Childless Aristocrats.
Inheritance and the extensive margin of fertility

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Childless Aristocrats. Inheritance and the extensive margin of fertility*

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Abstract

This paper uses genealogical data of British aristocrats to show that inheritances can affect childlessness rates. We study settlements, a contract restricting heirs' powers and settling bequests for yet-to-be-born generations. This arrangement pushed childlessness to the "natural" rate of 2.4%, ensuring aristocratic dynasties' survival. Our estimation exploits that settlements were signed at the heir's wedding if the family head lived until this date. The heir's birth order, hence, provides as-good-as-random assignment into settlements. Next, we develop a theory that reproduces our findings and provides two novel results: exponential discounting cannot rationalize inheritance systems restricting heirs (settlements, trusts...); and inheritance systems can emerge endogenously when fertility concerns exist.

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1 Introduction

Inheritance systems have long attracted the attention of economists. For example, Adam Smith gave a scathing criticism of primogeniture and entailment of the land: He argued that these laws exacerbated inequality, "making beggars" of all but the first-born.[1] This intuition has carried over to modern work. Several studies argue that inheritance systems have important effects for fiscal policy (Barro 1974), inequality (Stiglitz 1969; Chu 1991; Piketty 2011), or economic growth and the transition to modern, democratic societies (Bertocchi 2006).

However, what effect inheritance has on inequality, social mobility, or economic growth depends crucially on fertility choices. For example, a standard implication of models of intergenerational transfers is that if the very rich have more (less) children, inheritances seemingly reduce (increase) inequality (see, e.g., Stiglitz 1969; Atkinson and Harrison 1978). Despite the central role of fertility, one common feature in the analysis of inheritance is to treat fertility as *exogenous* (Abel 1987; Weil 1987) or, more recently, to consider endogenous fertility decisions only on the *intensive margin*—i.e., the number of children (Cordoba and Ripoll 2016). In contrast, the relation between inheritance systems and the *extensive margin* of fertility—i.e., the decision to have children or not—remain unexplored. This is surprising as the economic effects of any inheritance system and, in particular, of primogeniture or entailment, crucially hinge on the production of an heir.

In this paper, we study the relation between inheritance systems and the extensive margin of fertility through the lenses of settlements—the common inheritance system among British aristocrats. Settlements enforced primogeniture, but restricted the male heirs powers by entailing the family estates for one generation. This arrangement guaranteed that a large portion of the inheritance would pass down to the yet-to-be-born descendants of the direct family lineage, which likely created an additional incentive to produce an heir. Using genealogical data between 1650 and 1882, we find that families signing a settlement were c. 15 percentage points more likely to have children. Given that the average childlessness rate among peers was 17 percent, settlements increased by 83.5 percent the extensive margin of fertility, pushed childlessness rates close to the "natural" rate of 2.4 percent (Tietze 1957),[2] and hence, contributed to the survival of noble dynasties. In contrast, we find that settlements did not affect the intensive margin

[1] Smith 1776 [1937], book III, chapter II.

[2] The "natural" rate corresponds to that of Hutterites, who marry young, do not divorce, have access to modern health care, etc.

of fertility—the number of children by mothers.[3]

Our empirical setting offers several advantages. First, the institutional background of settlements can be used to address endogeneity concerns. Specifically, a father and his heir had to sign/renew a settlement every generation. Because of institutional constraints and tradition, the renewal was done at the heir's wedding day (Bonfield 1979). Thus, when the father died (exogenously) before his heir's wedding, a settlement was not signed. This generates as-good-as-random assignment of families into settlements. Our source of exogenous variation is the heir's birth order. The idea is that in families in which the heir is only born after several daughters, the father will be relatively older than his heir, exogenously decreasing the probability to live until his heir's wedding. In our context, it is unlikely that birth order had a direct effect on later fertility, for example, through breastfeeding (Jayachandran and Kuziemko 2011). The reason is that women in the aristocracy typically hired wet nurses to breastfeed their children (Fildes 1986: 193). The second advantage of our empirical setting is that it allows us to conduct placebo tests to validate our results. Unlike settlements in England and Ireland, Scottish entails were perpetual, i.e., they did not have to be renewed upon the heir's wedding (Habakkuk 1994: 6). We estimate our IV model for a comparable sample of women who should not be affected by settlements because they married a Scottish heir, or because they did not marry an heir. Our estimates are close to zero for these populations who did not use settlements. This suggests that our empirical model captures the effect of settlements and not other confounding factors. Third, settlements remained stable in their form between 1650 and 1881. Fourth, our historical setting allows to examine implications over the long-run to which modern data remains silent. This is important because inheritance systems which, like settlements, place restrictions on heirs are increasingly popular today. For example, trusts are widely used among the top one percent (Wolff and Gittleman 2014). In our context, we show that settlements were crucial for the survival of the British aristocracy in a time when strong demographic pressures threatened the extinction of these lineages. Around the 1600s, forty percent of all married women in the aristocracy were childless. Settlements reversed this pattern.

In sum, the first result of the paper is that settlements moved the British aristocracy to a higher fertility regime. This implies that settlements contributed to the perpetuation of elite lineages not only by entailing the land or favoring

[3]The reason is that settlements protected the heir's bequest, but were less stringent with regards to the bequest of other offspring. Hence, this contract should alter fertility incentives on the extensive margin rather than in the intensive margin.

primogeniture as suggested by Adam Smith, Alexis de Tocqueville, or Karl Marx, but also through changing fertility incentives.

