}$. Also, recall $\vartheta_{it} = -1$, $\nu_{it} = C_{it}^{-\sigma}$, and $\lambda_{it} = \frac{\theta}{1-\theta}C_{it}^{1-\sigma}$. Finally, the budget constraint implies

$$\mathbb{E}\rho_i Z_t\left(P_{it}^*\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} Y_t^* - P_t^*\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{-\theta} C_{it}\right) = 0,$$

since ρ_i is just a constant. After using all of these conditions, we arrive at

$$\frac{\mathrm{d}\mathcal{L}^{nUS}}{\mathrm{d}\gamma}\Big|_{\gamma=0} = \mathbb{E}\left[\frac{1}{1-\theta}A_{it}^{\frac{1}{\sigma}-1} - A_{it}^{-1}\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} Y_t^* - \frac{\theta}{1-\theta}A_{it}^{\frac{1}{\sigma}-1}\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{1-\theta}\right].$$

U.S. Recall from Section A.4.2 that the only differences between the U.S. problem and the non-U.S. problem are: 1) $\mathcal{E}_{it} = 1$, 2) there is an additional global constraint, and 3) the U.S. chooses global variables Y_t^*, Z_t, and P_t^*. Therefore, all global terms drop out from $\mathrm{d}\mathcal{L}^{US}/\mathrm{d}\gamma$ due to the envelope theorem. Next, recall from condition (A10) that the global constraint does not bind at the autarky point $\gamma = 0$. Crucially, the autarky limit also implies that the optimal U.S. policy is exactly the same as the non-U.S. policy and stabilizes domestic marginal costs. Therefore, repeating the same steps as above results in the same expression up to the $\mathcal{E}_{it} = 1$.

Difference Denote all U.S. variables with a subscript i and all variables of a non-U.S. country with j. Use the ex-ante symmetry of all non-U.S. countries so that $P_t^* = P_{jt}^*$, but keep $P_t^* \neq P_{it}^*$. Assume that shocks in all countries are identically distributed and hence, $\mathbb{E}A_{it}^{\frac{1}{\sigma}-1} = \mathbb{E}A_{jt}^{\frac{1}{\sigma}-1}$. Then the difference in welfare becomes

$$\frac{\mathrm{d}\left(\mathcal{L}^{US} - \mathcal{L}^{nUS}\right)}{\mathrm{d}\gamma}\Big|_{\gamma=0} = \mathbb{E}\left[\left(A_{jt}^{-1} - A_{it}^{-1}\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}\right) Y_t^* + \frac{\theta}{1-\theta}\left(A_{jt}^{\frac{1}{\sigma}-1}\left(\frac{\mathcal{E}_{jt}P_t^*}{P_{jt}}\right)^{1-\theta} - A_{it}^{\frac{1}{\sigma}-1}\left(\frac{P_t^*}{P_{it}}\right)^{1-\theta}\right)\right].$$

To get rid of the nominal exchange rate $\mathcal{E}_{jt}$, use the risk-sharing condition, which in a static model with ex-ante symmetric non-U.S. countries implies $\mathcal{E}_{jt}C_{jt}^{-\sigma}/P_{jt} = Z_t$. For the U.S., the same condition reduces to $C_{it}^{-\sigma}/P_{it} = \Lambda_i Z_t$, where Λ_i is a constant that describes the wealth of the U.S. relative to the rest of the world.

Combined with the marginal cost stabilization, this condition implies $P_{it}A_{it}\Lambda_i Z_t = 1$. Substitute these risk-sharing conditions along with $\frac{1}{\sigma} = \theta$ into the definition of the global demand:

$$Y_t^* \equiv \int \left(\frac{\mathcal{E}_{jt}P_t^*}{P_{jt}}\right)^{-\theta} C_{jt}\mathrm{d}j = P_t^{*-\theta} P_{it}^{\theta}\Lambda_i^{\theta} A_{it}^{\theta}.$$

After using these conditions, the welfare difference reduces to

$$\frac{\mathrm{d}\left(\mathcal{L}^{US} - \mathcal{L}^{nUS}\right)}{\mathrm{d}\gamma}\,|_{\gamma=0} = \mathbb{E}\left[\left(A_{it}^{\theta}A_{jt}^{-1} - A_{it}^{\theta-1}\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}\right) P_t^{*-\theta}P_{it}^{\theta}\Lambda_i^{\theta} + \frac{\theta}{\theta-1}\left(\frac{P_t^*}{P_{it}}\right)^{1-\theta} A_{it}^{\theta-1}\left(1-\Lambda_i^{\theta-1}\right)\right].$$

To get rid of prices P_t^* and P_{it}^*, we use the U.S. export price setting, which under domestic marginal cost stabilization is just $P_{it}^* = \frac{\varepsilon}{\varepsilon-1}P_{iit}$, and the non-U.S. export price setting (see Section A.4.2), which under the optimal policy collapses to

$$\mathbb{E}\left(\mathcal{E}_{jt}P_t^* - \frac{\varepsilon}{\varepsilon-1}P_{jjt}\right)\frac{C_{jt}^{-\sigma}}{P_{jt}}Y_t^* = 0.$$

Once again, substitute in the risk-sharing, other conditions from above, and $\frac{1}{\sigma} = \theta$ to simplify this expression to

$$P_t^* = P_{it}^*\Lambda_i \frac{\mathbb{E}A_{it}^{\theta}A_{jt}^{-1}}{\mathbb{E}A_{it}^{\theta-1}}.$$

To get rid of the wealth constant Λ_i, we use the U.S. budget constraint

$$\mathbb{E}Z_t\left(P_{it}^*\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} Y_t^* - P_t^*\left(\frac{P_t^*}{P_{it}}\right)^{-\theta} C_{it}\right) = 0,$$

which after the same manipulations reduces to

$$\Lambda_i = \left(\frac{\mathbb{E}A_{it}^{\theta}A_{jt}^{-1}}{\mathbb{E}A_{it}^{\theta-1}}\right)^{\frac{1-\varepsilon}{\theta+\varepsilon-1}}.$$

Using all these conditions results in

$$\frac{\mathrm{d}\left(\mathcal{L}^{US} - \mathcal{L}^{nUS}\right)}{\mathrm{d}\gamma}\,|_{\gamma=0} = \left(\frac{\theta}{\theta-1}\frac{\varepsilon}{\varepsilon-1} - 1\right)\left(1 - \left(\frac{\mathbb{E}A_{it}^{\theta}A_{jt}^{-1}}{\mathbb{E}A_{it}^{\theta-1}}\right)^{\frac{(1-\varepsilon)(\theta-1)}{\theta+\varepsilon-1}}\right)\frac{\left(\mathbb{E}A_{it}^{\theta}A_{jt}^{-1}\right)^{\frac{\theta}{\theta+\varepsilon-1}}}{\left(\mathbb{E}A_{it}^{\theta-1}\right)^{\frac{1-\varepsilon}{\theta+\varepsilon-1}}}\left(\frac{P_t^*}{P_{it}}\right)^{-\theta}.$$

As long as $\theta > 0$ and $\varepsilon > 1$, this difference is non-negative whenever $\mathbb{E}A_{it}^{\theta-1} \leq \mathbb{E}A_{it}^{\theta}A_{jt}^{-1}$. Take a second-order approximation to express this condition as $-2\theta\left(\mathbb{E}a_{it}^2 - \mathbb{E}a_{it}a_{jt}\right) \leq 0$, which is true since $\mathbb{E}a_{it}a_{jt} \leq \mathbb{E}a_{it}^2$.