The second contribution of the paper is to present a novel theory of intergenerational transmission of wealth that (1) rationalizes the effect of settlements on fertility and (2) shows that such inheritance systems can emerge endogenously in response to concerns over the survival of the dynasty. At first sight, it is not obvious why settlements should affect fertility. In a classic Barro model of wealth transmission with exponential discounting, incentives over fertility and bequests are aligned across generations. Hence, a contract like settlements—which restricts heirs' powers to decide over future bequests—should be innocuous. Our model departs from the classic assumption of exponential discounting. Instead, we adapt the idea of hyperbolic discounting *across generations*, as was first introduced by (Phelps and Pollak 1968). This type of discounting implies that individuals have strong dynastic preferences, as they do not value their childrens well-being significantly more than that of the future generations. Under this assumption, settlements can change fertility incentives and resolve intergenerational time inconsistencies. The economic intuition is simple: when an individual is subject to a settlement, he cannot appropriate the bequest settled for the next generation (e.g., by selling parts of the family estate). The only way in which he can derive utility from this settled wealth is by continuing the family line. This effect will be larger for families with a stronger degree of hyperbolicity (i.e., "dynastic preference").

The second result of our theory is that settlements—or, more generally, inheritance systems which place restrictions on heirs—can emerge endogenously in response to concerns over the continuation of the dynasty. Intuitively, it is far from obvious why an heir would agree to sign a settlement, renouncing to freely dispose of the dynasty's wealth and to decide over next generation's bequests. We show that a settlement is welfare improving for *all* the members of a dynasty with hyperbolic preferences (i.e., "dynastic preference"). On the one hand, the family head is better off as settlements ensure the continuation of the dynasty. On the other hand, the heir is also *ex ante* better off. Under a settlement, he can credibly commit to have children, which guarantees that a larger share of the family wealth will trickle down from the family head. Hence, the family head and the heir agree to sign a settlement as a result of their optimal decisions—even if this restricts the heir's powers in the management and control of the dynasty's wealth.[4]

[4]Specifically, we obtain these results by modeling three generations of the same dynasty that decide sequentially over consumption, bequests, and fertility. We compare the results of a regime where every generation decides the bequest of the next generation to a settlement regime where the first generation decides the bequests of the next two generations.

Relative to the existing literature, we make the following contributions. First, our paper is the first to provide empirical evidence showing that inheritance systems can change fertility incentives on the extensive margin, and hence, contribute to survival of family lineages. There is a growing literature showing that the extensive margin of fertility—i.e., the decision to have children or not—can respond differently to economic changes than the intensive margin of fertility—i.e., the number of children.[5] To the extent of our knowledge, this paper is the first to incorporate the dichotomy between the extensive and the intensive margin of fertility to the study of inheritance. This is an important step, as the economic effects of any inheritance system crucially hinge on the production of an heir.

Second, the bequests literature usually treats inheritance systems as exogenously given (see (Chu 1991) and references therein).[6] We show that intergenerational concerns intrinsic to the bequests problem can shape inheritance systems. Specifically, we propose a new theory where inheritance systems that restrict heirs emerge as a result of the family head's concerns over the survival of the dynasty and the heir's optimal decisions.

Our third contribution is to show that the classic assumption of exponential discounting across generations (Barro 1974) is hard to reconcile with a wide range of historical and modern inheritance systems that restrict heirs' powers in the management and control of the dynasty's wealth; e.g., settlements (England), trusts, fee tails (United States), entails (Scotland), *majorat* (France), *mayorazgo* (Spain), and *ordynacja* (Poland). Among these, trusts are increasingly popular among the very rich (Wolff and Gittleman 2014). Our results highlight the importance of hyperbolic discounting across generations (i.e., "dynastic" preferences) for these arrangements. Hyperbolic discounting has been used to explain savings decisions (Laibson, Repetto, and Tobacman 1998; Diamond and Köszegi 2003), addictive behavior (Gruber and Köszegi 2001), or fertility (Wrede 2011; Wigniolle 2013) *of individuals*. We apply the idea of hyperbolic discounting *across generations*, in line with the seminal paper by Phelps and Pollak (1968).

Finally, we add to the large literature on inheritance systems by presenting settlements, which, despite receiving a lot of attention from contemporaries like Adam Smith, Alexis de Tocqueville, or Karl Marx, are seldom considered by mod-

[5]See Aaronson, Lange, and Mazumder (2014), Baudin, de la Croix, and Gobbi (2015), Baudin, de la Croix, and Gobbi (2018), Brée and de la Croix (2016), and de la Croix, Schneider, and Weisdorf (2017).

[6]Notable exceptions are Chu (1991) and Grieco and Ziebarth (2015). They show that primogeniture can emerge endogenously as a result of, respectively, concerns over the economic survival of the dynasty and insurance against income shocks.

ern economists. So far, the study of settlements focuses on its functioning and has a descriptive nature (Habakkuk 1950; Bonfield 1979; English and Saville 1983).[7] We show that, as suggested by Adam Smith, settlements contributed to the perpetuation of elite lineages. Our results, however, suggest that they did so not only by entailing the land or favoring primogeniture, but also through changing fertility incentives. This challenges the common wisdom that fertility and inequality are negatively associated (Deaton and Paxson 1997; Kremer and Chen 2002; de la Croix and Doepke 2003). In contrast, our results suggest that an increase in the extensive margin of fertility can contribute to the survival of elites.[8]

The article proceeds as follows. Section 2 describes settlements and the data. Section 3 presents reduced-form estimates on the effect of settlements on fertility. Section 4 provides robustness checks for the empirical results. In Section 5, we present our model of inheritance and fertility. Finally, Section 6 concludes.

2 Institutional Setting and Data

2.1 Settlements

How did settlements came into being? Before 1650, settlements were used to set widowhood provisions but not to entail the land. The reason is that a landowner subject to a settlement could easily sell parts of the family estate because nobody defended the interest of the beneficiary, that is, his under-aged or yet-to-be-born son (Habakkuk 1994: p. 7). This changed during the interregnum period with the introduction of trustees, whose role was to defend the interest of these beneficiaries.

Settlements developed during the Interregnum period for reasons unrelated to fertility. After the Civil War, both Royalist and Parliamentarian landowners were afraid of expropriation in case events turned the tide in favor of the opposing side. Settlements ensured their family estates would not be lost. Note that when a landowner signed a settlement, the beneficiary of his estate was no longer him but his heir, most likely an under-aged or yet-to-be-born son who had obviously not taken sides, and thus, who could not be expropriated (Habakkuk 1994: p. 12).

Although the threat of expropriation eventually disappeared, settlements became widely used by the aristocracy to entail the land and to set wife's and younger

[7]The debate is centered on whether settlements were operational given that many family heads died before their heir's wedding, i.e., when settlements were typically signed.

[8]Additional, non-exclusive explanations for the consolidation of elites in pre-modern Europe are institutional capture (Acemoglu 2008; Allen 2009), primogeneiture (Bertocchi 2006), or marriage (Goñi 2018; Marcassa, Pouyet, and Trégouët 2017).

children's provisions. The prevalence of settlements amongst aristocrats is evident both geographically and in terms of social convention. According to Habakkuk (1950), "about one-half of the land of England was held under strict settlement in the mid-eighteenth century." Similarly,

> the full force of social convention and family custom ... [made it such that] ... only an unusually independent or unusually irresponsible young man ... would be able to stand up to such psychological pressures. (Stone and Stone 1984: p.78)

The typical settlement was signed between a father and his heir at the latter's wedding. With the settlement, the heir limited his interest in the estate to that of a life-tenant, ensuring that the family estate would descend unbroken to the first-born son of his marriage (Habakkuk 1950). In other words, settlements restricted heirs powers over the family estates and settled a large portion of the inheritance for yet-to-be-born generations. In order to convince an heir to make such a sacrifice, the father usually transferred him an income to support his household until he inherited the estate. Although settlements were only valid for a generation, *de facto* they operated as a permanent entailment of the land, as settlements were renewed every generation. For settlements to operate in this fashion, however, it was crucial for the father to live until his heir's wedding (Bonfield 1979).

This demographic aspect of settlements is illustrated by the cases of the Brudenell and Craven families. Robert Brudenell, Earl of Cardigan, settled his estates at his heir's wedding in 1668. In contrast, the sixth Lord Craven died when his heir was barely eighteen. As no settlement was signed, the heir could sell large parts of the family estate and did not marry until age 37. He also broke social rules by marrying an actress, Louisa Brunton (Habakkuk 1994: 19, 45, 46).

Settlements were signed at the heir's wedding day for two reasons: First, because the negotiation of settlements also involved the bride's family, who had an interest on the allowances set for her and for the couple's younger children.[9] Second, because as explained above settlements were originally only used to fix these provisions. Although settlements eventually evolved into a contract entailing the land, the date of the signing did not change. In some cases, the signing of the settlement was moved forward to the heir's majority. In Section 4 we show that our results are robust to this.

Importantly, settlements were prevalent in England, Wales, and Ireland, but

[9] We recognize the importance of allowances in the negotiation of settlements. However our analysis focuses on settlements as a legal instrument to entail the land and ensure the integrity of family estates.

not in Scotland. There, land could be entailed *ad perpetuum*. What frustrated the introduction of such permanent entails in England is not clear. Habakkuk (1994: p. 18) suggests that the reasons are unrelated to demographic aspects. He argues that the reason was the strong bias of English Common Law judges for the free alienability of land. In the empirical analysis, we exploit this institutional differences between England and Scotland to conduct placebo tests.

Settlements came to an end with the Settled Land Act in 1882. In the midst of a great debate about landownership inequality, Parliament established that settlements could not prevent the life tenant to sell parts of the land, as long as he obtained the best price and the profits from the sell were settled—that is, the money had to pass down untouched to the next generation (Habakkuk 1994: 1).

2.2 Data

We use genealogical data on the British peerage collected by Hollingsworth (1964). The dataset covers the entire period in which settlements were prevalent (1650-1882) and provides demographic information on c. 1,500 peer heirs and their wives. Unfortunately, the entries from the Hollingsworth (2001) dataset are not linked across generations. To resolve this, we manually matched each entry in the database to their father's entry. This subsection describes the original Hollingsworth (2001) dataset, the process of matching parents to offspring, and presents descriptive statistics.