A.4.4 Proof of Proposition 7

Consider the following CES demand structure with heterogeneous import shares γ_{ji}:

$$(1-\gamma)^{\frac{1}{\theta}} \left(\frac{C_{iit}}{C_{it}}\right)^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} \left(\frac{C^*_{it}}{C_{it}}\right)^{\frac{\theta-1}{\theta}} = 1, \quad \int \gamma_{ji}^{\frac{1}{\varepsilon}} \left(\frac{C_{jit}}{C^*_{it}}\right)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}j = 1,$$

where $\int \gamma_{ji} \mathrm{d}j = 1$ for every i.[40] The import price index is then country-specific and we denote it with $\mathcal{P}^*_{it}$:

$$\int \gamma_{ji} \left(\frac{P^*_{jt}}{\mathcal{P}^*_{it}}\right)^{1-\varepsilon} \mathrm{d}j = 1,$$

where P^*_{jt} is, as before, the dollar price of goods exported from country j.

Note that the export Phillips curve (7) drops out from the policy problem as the state-dependent production subsidies to exporters $\{\tau^*_{it}\}$ allow the planner to implement any feasible export prices. Similarly, we drop the no-arbitrage condition (3) due to the presence of state-contingent taxes $\{\tau^c_{it}\}$ that can implement any feasible portfolio allocation. The policy problem can then be written as

$$\max_{\left\{C_{it}, X_{it}, L_{it}, N_{it}, P_{iit}, P_{it}, P^*_{it}, \mathcal{P}^*_{it}, \mathcal{E}_{it}, \pi_{iit}, \pi^*_{it}, \left\{B^h_{it+1}\right\}_h\right\}_{it}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \int U\left(C_{it}, N_{it}, \xi_{it}\right) \mathrm{d}i$$

$$\text{s.t. } A_{it} F\left(L_{it}, X_{it}\right) = (1-\gamma) \left(\frac{P_{iit}}{P_{it}}\right)^{-\theta} \left(C_{it} + X_{it}\right) + \gamma \int \gamma_{ij} \left(\frac{P^*_{it}}{\mathcal{P}^*_{jt}}\right)^{-\varepsilon} \left(\frac{\mathcal{E}_{jt} \mathcal{P}^*_{jt}}{P_{jt}}\right)^{-\theta} \left(C_{jt} + X_{jt}\right) \mathrm{d}j,$$

$$\begin{aligned}
&\sum_{h \in H_t} \mathcal{Q}^h_t B^h_{it+1} - \sum_{h \in H_{t-1}} \left(\mathcal{Q}^h_t + D^h_t\right) B^h_{it} \\
&\quad = P^*_{it} \gamma \int \gamma_{ij} \left(\frac{P^*_{it}}{\mathcal{P}^*_{jt}}\right)^{-\varepsilon} \left(\frac{\mathcal{E}_{jt} \mathcal{P}^*_{jt}}{P_{jt}}\right)^{-\theta} \left(C_{jt} + X_{jt}\right) \mathrm{d}j - \gamma \mathcal{P}^*_{it} \left(\frac{\mathcal{E}_{it} \mathcal{P}^*_{it}}{P_{it}}\right)^{-\theta} \left(C_{it} + X_{it}\right) + \psi_{it},
\end{aligned}$$

$$\frac{X_{it}}{L_{it}} = g\left(\frac{-U_{Nit}}{U_{Cit}}\right), \quad L_{it} = N_{it} - \frac{\varphi}{2}(1-\gamma)\pi^2_{iit} - \frac{\varphi}{2}\gamma \pi^{*2}_{it},$$

$$(1-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{1-\theta} + \gamma \left(\frac{\mathcal{E}_{it} \mathcal{P}^*_{it}}{P_{it}}\right)^{1-\theta} = 1, \quad \int \gamma_{ji} \left(\frac{P^*_{jt}}{\mathcal{P}^*_{it}}\right)^{1-\varepsilon} \mathrm{d}j = 1,$$

$$\pi_{iit} = \frac{P_{iit}}{P_{iit-1}} - 1, \quad \pi^*_{it} = \frac{P^*_{it}}{P^*_{it-1}} - 1,$$

$$\begin{aligned}
\pi_{iit}\left(\pi_{iit}+1\right)\left(-U_{Nit}\right) &= \beta \mathbb{E}_t \pi_{iit+1}\left(\pi_{iit+1}+1\right)\left(-U_{Nit+1}\right) \\
&\quad - \kappa \left(\frac{P_{iit}}{P_{it}} - \frac{h\left(\frac{-U_{Nit}}{U_{Cit}}\right)}{A_{it}}\right) U_{Cit} \left(\frac{P_{iit}}{P_{it}}\right)^{-\theta} \left(C_{it} + X_{it}\right),
\end{aligned}$$

[40] The heterogeneity in home bias $1-\gamma$ does not affect the results and is omitted to simplify the notation.

$$\mathbb{E}\pi_{ii0}\left(\pi_{ii0}+1\right)\left(-U_{Ni0}\right)=0, \qquad \int B^h_{it+1}\mathrm{d}i=0,$$

where we the last constraint is the global market clearing condition for assets (11).

Similarly to the proof of Proposition 1 in Section A.2.2, denote the Lagrange multipliers corresponding to these constraints with $\nu_{it}, \rho_{it}, \eta_{it}, \vartheta_{it}, \lambda_{it}, \lambda^*_{it}, \zeta_{it}, \zeta^*_{it}, \mu_{it}, \mu_i, \varsigma^h_t$. Guess and verify later that some of the constraints do not bind, $\eta_{it}=\zeta_{it}=\mu_{it}=\mu_i=0$, and take the first-order conditions:

- wrt C_{it}:

$$U_{Cit}-\nu_{it}\left(1-\gamma\right)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}-\int\nu_{jt}\gamma\gamma_{ji}\left(\frac{P^*_{jt}}{\mathcal{P}^*_{it}}\right)^{-\varepsilon}\left(\frac{\mathcal{E}_{it}\mathcal{P}^*_{it}}{P_{it}}\right)^{-\theta}\mathrm{d}j$$
$$-\rho_{it}\gamma\mathcal{P}^*_{it}\left(\frac{\mathcal{E}_{it}\mathcal{P}^*_{it}}{P_{it}}\right)^{-\theta}+\int\rho_{jt}\gamma P^*_{jt}\gamma_{ji}\left(\frac{P^*_{jt}}{\mathcal{P}^*_{it}}\right)^{-\varepsilon}\left(\frac{\mathcal{E}_{it}\mathcal{P}^*_{it}}{P_{it}}\right)^{-\theta}\mathrm{d}j=0,$$

- wrt X_{it}:

$$\nu_{it}A_{it}F_X\left(L_{it},X_{it}\right)-\nu_{it}\left(1-\gamma\right)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}-\int\nu_{jt}\gamma\gamma_{ji}\left(\frac{P^*_{jt}}{\mathcal{P}^*_{it}}\right)^{-\varepsilon}\left(\frac{\mathcal{E}_{it}\mathcal{P}^*_{it}}{P_{it}}\right)^{-\theta}\mathrm{d}j$$
$$-\rho_{it}\gamma\mathcal{P}^*_{it}\left(\frac{\mathcal{E}_{it}\mathcal{P}^*_{it}}{P_{it}}\right)^{-\theta}+\int\rho_{jt}\gamma P^*_{jt}\gamma_{ji}\left(\frac{P^*_{jt}}{\mathcal{P}^*_{it}}\right)^{-\varepsilon}\left(\frac{\mathcal{E}_{it}\mathcal{P}^*_{it}}{P_{it}}\right)^{-\theta}\mathrm{d}j=0,$$

- wrt L_{it}:

$$\nu_{it}A_{it}F_L\left(L_{it},X_{it}\right)+\vartheta_{it}=0,$$

- wrt N_{it}:

$$U_{Nit}=\vartheta_{it},$$

- wrt P_{iit}:

$$\nu_{it}\theta P^{-1}_{iit}\left(1-\gamma\right)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right)-\lambda_{it}\left(1-\gamma\right)\left(1-\theta\right)P^{-1}_{iit}\left(\frac{P_{iit}}{P_{it}}\right)^{1-\theta}=0,$$

- wrt P_{it}:

$$-\nu_{it}\left(1-\gamma\right)\theta P^{-1}_{it}\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right)-\rho_{it}\gamma\mathcal{P}^*_{it}\theta P^{-1}_{it}\left(\frac{\mathcal{E}_{it}\mathcal{P}^*_{it}}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right)$$
$$-\int\nu_{jt}\gamma\gamma_{ji}\left(\frac{P^*_{jt}}{\mathcal{P}^*_{it}}\right)^{-\varepsilon}\theta P^{-1}_{it}\left(\frac{\mathcal{E}_{it}\mathcal{P}^*_{it}}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right)\mathrm{d}j$$
$$+\int\rho_{jt}\gamma P^*_{jt}\gamma_{ji}\left(\frac{P^*_{jt}}{\mathcal{P}^*_{it}}\right)^{-\varepsilon}\theta P^{-1}_{it}\left(\frac{\mathcal{E}_{it}\mathcal{P}^*_{it}}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right)\mathrm{d}j$$
$$-\lambda_{it}\left(\theta-1\right)P^{-1}_{it}\left[\left(1-\gamma\right)\left(\frac{P_{iit}}{P_{it}}\right)^{1-\theta}+\gamma\left(\frac{\mathcal{E}_{it}\mathcal{P}^*_{it}}{P_{it}}\right)^{1-\theta}\right]=0,$$

- wrt P^*_{it}:

$$\nu_{it}\varepsilon P^{*-1}_{it}\gamma\int\gamma_{ij}\left(\frac{P^*_{it}}{\mathcal{P}^*_{jt}}\right)^{-\varepsilon}\left(\frac{\mathcal{E}_{jt}\mathcal{P}^*_{jt}}{P_{jt}}\right)^{-\theta}(C_{jt}+X_{jt})\,\mathrm{d}j$$
$$+\rho_{it}\gamma(1-\varepsilon)\int\gamma_{ij}\left(\frac{P^*_{it}}{\mathcal{P}^*_{jt}}\right)^{-\varepsilon}\left(\frac{\mathcal{E}_{jt}\mathcal{P}^*_{jt}}{P_{jt}}\right)^{-\theta}(C_{jt}+X_{jt})\,\mathrm{d}j$$
$$+\int\lambda^*_{jt}\gamma_{ij}\left(\frac{P^*_{it}}{\mathcal{P}^*_{jt}}\right)^{1-\varepsilon}(1-\varepsilon)\,P^{*-1}_{it}\mathrm{d}j-\zeta^*_t\frac{1}{P^*_{it-1}}+\beta\mathbb{E}_t\zeta^*_{t+1}\frac{P^*_{it+1}}{P^{*2}_{it}}=0,$$

- wrt $\mathcal{P}^*_{it}$:

$$-\int\nu_{jt}\gamma\gamma_{ji}(\varepsilon-\theta)\,\mathcal{P}^{*-1}_{it}\left(\frac{P^*_{jt}}{\mathcal{P}^*_{it}}\right)^{-\varepsilon}\left(\frac{\mathcal{E}_{it}\mathcal{P}^*_{it}}{P_{it}}\right)^{-\theta}(C_{it}+X_{it})\,\mathrm{d}j+\lambda^*_{it}(\varepsilon-1)\,\mathcal{P}^{*-1}_{it}\int\gamma_{ji}\left(\frac{P^*_{jt}}{\mathcal{P}^*_{it}}\right)^{1-\varepsilon}\mathrm{d}j$$
$$-\rho_{it}\gamma(1-\theta)\left(\frac{\mathcal{E}_{it}\mathcal{P}^*_{it}}{P_{it}}\right)^{-\theta}(C_{it}+X_{it})-\lambda_{it}\gamma(1-\theta)\,\mathcal{P}^{*-1}_{it}\left(\frac{\mathcal{E}_{it}\mathcal{P}^*_{it}}{P_{it}}\right)^{1-\theta}$$
$$+\int\rho_{jt}\gamma P^*_{jt}\gamma_{ji}(\varepsilon-\theta)\,\mathcal{P}^{*-1}_{it}\left(\frac{P^*_{jt}}{\mathcal{P}^*_{it}}\right)^{-\varepsilon}\left(\frac{\mathcal{E}_{it}\mathcal{P}^*_{it}}{P_{it}}\right)^{-\theta}(C_{it}+X_{it})\,\mathrm{d}j=0,$$