The original Hollingsworth (1964) dataset is based on peerage records, which contain biographical entries for members of the aristocracy. Hollingsworth tracked all peers who died in 1603–1938 (primary universe) and their offspring (secondary universe).[10] In 2001, the Cambridge Group for the History of Population and Social Structure re-digitized the 30,000 original index sheets. In its current form, the data comprise c. 26,000 individuals. Each entry provides the date of birth, marriage, and death, as well as a variable indicating its accuracy. It also states social status, title, whether he was heir-apparent at age 15, parents' social status, and whether a title is an English, Scottish, or Irish peerage. Social status comprises five categories: (1) duke, earl, or marquis, (2) baron or viscount, (3) baronet, (4) knight, and (5) commoner. If the individual was married, we also know the spouses' date of birth, date of death, and social status. Each entry also lists the name and the date of birth of the children born to this marriage.

The entries from the original Hollingsworth (2001) dataset are not linked across

[10]For a list of all the peerage records used, see Hollingsworth (1964), Appendix I.

generations. Hence, we matched each entry in the database to their parents entry. First, we match non-heirs (i.e., peers' daughters and younger sons) to their parents exploiting the fact that the reference number identifying non-heirs is a consecutive number of their father's reference number. The matching of heirs is less trivial: we match an entry C (children) to entry P (parent) if the information in entry C corresponds to what entry P reports about P's children. Specifically, we match entries according to the variables surname, name, date of birth, and accuracy. We perform four iterations in which matches are produced according to different combinations of these variables. At each iteration, we remove matched entries and we check double matches manually using information from `thepeerage.com`, an online genealogical survey of the peerage of Britain. We also use this webpage to double check matches in which the father's and children's surname display a Levenshtein distance above 1.[11] Finally, we match the remaining 1,503 unmatched heirs to their parents manually using `thepeerage.com`. Overall, we match 98.25 percent of the 26,499 entries in the dataset to their parents. Appendix A provides a detailed description of the matching process.

Table 1 presents descriptive statistics for c. 1,500 peer heirs marrying in 1650–1882, and their wives. This is the main sample used in our empirical analysis. On average, 17 percent of married heirs remained childless. Admittedly, peers had children out of wedlock. Therefore, our childlessness rates might be an overestimate. However, illegitimate children did not inherit and therefore are not relevant for our analysis. Those who were not childless had, on average, 5.64 children. Wives were younger than husbands at marriage (22 versus 27 years old) and died at a similar age (60 versus 58 years). Around 50 percent of them had girls as the last child, indicating that on average parents did not stop having children after producing son.[12] Regarding socio-economic status, 63 percent of the individuals were heirs to dukedom, an earldom, or a marquisate. Forty-five percent are heirs to an English peerage, 31 percent to an Irish peerage, and 24 percent to a Scottish peerage, where settlements were not prevalent. In Section 3 we will exploit this sub-sample for a placebo test. Finally, 56 percent of the heirs married before their father's death. Since settlements were typically signed at the heir's wedding, this suggests that around 56 percent of our sample signed a settlement. In the next section, we will use this proxy for settlements to gauge their impact on childlessness and on completed fertility, i.e., number of births.

[11]The Levenshtein distance measures the minimum number of single-character edits required to change one surname into the other.

[12]For this variable, the sample is reduced to 899 because it considers heirs who had at least a child who (1) also appears in the Hollingsworth (2001) database and (2) who could be matched.

3 The effect of settlements on fertility

In this section we show that settlements reduced childlessness in the British peerage. We begin by describing historical trends in childlessness for the peerage, other European aristocracies, and the general population. Next, we present OLS estimates showing that families which failed to sign a settlement were more likely to be childless. To establish causality, we pursue two strategies: We exploit exogenous variation in (male) heirs birth order, which affects a family heads probability to live until his heirs wedding—when settlements were signed/renewed. Second, we perform placebo tests by estimating our model for non-heirs and Scottish heirs. We find zero effects for these populations who did not use settlements.

3.1 Historical trends

Compared to the general population, the British aristocracy had more children but a considerably higher childlessness rate. Figure 1 plots the average fertility of mothers (left panel) and childlessness rates (right panel), for all peers' daughters first-marrying between ages 15 and 35 in 1600–1959. Dots illustrate the corresponding estimates for the general population.[13] On average, mothers in the aristocracy had between 4 and 5 children before the 1800s. The peerage experienced a demographic transition around 1810, eighty years earlier than the general population. This is consistent with previous research on the fertility of wealthy individuals (Clark and Cummins 2009).

In contrast, marital childlessness rates amongst aristocrats were astonishingly high. For example, in the 1600s between 30 and 40 percent of all married women in the aristocracy were childless. In the general population, the corresponding rate was only c. 10 percent. The rate of childlessness in the peerage was high also in comparison to other European aristocracies. For example, Pedlow (1982) and Lévy and Henry (1960) show that childlessness rates among the aristocracy of Hesse-Kassel (Germany) and of France were, respectively, 5 and 9 percent in 1650-99 (see Appendix B, Table B.I).[14]

The high rates of childlessness in the peerage in 1600 posed a threat for the survival of aristocratic dynasties. By 1650, however, childlessness rates started

[13]Estimates for the general population are from de la Croix, Schneider, and Weisdorf (2017), Galor (2011), Anderson (1998) and Wrigley et al. (1997).

[14]These comparisons have to be taken with grain of salt. First, Pedlow (1982) and Lévy and Henry (1960) base their estimates on a few observations. Second, the sample of French nobles is women marrying before 20, probably selecting women who married close relatives in pre-arranged marriages, which could affect childlessness rates (Goñi 2014).

to decline and by 1850 they reached 10 percent, the level for the general population (de la Croix, Schneider, and Weisdorf 2017). This trend coincides with the introduction of settlements. Next, we show that settlements crucially moved the peerage to a higher fertility regime, ensuring the survival of aristocratic dynasties.

3.2 OLS estimates

Here we show that settlements reduced childlessness rates in the British peerage. Ideally, we would like to compare fertility outcomes in families that signed a settlement to the outcomes of similar families who did not sign it. Unfortunately, we do not observe who signed a settlement and who did not. To resolve this issue, we exploit that, because of institutional constraints and tradition, settlements were signed at the heir's wedding.[15] In other words, for a settlement to be signed, it was crucial for the father to live until his heir's wedding (Bonfield 1979). If the father died before that date, the heir would not be subject to a settlement; he could dispose of the family estate at will, sell parts of it, and decide over the next generation's bequest. We use the fact that a father lived (did not live) until his heir's wedding as a proxy for the presence (absence) of a settlement. Formally, we estimate:

$$\chi_{i,j,b,q} = \beta \cdot S_i + \mu_j + \mu_b + \mu_q + \mathbf{X}'_{i,j,b,q}\gamma + \epsilon_{i,j,b,q} \ . \qquad (1)$$

Our unit of observation is a matrimony where the husband, i, is heir to a peerage. χ equals one if the matrimony did not have any children and equals zero otherwise. As explained above, our proxy for settlements, S, indicates if i's father lived until his heir's wedding. The coefficient β captures the association between settlements and childlessness. Following Galor and Klemp (2014), we include fixed effects for the (husband's) family, μ_j, and cluster standard errors by family. That is, we identify the effect of settlements using variation in fertility among members of the same lineage. This will capture any genetic, cultural, religious, or socio-economic predisposition towards childlessness among these genetically related individuals. In addition, childlessness may be affected by the socio-economic and demographic conditions during one's lifetime. To capture such lifecycle effects, we include birth year dummies, μ_b, and dummies indicating the quarter-century in which the marriage took place, μ_q. Finally, the vector $\mathbf{X}$ includes covariates that may

[15] As explained in Section 2, settlements were signed at the heir's wedding day for two reasons: First, because they also included provisions for the bride's family which had to be negotiated around the wedding date. Second, because originally settlements were only used to fix these provisions. Although settlements evolved into a contract entailing the land, the date of the signing did not change.

also affect childlessness rates such as social status of the spouses, wife's age at marriage, spouses' age at death, history of stillbirths in the husband's family, and the number of siblings of the husband. The latter accounts for the allowances for siblings, typically specified in the settlement.

Table 2 presents the results of estimating Equation (1) for all matrimonies where the husband is a peer or a peer heir between 1650 and 1882 using OLS.[16] There is a strong, significant association between settlements and childlessness. Signing a settlement is associated with a decrease in the probability of being childless by 4 to 8 percentage points. Results are robust to the inclusion of covariates that may also affect childlessness, like the social status of spouses, the wife's age at marriage, or the ratio of stillbirths to live births in the husbands family (cols. 2 and 3). The precision of the model increases when we include family fixed effects to control for unobserved heterogeneity in terms of genetic preconditions, culture, or social-economic position. Finally, by including dummies for birth year and quarter-century of the marriage we control for life-cycle conditions (col. 4).

These results suggest that settlements altered incentives in the extensive margin of fertility. The rationale is that an individual's decision to have children or not depends on the wealth he can bequeath them. If the family estate is broken, parts of it have been sold or mortgaged, etc. the likelihood to have children may be lower, as the dynasty's wealth is be substantially reduced. Signing a settlement prevents this, and hence, reduces childlessness rates. Note that, since primogeniture prevailed in England, settlements protected the heir's bequest more, and hence, should affect the production of an heir more than the production of a second, third, fourth, etc. offspring. That is, we expect settlements to alter fertility incentives in the extensive margin, but not in the intensive margin.

Table 2, column (5) confirms this. It presents results of poisson regressions[17] of Equation (1)'s form, with the number of births as dependent variable. To explain away the effect of settlements on childlessness, we restrict the sample to matrimonies having at least one child. Results suggest that signing a settlement did not significantly affect the intensive margin of fertility: our proxy for settlements is not significantly associated with he number of live births, conditional on having at least one child. The estimates are small in magnitude: a coefficient of 0.036 indicates that an heir signing a settlement is expected to give birth to 3.6 percent

[16] Our preferred specification is a linear probability model. The reason is that it is more flexible in dealing with fixed effects—which in our case are crucial to control for genetic, cultural, and religious unobserved factors affecting fertility at the family level. That said, our baseline results are robust to using non-linear probit or logit models (results are available upon request).

[17] Poisson regression is the standard model for count data like the number of live births.

more children than what he would have if he had not signed a settlement. Given that, conditional on not being childless, the average number of births of an heir's wife is 5.2, this effect is equivalent to having 0.19 more children.[18]

Altogether, the evidence indicates a strong correlation between settlements and childlessness, but not with the number of births. In other words, settlements are associated with the extensive margin of fertility, but not with the intensive margin. Next, we provide evidence suggesting that the effect of settlements on childlessness is causal.

3.3 IV estimates

Here we estimate the causal effect of settlements on childlessness using an instrumental variables approach. Whether a settlement was signed or not depends on many factors, some of which might be endogenous to childlessness. Specifically, it could be that heirs with certain characteristics that are correlated to childlessness may choose not to sign a settlement by delaying marriage until their father's death. We exploit exogenous variation in our proxy for settlements—i.e., whether a father lived until the heir's wedding or not—coming from the heir's birth order.