- wrt $\mathcal{E}_{it}$:

$$\int\nu_{jt}\gamma\gamma_{ji}\left(\frac{P^*_{jt}}{\mathcal{P}^*_{it}}\right)^{-\varepsilon}\theta\mathcal{E}^{-1}_{it}\left(\frac{\mathcal{E}_{it}\mathcal{P}^*_{it}}{P_{it}}\right)^{-\theta}(C_{it}+X_{it})\,\mathrm{d}j+\rho_{it}\gamma\mathcal{P}^*_{it}\theta\mathcal{E}^{-1}_{it}\left(\frac{\mathcal{E}_{it}\mathcal{P}^*_{it}}{P_{it}}\right)^{-\theta}(C_{it}+X_{it})$$
$$-\int\rho_{jt}\gamma P^*_{jt}\gamma_{ji}\left(\frac{P^*_{jt}}{\mathcal{P}^*_{it}}\right)^{-\varepsilon}\theta\mathcal{E}^{-1}_{it}\left(\frac{\mathcal{E}_{it}\mathcal{P}^*_{it}}{P_{it}}\right)^{-\theta}(C_{it}+X_{it})\,\mathrm{d}j-\lambda_{it}\gamma(1-\theta)\,\mathcal{E}^{-1}_{it}\left(\frac{\mathcal{E}_{it}\mathcal{P}^*_{it}}{P_{it}}\right)^{1-\theta}=0,$$

- wrt B^h_{it+1}:

$$\varsigma^h_t-\rho_{it}\mathcal{Q}^h_t+\beta\mathbb{E}_t\rho_{it+1}\left(\mathcal{Q}^h_{t+1}+D^h_{t+1}\right)=0,$$

- wrt $\mathcal{Q}^h_t$:

$$-\int\left[\rho_{it}\left(B^h_{it+1}-B^h_{it}\right)\right]\mathrm{d}i=0,$$

- wrt π_{iit}:

$$\vartheta_{it}\varphi(1-\gamma)\,\pi_{iit}=0,$$

- wrt π^*_{it}:

$$\vartheta_{it}\varphi\gamma\pi^*_{it}+\zeta^*_{it}=0.$$

Subtract the FOC wrt X_{it} from the FOC wrt C_{it} to get $U_{Cit}=\nu_{it}A_{it}F_X(L_{it},X_{it})$. This condition together with the FOCs wrt L_{it} and N_{it} imply the same relative input choice as in (5), confirming our guess that $\eta_{it}=0$.

Next, combine the FOCs wrt P_{it} and $\mathcal{E}_{it}$ to obtain $\lambda_{it} = -\nu_{it}\frac{\theta}{\theta-1}\frac{P_{it}}{P_{iit}}(C_{it} + X_{it})$ and

$$\nu_{it}\frac{\mathcal{E}_{it}\mathcal{P}^*_{it}}{P_{iit}} - \int \nu_{jt}\gamma_{ji}\left(\frac{P^*_{jt}}{\mathcal{P}^*_{it}}\right)^{-\varepsilon} \mathrm{d}j - \rho_{it}\mathcal{P}^*_{it} + \int \rho_{jt}P^*_{jt}\gamma_{ji}\left(\frac{P^*_{jt}}{\mathcal{P}^*_{it}}\right)^{-\varepsilon} \mathrm{d}j = 0.$$

Use this condition along with the FOC wrt C_{it} to arrive at $\nu_{it} = U_{Cit}\frac{P_{iit}}{P_{it}}$. This condition and the FOCs wrt L_{it} and N_{it} immediately imply marginal cost stabilization $\pi_{iit} = 0$, that is condition (A6). Domestic price setting conditions are trivially satisfied, and this confirms our guess of $\zeta_{it} = \mu_{it} = \mu_i = 0$. It is also straightforward to verify that the FOC wrt P_{iit} holds. Now we can use the FOC wrt C_{it} to derive the following optimality condition:

$$\left(U_{Cit}\frac{\mathcal{E}_{it}}{P_{it}} - \rho_{it}\right)\mathcal{P}^*_{it} = \int \left(U_{Cjt}\frac{P_{jjt}}{P_{jt}} - \rho_{jt}P^*_{jt}\right)\gamma_{ji}\left(\frac{P^*_{jt}}{\mathcal{P}^*_{it}}\right)^{-\varepsilon} \mathrm{d}j. \tag{A11}$$

In the case of complete markets, ρ_i becomes a time-invariant constant because there is just one inter-temporal budget constraint. Since the global planner treats equally all ex-ante symmetric non-U.S. countries, it follows $\rho_i = \rho_j$. Together with condition (A11) and the price index constraint, this implies

$$U_{Cit}\frac{\mathcal{E}_{it}}{P_{it}} = \int U_{Cjt}\frac{\mathcal{E}_{jt}}{P_{jt}}\frac{P_{jjt}}{\mathcal{E}_{jt}P^*_{jt}}\gamma_{ji}\left(\frac{P^*_{jt}}{\mathcal{P}^*_{it}}\right)^{1-\varepsilon} \mathrm{d}j. \tag{A12}$$

Adding capital controls and using normalization from footnote 15, the risk-sharing condition (A9) becomes

$$U_{Cit}\frac{\mathcal{E}_{it}}{P_{it}} = (1 - \tau^c_{it})\, Z_t.$$

Thus, the optimality condition (A12) can be rewritten as

$$1 - \tau^c_{it} = \int \left(1 - \tau^c_{jt}\right)\gamma_{ji}\left(\frac{P^*_{jt}}{\mathcal{P}^*_{it}}\right)^{1-\varepsilon} \frac{P_{jjt}}{\mathcal{E}_{jt}P^*_{jt}}\mathrm{d}j,$$

giving the optimal choice of capital controls under complete markets.

Now integrate equation (A12) over all countries i,

$$\int \frac{U_{Cjt}\mathcal{E}_{jt}}{P_{jt}}\left(\int \gamma_{ji}\left(\frac{P^*_{jt}}{\mathcal{P}^*_{it}}\right)^{1-\varepsilon} \frac{P_{it}}{U_{Cit}\mathcal{E}_{it}}\mathrm{d}i\right)\frac{P_{jjt}}{\mathcal{E}_{jt}P^*_{jt}}\mathrm{d}j = 1.$$

This condition shows that the optimal U.S. policy under cooperation stabilizes the average law-of-one-price deviations, $\frac{P_{jjt}}{\mathcal{E}_{jt}P^*_{jt}}$. If trade flows are symmetric $\gamma_{ji} = 1$, equation (A12) simplifies further and implies that $U_{Cit}\mathcal{E}_{it}/P_{it}$ are equalized across countries. Then the U.S. monetary policy reduces to

$$\int \left(\frac{P^*_{jt}}{\mathcal{P}^*_{t}}\right)^{1-\varepsilon} \frac{P_{jjt}}{\mathcal{E}_{jt}P^*_{jt}}\mathrm{d}j = 1.$$

Finally, check the optimal dynamics of export prices. Assume symmetric trade flows with $\gamma_{ji} = 1$, and use the FOCs wrt P^*_{it}, $\mathcal{P}^*_{it}$, and π^*_{it}, along with the optimality condition (A11), to derive

$$\pi^*_{it}\left(\pi^*_{it}+1\right)\left(-U_{Nit}\right)-\beta\mathbb{E}_t\pi^*_{it+1}\left(\pi^*_{it+1}+1\right)\left(-U_{Nit+1}\right)$$
$$+\frac{\varepsilon}{\varphi}\int\mathcal{E}_{it}P^*_{it}\left[\frac{P_{iit}}{\mathcal{E}_{it}P^*_{it}}-1-\frac{1}{\varepsilon}\left(\frac{U_{Cjt}\mathcal{E}_{jt}}{U_{Cit}\mathcal{E}_{it}}\frac{P_{it}}{P_{jt}}-1\right)\right]\frac{U_{Cit}}{P_{it}}\left(\frac{P^*_{it}}{\mathcal{P}^*_{t}}\right)^{-\varepsilon}\left(\frac{\mathcal{E}_{jt}\mathcal{P}^*_{t}}{P_{jt}}\right)^{-\theta}\left(C_{jt}+X_{jt}\right)\mathrm{d}j=0.$$

Note that this expression does not rely on full risk sharing and holds for an arbitrary structure of the asset markets.

A.4.5 Proof of Corollary 7.1

Consider a continuum of non-U.S. economies with the total measure of zero. It is sufficient to show that the welfare of the union is higher than the welfare of its members under no cooperation for the limiting case with perfectly correlated shocks — by continuity, the same is then true if the shocks are sufficiently highly correlated. In this limit, without loss of generality, we can model the policy problem of the monetary union as a problem of a single country with some share of its internal goods priced in dollars. Therefore, the policy problem becomes similar to the one in Sections 4.3 and A.3.3, and it can be written as

$$\max_{\left\{C_{it},X_{it},L_{it},N_{it},P_{iit},P^*_{iit},P^*_{it},P_{it},\mathcal{E}_{it},\pi_{iit},\pi^*_{iit},\pi^*_{it},\left\{B^h_{it+1}\right\}_h\right\}_t}\mathbb{E}\sum_{t=0}^{\infty}\beta^t U\left(C_{it},N_{it},\xi_{it}\right)$$

$$\text{s.t. } A_{it}F\left(L_{it},X_{it}\right)=\left(1-\gamma^*-\gamma\right)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right)+\gamma^*\left(\frac{\mathcal{E}_{it}P^*_{iit}}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right)+\gamma\left(\frac{P^*_{it}}{P^*_{t}}\right)^{-\varepsilon}Y^*_t,$$

$$\sum_{h\in H_t}\mathcal{Q}^h_t B^h_{it+1}-\sum_{h\in H_{t-1}}\left(\mathcal{Q}^h_t+D^h_t\right)B^h_{it}=\gamma P^*_{it}\left(\frac{P^*_{it}}{P^*_{t}}\right)^{-\varepsilon}Y^*_t-\gamma P^*_t\left(\frac{\mathcal{E}_{it}P^*_{t}}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right)+\psi_{it},$$

$$\frac{X_{it}}{L_{it}}=g\left(\frac{-U_{Nit}}{U_{Cit}}\right),\quad L_{it}=N_{it}-\frac{\varphi}{2}\left(1-\gamma^*-\gamma\right)\pi^2_{iit}-\frac{\varphi}{2}\gamma^*\pi^{*2}_{iit}-\frac{\varphi}{2}\gamma\pi^{*2}_{it},$$

$$\beta\mathbb{E}_t\frac{U_{Cit+1}}{U_{Cit}}\frac{\mathcal{E}_{it+1}}{P_{it+1}}\frac{P_{it}}{\mathcal{E}_{it}}\frac{\mathcal{Q}^h_{t+1}+D^h_{t+1}}{\mathcal{Q}^h_t}=1,$$

$$\left(1-\gamma^*-\gamma\right)\left(\frac{P_{iit}}{P_{it}}\right)^{1-\theta}+\gamma^*\left(\frac{\mathcal{E}_{it}P^*_{iit}}{P_{it}}\right)^{1-\theta}+\gamma\left(\frac{\mathcal{E}_{it}P^*_{t}}{P_{it}}\right)^{1-\theta}=1,$$

$$\pi_{iit}=\frac{P_{iit}}{P_{iit-1}}-1,\quad\pi^*_{iit}=\frac{P^*_{iit}}{P^*_{iit-1}}-1,\quad\pi^*_{it}=\frac{P^*_{it}}{P^*_{it-1}}-1,$$

$$\pi_{iit}\left(\pi_{iit}+1\right)\left(-U_{Nit}\right)=\beta\mathbb{E}_t\pi_{iit+1}\left(\pi_{iit+1}+1\right)\left(-U_{Nit+1}\right)$$
$$-\kappa\left(\frac{P_{iit}}{P_{it}}-\frac{h\left(\frac{-U_{Nit}}{U_{Cit}}\right)}{A_{it}}\right)U_{Cit}\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right),$$

$$\pi_{iit}^{*}\left(\pi_{iit}^{*}+1\right)\left(-U_{Nit}\right)=\beta\mathbb{E}_{t}\pi_{iit+1}^{*}\left(\pi_{iit+1}^{*}+1\right)\left(-U_{Nit+1}\right)$$

$$-\kappa\left(\frac{\mathcal{E}_{it}P_{iit}^{*}}{P_{it}}-\frac{\varepsilon}{\varepsilon-1}\frac{h\left(\frac{-U_{Nit}}{U_{Cit}}\right)}{A_{it}}\right)U_{Cit}\left(\frac{\mathcal{E}_{it}P_{iit}^{*}}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right),$$

$$\pi_{it}^{*}\left(\pi_{it}^{*}+1\right)\left(-U_{Nit}\right)=\beta\mathbb{E}_{t}\pi_{it+1}^{*}\left(\pi_{it+1}^{*}+1\right)\left(-U_{Nit+1}\right)$$

$$-\kappa\left(\frac{\mathcal{E}_{it}P_{it}^{*}}{P_{it}}-\frac{\varepsilon}{\varepsilon-1}\frac{h\left(\frac{-U_{Nit}}{U_{Cit}}\right)}{A_{it}}\right)U_{Cit}\left(\frac{P_{it}^{*}}{P_{t}^{*}}\right)^{-\varepsilon}\frac{Y_{t}^{*}}{\gamma},$$

$$\mathbb{E}\pi_{ii0}\left(\pi_{ii0}+1\right)\left(-U_{Ni0}\right)=0,\quad \mathbb{E}\pi_{ii0}^{*}\left(\pi_{ii0}^{*}+1\right)\left(-U_{Ni0}\right)=0,\quad \mathbb{E}\pi_{i0}^{*}\left(\pi_{i0}^{*}+1\right)\left(-U_{Ni0}\right)=0.$$

It is straightforward, though tedious, to take the first-order conditions and to verify that $\pi_{iit} = 0$ is not the solution. Now consider a single small open economy outside of the monetary union in a similar environment. It follows from our previous analysis that such an economy chooses $\pi_{iit} = 0$ as its optimal policy. Different small open economies with perfectly correlated shocks follow the same policy and also have exactly the same allocation. This implies that the bilateral exchange rates across these economies are equal to one. Therefore, it is feasible for the monetary union to implement exactly the same monetary policy as each of its members would choose on its own. However, it is optimal for the union to choose a different policy. Thus, the welfare of a country within a union is higher than the welfare of a country outside of the union.