The intuition of our instrument is simple. Families who decide to have an heir cannot control the gender of any birth. In some families, an heir might not be born until the second or third birth. Therefore, the age difference between father and heir will be larger, which exogenously decreases the father's probability to live until his heir's wedding. In contrast, in families in which the first birth is a son, the father will be younger, more likely to live until this son's wedding, and hence, more likely to sign a settlement.

Formally, we treat our proxy for settlements, S, as an endogenous variable:

$$S_i = \sum_{n=2}^{15} \beta_n \mathbb{I}(r_i = n) + \beta_z Z_i + \mu_q + \mathbf{X}'_{i,q}\gamma + \epsilon_{i,q} \ , \tag{2}$$

where S_i indicates if i's father lived until i's wedding. That is, it is equal to one when i is likely subject to a settlement and equal to zero otherwise. Our principal instrument is $r_{i,q}$, the birth order of the heir i. The indicator function $\mathbb{I}$ is equal to one when $r_i = n$ and zero otherwise. We also include the age at death of i's father, Z, which obviously affects S without regard to i's birth order. As in equation (1), μ_q are marriage quarter-century fixed effects; and $\mathbf{X}$ is a vector of

[18] In addition to this evidence, note that historical trends suggest that peers and commoners (who did not use settlements) present a comparable record for the number of births (Figure 1).

covariates including social status of the spouses, wife's age at marriage, spouses' age at death, history of stillbirth in the husband's family, and the total number of siblings of the heir.

The causal effect of settlements on childlessness is captured by coefficient β in:

$$\chi_{i,j,b,d} = \beta \hat{S}_i + \mu_j + \mu_b + \mu_q + \mathbf{X}'_{i,j,b,q}\gamma + \epsilon_{i,j,b,q} \ . \tag{3}$$

where $\hat{S}_i$ is the value of S_i estimated from Equation (2), and μ_j and μ_b are, respectively, family and birth year fixed effects.

Table 3 presents the instrumental variables' results. First-stage estimates show that, relative to first-born heirs, later-born heirs were less likely to marry before their father's death, and hence, to sign a settlement. For example, a third-born heir was 10 percentage points less likely to sign a settlement, a fourth-born heir 11.9 percentage points, etc. The remaining covariates have expected signs: the probability of signing a settlement increases in the father's live expectancy and the family's social status, and decreases with wife's age at marriage.

Second-stage estimates show that settlements had a negative, causal effect on childlessness. An heir marrying before his father's death and, thus, signing a settlement, was 14.7 percentage points less likely to be childless. The estimated effect is sizable. Given that the average childlessness rate for heirs was 17.6%, settlements increased by 83.5% the extensive margin of fertility, pushing childlessness rates close to the "natural" rate of 2.4 percent (Tietze 1957). Note that the bias affecting the OLS results is an attenuation bias. A possible explanation is that if the father lived until the heir's wedding, he likely influenced the choice of a bride. In other words, the heir might have enjoyed less freedom when choosing his bride. If such marriages have less children (e.g., because they socially convenient marriages rather than love matches), this could explain the attenuation bias in our OLS specification, corrected by the IV model.

Covariates have expected signs. For example, marrying an older wife significantly increases the probability of not having children. Note that the effect is much lower than that of settlements. In detail, to match the estimated effect of settlements on childlessness one would have to marry a wife aged 12 years younger.

In sum, the evidence shows that settlements had a negative, large causal effect on childlessness. Heirs born after several daughters were exogenously less likely to marry before their father died. That is, they were exogenously less likely to sign a settlement, and thus, could dispose of the family estates at will, sell parts of it, and decide over the next generation's bequest. As a result, their rates of childlessness were high. In contrast, first-born heirs were exogenously more likely to sign a settlement, which reduced their childlessness rates.

Next, we provide evidence supporting our identification strategy. Our identifying assumptions are that the instrument is relevant and that the exclusion restriction is satisfied. Since we estimated a triangular IV model, we also show evidence for the validity of our triangular IV specification.

First stage results confirm that the birth order of the heir is a relevant instrument for our proxy for settlements: in families in which the heir is born after one or two daughters, the father is older and thus the likelihood that he survives until the heirs' wedding is smaller than if the heir is his first-born child. Furthermore, F-stats are large enough to rule out concerns about weak instruments.

The validity of our identification strategy, hence, rests on the exclusion restriction. The exclusion restriction would be violated if the heir's birth order affects childlessness through channels other than the probability of signing a settlement. A potential concern is that birth order is associated with breast-feeding. In developing economies, it has been shown that breastfeeding increases with birth order, as mothers make use of the contraceptive properties of nursing when they hit the desired family size (Jayachandran and Kuziemko 2011). Since breast-feeding confers health benefits, low-birth-order heirs may be healthier, and hence, less likely to be childless. This scenario is unlikely in our historical context. Women in the aristocracy typically did not breastfeed their children; the common practice was to hire wet nurses (Fildes 1986: 193).[19] In other words, it is unlikely that breastfeeding is associated with birth order amongst aristocrats.

Finally, note that our main specification is a triangular IV model in which not all the first-stage covariates are included in the second-stage.[20] In detail, we include father's age at death in the first-stage but do not consider it to affect childlessness in the second-stage. The implicit assumption is that father's age at death does not have a direct effect on childlessness other than affecting the probability of signing a settlement. This assumption would be violated, for example, if an early age at death of the father reflects poor health conditions that are transmitted across generations. This scenario is unlikely for three reasons. First, we include the history of stillbirths in the second stage and estimate all the effects using family fixed effects. This captures any genetic predisposition towards childless-

[19]Moreover, the mechanism highlighted for developing countries is that women would like to limit the number of children because of budget constraints considerations. British Aristocrats were extremely wealthy and therefore did not face the same problem.

[20]To fit this model, we estimate the recursive equation system (2) and (3) by maximum likelihood using the STATA user-written command cmp (Roodman 2015).

ness. Conditional on these covariates, father's age at death likely does not affect childlessness. Second, we test the exogeneity of father's age at death formally by conducting Sargan-Hansen tests. Results suggest that, conditional on birth order being a valid instrument, father's age at death is exogenous to childlessness rates. Finally, the triangular structure of our IV model is not driving our results. In Appendix B we estimate a classic IV model (i.e., including all second-stage covariates in the first stage) and show that our main results are robust.

A final caveat is that we do not observe which families signed a settlement and which did not. Our estimates are based on a proxy that exploits whether the family head died before or after his heir's wedding. In the next section, we address this by analyzing two populations for whom we do observe that settlements were not signed: non-heirs and Scottish heirs.

3.4 Placebo tests

So far, we have shown that peerage families in which the father lived until the heir's wedding, that is, families which likely signed a settlement, were less likely to be childless. We interpreted this as evidence that settlements reduced childlessness rates. However, since we do not observe which families signed a settlement and which did not, our interpretation crucially hinges on the assumption that our proxy for settlements does not affect fertility through other channels. That is, that the survival of the father until the heir's wedding does not affect fertility through channels other than the settlement. We plausibly addressed this by controlling for genetic and socio-economic factors that are likely correlated with the survival of the father and the next generation's fertility. To further validate our interpretation of the results, here we use data from two populations that did not use settlements: non-heirs and Scottish heirs.

Specifically, we conduct two placebo tests. We estimate the instrumental variables system in Equations (2) and (3) with a comparable sample of matrimonies who should not be affected by settlements because (a) the husband was not an heir, or because (b) the husband was heir to a Scottish peerage. Unlike settlements in England and Ireland, Scottish entails were perpetual, i.e., they did not had to be renewed every generation at the heir's wedding (Habakkuk 1994: 6). If our proxy—i.e., whether father lived (did not live) until his heir's wedding—only affects fertility through settlements, we should find a zero effect for these populations that did not use settlements. If our estimation captures confounding factors correlated with fertility, the estimates will also be negative for these placebo samples. Similarly, this placebo test can be used to assess the validity of the exclusion restriction. If the birth order of the heir (or the father's age at death) only affects childlessness through our proxy for settlements—we should find no effect for these populations.

Table 4 presents the results of these placebo tests. The effect of our proxy on childlessness is much smaller and not significantly different from zero for non-heirs (col. 2).[21] In other words, for those who did not inherit the family estates, our proxy rightly indicates that settlements did not affect their choice of having children. A Wald test confirms that the estimated coefficients are significantly different from the baseline effect for the sample of heirs (col. 1). Hence, our proxy (and our instruments) do not seem to have a direct effect on childlessness other than that operating through settlements.

We find similar results when we compare heirs to an English or Irish peerage (col. 3) to heirs to a Scottish peerage—who did not renew settlements every generation (col. 4). Signing a settlement decreases the probability of being childless by 16 percentage points in matrimonies where the husband was an English or Irish heir. The childlessness rates of Scottish heirs is not affected by the fact that the father lived until his heir's wedding or not: the coefficient is small and not significantly different from zero. The Wald test rejects that the effect is the same across populations. Note that, compared to the results in columns (1) and (2), the Wald test is weaker. This may be the result of the measurement error: on the one hand, there are fewer Scottish peers, so the regression is estimated with fewer observations. On the other hand, Scottish peers could held land and titles in England too, so some of them might have been subject to settlements.

Note that the Wald tests in columns (3) and (4) can be interpreted as difference-in-differences estimators. Specifically, the Wald test captures the differential effect of our proxy for signing a settlement on English and Irish heirs (treatment group) versus non-heirs or Scottish heirs (control group). In this difference-in-differences framework, the control group washes away any factor other than settlements that is correlated with our proxy (or our instruments) and may affect fertility. Results suggest that childlessness rates were reduced only for those who signed a settlement; i.e., heirs in England and Ireland whose father lived until their wedding. In contrast, for non-heirs or heirs to a Scottish peerage, marrying before or after their father's death does not seem to affect childlessness. This strongly suggests that our estimation captures the effect of settlements on fertility.

[21] Note that in this case the instrument is the birth order of the family heir, that is, the birth order of the husband's older brother.

4 Robustness and extension

This section examines the robustness of our results and presents an extension of the analysis. First, we consider the possibility that settlements were signed at the heir's majority. Second, we estimate an alternative IV model exploiting variation in the gender of the first birth. Third, we explore whether the socioeconomic changes triggered by the Industrial Revolution affect our estimates.