A.5 Calvo pricing

Equilibrium conditions In contrast to the baseline model, the Calvo friction generates price dispersion, which affects all aggregate quantities. In particular, the market clearing condition (8) becomes

$$A_{it}F\left(L_{it},X_{it}\right)=\left(1-\gamma\right)\Delta_{iit}\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right)+\gamma\Delta_{it}^{*}\left(\frac{P_{it}^{*}}{P_{t}^{*}}\right)^{-\varepsilon}Y_{t}^{*},$$

where

$$\Delta_{iit}\equiv\int\left(\frac{P_{iit}\left(\omega\right)}{P_{iit}}\right)^{-\varepsilon}\mathrm{d}\omega\qquad\text{and}\qquad\Delta_{it}^{*}\equiv\int\left(\frac{P_{it}^{*}\left(\omega\right)}{P_{it}^{*}}\right)^{-\varepsilon}\mathrm{d}\omega.$$

Then each price index has a non-trivial dynamics:

$$P_{it}^{*1-\varepsilon}=\lambda P_{it-1}^{*1-\varepsilon}+\left(1-\lambda\right)\tilde{P}_{it}^{*1-\varepsilon},\tag{A13}$$

where a fraction $1-\lambda$ of firms can adjust their prices in a given period and $\tilde{P}_{it}^{*}$ is the price that they choose. Solving for the dynamics of price dispersion yields

$$\Delta_{it}^{*}=\lambda\Delta_{it-1}^{*}\Pi_{it}^{*\varepsilon}+\left(1-\lambda\right)\left(\frac{1}{1-\lambda}-\frac{\lambda}{1-\lambda}\Pi_{it}^{*\varepsilon-1}\right)^{\frac{-\varepsilon}{1-\varepsilon}},\tag{A14}$$

where $\Pi_{it}^{*}\equiv P_{it}^{*}/P_{it-1}^{*}$ is the (gross) inflation rate. Expressions for domestic variables Δ_{iit} and Π_{iit} are similar.

To derive the export price-setting condition, start with a problem of an exporter:

$$\max_{\tilde{P}^*_{it}} \mathbb{E}_t \sum_{k=0}^{\infty} \Theta_{it,t+k} \lambda^k \left(\mathcal{E}_{it+k} \tilde{P}^*_{it} - \tau^*_i MC_{it+k} \right) \left(\frac{\tilde{P}^*_{it}}{P^*_{t+k}} \right)^{-\varepsilon} Y^*_{t+k}.$$

The first-order condition determines $\tilde{P}^*_{it}$:

$$\mathbb{E}_t \sum_{k=0}^{\infty} \Theta_{it,t+k} \lambda^k \left(\mathcal{E}_{it+k} \tilde{P}^*_{it} - \frac{\varepsilon \tau^*_i}{\varepsilon - 1} MC_{it+k} \right) \left(\frac{\tilde{P}^*_{it}}{P^*_{t+k}} \right)^{-\varepsilon} Y^*_{t+k} = 0. \tag{A15}$$

As before, we assume that there is no export subsidy, $\tau^*_i = 1$. The domestic price-setting condition is similar, but monopolistic distortion in local markets is eliminated with the production subsidy, $\frac{\varepsilon \tau_i}{\varepsilon - 1} = 1$. As before, we assume that in period zero, all firms can adjust their prices, which implies no price dispersion, $\Delta_{ii0} = \Delta^*_{i0} = 1$.

For the analysis below, it is convenient to express condition (A15) in a recursive form. First, rewrite it as

$$\tilde{P}^*_{it} F_t = \frac{\varepsilon}{\varepsilon - 1} \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \lambda^k MC_{it+k} \frac{U_{Cit+k}}{P_{it+k}} P^{*\varepsilon}_{t+k} Y^*_{t+k}, \quad \text{where } F_t \equiv \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \lambda^k \frac{U_{Cit+k}}{P_{it+k}} \mathcal{E}_{it+k} P^{*\varepsilon}_{t+k} Y^*_{t+k}.$$

Then, isolate the first term from the sum on the right hand side and use the law of iterated expectation:

$$\tilde{P}^*_{it} F_t = \frac{\varepsilon}{\varepsilon - 1} MC_{it} \frac{U_{Cit}}{P_{it}} P^{*\varepsilon}_t Y^*_t + \beta \lambda \mathbb{E}_t \tilde{P}^*_{it+1} F_{t+1}, \tag{A16}$$

and note that F_t can also be written recursively as

$$F_t = \frac{U_{Cit}}{P_{it}} \mathcal{E}_{it} P^{*\varepsilon}_t Y^*_t + \beta \lambda \mathbb{E}_t F_{t+1}. \tag{A17}$$

Together, recursive equations (A16) and (A17) are equivalent to the inter-temporal price-setting condition (A15).

Optimal policy The planner's problem in a representative non-U.S. economy is

$$\max_{\left\{C_{it}, X_{it}, N_{it}, P_{iit}, \tilde{P}_{iit}, P_{it}, P^*_{it}, \tilde{P}^*_{it}, \mathcal{E}_{it}, \Pi_{iit}, \Pi^*_{it}, \Delta_{iit}, \Delta^*_{it}, \left\{B^h_{it+1}\right\}_h\right\}_t} \mathbb{E} \sum_{t=0}^{\infty} \beta^t U\left(C_{it}, N_{it}, \xi_{it}\right)$$

$$\text{s.t. } A_{it} F\left(N_{it}, X_{it}\right) = (1-\gamma) \Delta_{iit} \left(\frac{P_{iit}}{P_{it}} \right)^{-\theta} \left(C_{it} + X_{it}\right) + \gamma \Delta^*_{it} \left(\frac{P^*_{it}}{P^*_t} \right)^{-\varepsilon} Y^*_t,$$

$$\sum_{h \in H_t} \mathcal{Q}^h_t B^h_{it+1} - \sum_{h \in H_{t-1}} \left(\mathcal{Q}^h_t + D^h_t \right) B^h_{it} = \gamma \left[P^*_{it} \left(\frac{P^*_{it}}{P^*_t} \right)^{-\varepsilon} Y^*_t - P^*_t \left(\frac{\mathcal{E}_{it} P^*_t}{P_{it}} \right)^{-\theta} \left(C_{it} + X_{it}\right) \right] + \psi_{it},$$

$$\frac{X_{it}}{L_{it}} = g\left(\frac{-U_{Nit}}{U_{Cit}} \right), \quad \beta \mathbb{E}_t \frac{U_{Cit+1}}{U_{Cit}} \frac{\mathcal{E}_{it+1}}{P_{it+1}} \frac{P_{it}}{\mathcal{E}_{it}} \frac{\mathcal{Q}^h_{t+1} + D^h_{t+1}}{\mathcal{Q}^h_t} = 1,$$

$$(1-\gamma) \left(\frac{P_{iit}}{P_{it}} \right)^{1-\theta} + \gamma \left(\frac{\mathcal{E}_{it} P^*_t}{P_{it}} \right)^{1-\theta} = 1,$$

$$P_{iit}^{1-\varepsilon} = \lambda P_{iit-1}^{1-\varepsilon} + (1-\lambda)\tilde{P}_{iit}^{1-\varepsilon}, \quad P_{it}^{*1-\varepsilon} = \lambda P_{it-1}^{*1-\varepsilon} + (1-\lambda)\tilde{P}_{it}^{*1-\varepsilon},$$

$$\Delta_{iit} = \lambda \Delta_{iit-1}\Pi_{iit}^{\varepsilon} + (1-\lambda)\left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\Pi_{iit}^{\varepsilon-1}\right)^{\frac{-\varepsilon}{1-\varepsilon}},$$

$$\Delta_{it}^{*} = \lambda \Delta_{it-1}^{*}\Pi_{it}^{*\varepsilon} + (1-\lambda)\left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\Pi_{it}^{*\varepsilon-1}\right)^{\frac{-\varepsilon}{1-\varepsilon}},$$

$$\Pi_{iit} = \frac{P_{iit}}{P_{iit-1}}, \quad \Pi_{it}^{*} = \frac{P_{it}^{*}}{P_{it-1}^{*}},$$

$$\mathbb{E}_t \sum_{k=0}^{\infty} \Theta_{it,t+k}\lambda^k \left(\tilde{P}_{iit} - MC_{it+k}\right)\left(\frac{\tilde{P}_{iit}}{P_{iit+k}}\right)^{-\varepsilon}\left(\frac{P_{iit+k}}{P_{it+k}}\right)^{-\theta}(C_{it+k} + X_{it+k}) = 0,$$

$$\mathbb{E}_t \sum_{k=0}^{\infty} \Theta_{it,t+k}\lambda^k \left(\mathcal{E}_{it+k}\tilde{P}_{it}^{*} - \frac{\varepsilon}{\varepsilon-1}MC_{it+k}\right)\left(\frac{\tilde{P}_{it}^{*}}{P_{t+k}^{*}}\right)^{-\varepsilon} Y_{t+k}^{*} = 0,$$

$$\Delta_{ii0} = 1, \quad \Delta_{i0}^{*} = 1.$$

Denote the Lagrange multipliers on these constraints as ν_{it}, ρ_{it}, η_{it}, χ_{it}, λ_{it}, λ_{iit}, λ_{it}^{*}, ϑ_{iit}, ϑ_{it}^{*}, ζ_{iit}, ζ_{it}^{*}, μ_{iit}, μ_{it}^{*}, μ_{ii}, μ_{i}^{*}. Guess and verify later that some of the constraints are not binding: $\eta_{it} = \chi_{it} = \lambda_{iit} = \vartheta_{iit} = \zeta_{iit} = \mu_{iit} = \mu_{it}^{*} = \mu_{ii} = \mu_{i}^{*} = 0$. The first-order conditions are:

- wrt C_{it}:

$$U_{Cit} - \nu_{it}(1-\gamma)\Delta_{iit}\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta} + \rho_{it}\gamma P_t^{*}\left(\frac{\mathcal{E}_{it}P_t^{*}}{P_{it}}\right)^{-\theta} = 0,$$

- wrt X_{it}:

$$\nu_{it}A_{it}F_X(N_{it}, X_{it}) - \nu_{it}(1-\gamma)\Delta_{iit}\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta} + \rho_{it}\gamma P_t^{*}\left(\frac{\mathcal{E}_{it}P_t^{*}}{P_{it}}\right)^{-\theta} = 0,$$

- wrt N_{it}:

$$U_{Nit} + \nu_{it}A_{it}F_L(N_{it}, X_{it}) = 0,$$

- wrt P_{iit}:

$$\nu_{it}(1-\gamma)\theta\frac{1}{P_{iit}}\Delta_{iit}\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}(C_{it} + X_{it}) - (1-\gamma)\lambda_{it}(1-\theta)\frac{1}{P_{iit}}\left(\frac{P_{iit}}{P_{it}}\right)^{1-\theta} = 0,$$

- wrt P_{it}:

$$-\nu_{it}(1-\gamma)\Delta_{iit}\theta\frac{1}{P_{it}}\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}(C_{it} + X_{it}) + \rho_{it}\gamma P_t^{*}\theta\frac{1}{P_{it}}\left(\frac{\mathcal{E}_{it}P_t^{*}}{P_{it}}\right)^{-\theta}(C_{it} + X_{it})$$
$$-\lambda_{it}(\theta-1)\frac{1}{P_{it}}\left[(1-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{1-\theta} + \gamma\left(\frac{\mathcal{E}_{it}P_t^{*}}{P_{it}}\right)^{1-\theta}\right] = 0,$$

- wrt P_{it}^*:

$$\nu_{it}\gamma\Delta_{it}^*\varepsilon\frac{1}{P_{it}^*}\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}Y_t^* - \rho_{it}\gamma\left(1-\varepsilon\right)\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}Y_t^* + \lambda_{it}^*\left(1-\varepsilon\right)P_{it}^{*-\varepsilon}$$
$$-\beta\mathbb{E}_t\lambda_{it+1}^*\lambda\left(1-\varepsilon\right)P_{it}^{*-\varepsilon} - \zeta_{it}^*\frac{1}{P_{it-1}^*} + \beta\mathbb{E}_t\zeta_{it+1}^*\frac{P_{it+1}^*}{P_{it}^{*2}} = 0,$$

- wrt $\mathcal{E}_{it}$:

$$-\rho_{it}\gamma P_t^*\theta\frac{1}{\mathcal{E}_{it}}\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right) - \lambda_{it}\gamma\left(1-\theta\right)\frac{1}{\mathcal{E}_{it}}\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{1-\theta} = 0,$$

- wrt Π_{it}^*:

$$-\vartheta_{it}^*\lambda\Delta_{it-1}^*\varepsilon\Pi_{it}^{*\varepsilon-1} + \vartheta_{it}^*\varepsilon\lambda\left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\Pi_{it}^{*\varepsilon-1}\right)^{\frac{-\varepsilon}{1-\varepsilon}-1}\Pi_{it}^{*\varepsilon-2} + \zeta_{it}^* = 0,$$

- wrt Δ_{it}^*:

$$-\nu_{it}\gamma\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}Y_t^* + \vartheta_{it}^* - \beta\mathbb{E}_t\vartheta_{it+1}^*\lambda\Pi_{it+1}^{*\varepsilon} = 0,$$

- wrt B_{it+1}^h:

$$\rho_{it}\mathcal{Q}_t^h - \beta\mathbb{E}_{it}\rho_{t+1}\left(\mathcal{Q}_{t+1}^h + D_{t+1}^h\right) = 0.$$

The FOCs with respect to $\tilde{P}_{iit}$, $\tilde{P}_{it}^*$, Π_{iit}, and Δ_{iit} are omitted because they are trivially satisfied under our guess.

Combine the first three FOCs to get the relative demand for inputs (5), which verifies our guess $\eta_{it} = 0$. The FOC wrt $\mathcal{E}_{it}$ implies $\lambda_{it} = \rho_{it}\frac{P_{it}}{\mathcal{E}_{it}}\frac{\theta}{\theta-1}\left(C_{it}+X_{it}\right)$, so that the FOC wrt P_{it} along with the price index constraint turns to $\nu_{it}\Delta_{iit} = -\rho_{it}\frac{P_{iit}}{\mathcal{E}_{it}}$. Combine this with the FOC wrt C_{it} and arrive at $\rho_{it} = -\frac{U_{Cit}}{P_{it}}\mathcal{E}_{it}$, and thus $\nu_{it}\Delta_{iit} = U_{Cit}\frac{P_{iit}}{P_{it}}$. Combined with the FOC wrt N_{it}, the latter implies

$$\frac{P_{iit}}{P_{it}}A_{it}F_L\left(N_{it},X_{it}\right) = \Delta_{iit}\frac{-U_{Nit}}{U_{Cit}},$$

which is the same as (A6), that is the domestic marginal cost stabilization, as long as $\Delta_{iit} = \Pi_{iit} = 1$. Then the FOC wrt P_{iit} is satisfied. The FOC wrt B_{it+1}^h is also satisfied as it turns to the no-arbitrage condition (3), justifying our guess of $\chi_{it} = 0$.

To find the optimal dynamics of export prices, use the FOC wrt Π_{it}^* to express

$$\zeta_{it}^* = \vartheta_{it}^*\varepsilon\lambda\Pi_{it}^{*\varepsilon-1}\left(\Delta_{it-1}^* - \left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\Pi_{it}^{*\varepsilon-1}\right)^{\frac{-1}{1-\varepsilon}}\Pi_{it}^{*-1}\right),$$

and note that price dynamics constraints (A13) and (A14) can be used to show

$$\zeta_{it}^* = \vartheta_{it}^*\varepsilon\lambda\Pi_{it}^{*\varepsilon-1}\left(\Delta_{it-1}^* - \frac{P_{it}^*}{\tilde{P}_{it}^*}\Pi_{it}^{*-1}\right).$$

Plug this expression into the FOC wrt P_{it}^*,

$$\frac{1-\varepsilon}{\varepsilon\lambda}\left(\mathcal{E}_{it}P_{it}^* - P_{iit}\Delta_{it}^*\frac{\varepsilon}{\varepsilon-1}\right)\frac{U_{Cit}}{P_{it}}\gamma\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}Y_t^*$$
$$-\vartheta_{it}^*\Pi_{it}^{*\varepsilon}\left(\Delta_{it-1}^* - \frac{P_{it}^*}{\tilde{P}_{it}^*}\Pi_{it}^{*-1}\right) + \beta\mathbb{E}_t\vartheta_{it+1}^*\Pi_{it+1}^{*\varepsilon}\left(\Delta_{it}^* - \frac{P_{it+1}^*}{\tilde{P}_{it+1}^*}\Pi_{it+1}^{*-1}\right) = 0,$$

where we have guessed that $\lambda_{it}^* = \beta\lambda\mathbb{E}_t\lambda_{it+1}^*$.

Use price dynamics constraints (A13) and (A14) to show that

$$\Pi_{it}^{*\varepsilon}\left(\Delta_{it-1}^* - \frac{P_{it}^*}{\tilde{P}_{it}^*}\Pi_{it}^{*-1}\right) = \frac{1}{\lambda}\left(\Delta_{it}^* - \frac{P_{it}^*}{\tilde{P}_{it}^*}\right)$$

and substitute this equation into the previous expression to obtain

$$\frac{1-\varepsilon}{\varepsilon}\left(\mathcal{E}_{it}P_{it}^* - P_{iit}\Delta_{it}^*\frac{\varepsilon}{\varepsilon-1}\right)\frac{U_{Cit}}{P_{it}}\gamma\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}Y_t^*$$
$$-\Delta_{it}^*\left(\vartheta_{it}^* - \beta\lambda\mathbb{E}_t\vartheta_{it+1}^*\Pi_{it+1}^{*\varepsilon}\right) + \vartheta_{it}^*\frac{P_{it}^*}{\tilde{P}_{it}^*} - \beta\lambda\mathbb{E}_t\vartheta_{it+1}^*\Pi_{it+1}^{*\varepsilon-1}\frac{P_{it+1}^*}{\tilde{P}_{it+1}^*} = 0.$$

Simplify the FOC wrt Δ_{it}^*

$$\vartheta_{it}^* = U_{Cit}\frac{P_{iit}}{P_{it}}\gamma\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}Y_t^* + \beta\lambda\mathbb{E}_t\vartheta_{it+1}^*\Pi_{it+1}^{*\varepsilon}, \tag{A18}$$

and use it to express the previous equation as

$$\vartheta_{it}^*\frac{P_{it}^*}{\tilde{P}_{it}^*} = \frac{\varepsilon-1}{\varepsilon}\mathcal{E}_{it}P_{it}^*\frac{U_{Cit}}{P_{it}}\gamma\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}Y_t^* + \beta\lambda\mathbb{E}_t\vartheta_{it+1}^*\Pi_{it+1}^{*\varepsilon-1}\frac{P_{it+1}^*}{\tilde{P}_{it+1}^*}. \tag{A19}$$

In the end, the set of optimality conditions have collapsed to equations (A18) and (A19). Note that these two equations are equivalent to the private price-setting condition (A15), as they are equivalent to conditions (A16) and (A17). To see this, rewrite these optimality conditions in terms of a new variable $\tilde{F}_t$,

$$\tilde{F}_t \equiv \gamma^{-1}\frac{\varepsilon}{\varepsilon-1}P_{it}^{*\varepsilon}\frac{\vartheta_{it}^*}{\tilde{P}_{it}^*},$$

so that equations (A18) and (A19) become respectively,

$$\tilde{F}_t\tilde{P}_{it}^* = \frac{\varepsilon}{\varepsilon-1}P_{iit}\frac{U_{Cit}}{P_{it}}P_t^{*\varepsilon}Y_t^* + \beta\lambda\mathbb{E}_t\tilde{F}_{t+1}\tilde{P}_{it+1}^* \quad \text{and} \quad \tilde{F}_t = \mathcal{E}_{it}\frac{U_{Cit}}{P_{it}}P_t^{*\varepsilon}Y_t^* + \beta\lambda\mathbb{E}_t\tilde{F}_{t+1}.$$

The latter condition coincides with equation (A17) and hence, $\tilde{F}_t = F_t$. It follows that the former condition is the same as equation (A16) since $P_{iit} = MC_{it}$ due to domestic cost stabilization.

Thus, we have shown that there exists a set of Lagrange multipliers such that the system of the first-order conditions is satisfied under the optimal policy of $\Pi_{iit} = 1$, which completes the proof.