4.1 Settlements signed at heir's majority.

So far, our empirical strategy assumes that a settlement was signed if the family head lived until the wedding of his heir. Although most settlements were signed at the heir's wedding, some settlements were signed when the heir turned 21, the age of majority. The reason was that

> the father might find it advantageous to bargain with his eldest son before a marriage was in immediate prospect to avoid the pressure of the bride's family. (Habakkuk 1994: p. 26).

Here we show that assuming that settlements were signed at the heir's majority does not alter our main conclusions. Formally, we estimate the IV model in equations (2) and (3) with an alternative proxy for settlements, S_i, indicating if i's father lived until i's majority. This alternative approach has the advantage of disentangling the two purposes of a settlement: entailing the land and setting a provision for the wife. As reflected in Habakkuk's quote, settlements signed at the heir's majority would only reflect the former, while settlements signed at the heir's wedding may also reflect the interest of the bride's family for a larger allowance.

Table 5 presents our results using this alternative proxy. As before, we find that signing a settlement decreased the probability to be childless by 8 to 15 percentage points. The magnitude of the IV coefficient (col. 2) is not significantly different to that of Table 3.[22] The heir's birth order is also a relevant instrument under this alternative specification. First-stage results (Panel B) show that first-born heirs were more likely to turn 21 before their father's death than later-born heirs. Columns 3 to 5 present placebo tests using two populations for which we know settlements were not signed. The childlessness rates of non-heirs or heirs to

[22] As before, we find an attenuation bias in the OLS estimates. Our conjecture was that heirs who chose to delay marriage until their father's death were freer to chose a bride and, hence, may have had higher fertility. This conjecture is also valid here, as average age at marriage was was 28.7, significantly above the age of majority.

a Scottish peerage were not affected by the fact that the family head lived until his heir's majority or not. Wald tests reject the null hypothesis that the effects are the same for the sample of heirs vs. the sample of non-heirs and Scottish heirs. In other words, neither our proxy (nor our instruments) seem to have a direct effect on childlessness in the placebo group, strongly suggesting it captures the effect settlements. Finally, Column 6 suggests that our alternative proxy for settlements is not associated with the intensive margin of fertility.

Finally, note that settlements signed at the heir's majority were not influenced by the interest of the bride's family as much as settlements signed at the heir's wedding. The fact that we find similar results as before suggests that the effect of settlements on childlessness is the result of family interests to entail of land, and not the result of the bride's family interest in setting family provisions.

4.2 Alternative IV: gender of the first-born child.

Here we use an alternative instrument. We exploit exogenous variation in our proxy for settlements—i.e., the probability that a father lives until his heir's wedding—coming from the gender of the father's first child. One cannot manipulate a child's gender. In some families, the first-born will be a girl and the father will be older when his heir is born than what he would have been had his first-born been a boy. This decreases (exogenously) the probability of living until the heir's wedding, and hence, of signing a settlement.

Formally, we treat our proxy for settlements as an endogenous variable:

$$S_i = \beta_g G_i + \beta_z Z_{i,q} + \mu_q + \mathbf{X}^{'}_{i,q}\gamma + \epsilon_{i,q} \ , \tag{4}$$

where S_i indicates if i's father lived until his heir's wedding. Our instrument is the gender of the first birth, G, which is equal to one when the first-born child of i's father was a daughter. As before, we include the age at death of i's father, Z; marriage quarter-century fixed effects, μ_q; and a vector of covariates $\mathbf{X}$ (spouses' social status and age at death, wife's age at marriage, the history of stillbirths in the husband's family, and the number of siblings of the heir). The second stage takes the form of equation (3), where $\hat{S}$ is now estimated from equation (4).

This approach presents some advantages. In our main specification in Section 3, we used the birth order of the heir, that is, we exploit variation coming from the gender of *all* the births before an heir is produced. This instrument is correlated with family size, which could be problematic if, for example, larger families with many daughters to marry off became cash-constrained due to dowry payments. This scenario is unlikely—peerage families were extremely wealth (Rubinstein 1977). However, we use this alternative approach to fully rule-out such concerns, as the gender of the first-birth *alone* is not correlated with family size.

Table 6 (Panel B) presents the first-stage results. In families in which the first-born was a girl, the heir was eight percentage points less likely to marry before his father's death, and hence, to sign a settlement. Second-stage results (Panel A) are consistent with our previous findings: Signing a settlement decreased the probability of being childless by 14.6 percentage points. As before, childlessness rates were not affected for our placebo populations not using settlements, i.e., non-heirs (col. 3) and Scottish heirs (col. 5).

4.3 Settlements after the Industrial Revolution.

A natural question is whether the effect of settlements varies over time. Specifically, our time window (1650–1882) includes the Industrial Revolution, an event that triggered major economic and demographic changes. Whether this altered the effect of settlements on fertility is an open question. On the one hand, the value of land relative to industrial wealth likely decreased after the Industrial Revolution.[23] Aristocrats might have faced lower incentives to sign a settlement to consolidate their landholdings. This should weaken the effect of settlements on fertility. On the other hand, according to Doepke and Zilibotti (2008), the "fine tastes for leisure" of the landowners were not affected by the Industrial Revolution; they continued to live off their land rents. If this was the case, neither the incentive to sign a settlement nor its effects on fertility should be altered.

To answer this question, we split our sample before and after the Industrial Revolution. The estimated effects remain stable. Table 7, col. (1) presents our baseline IV-estimates. Heirs whose father lived until their wedding, that is, heirs who signed a settlement, were 14.8 percentage points less likely to be childless. In columns (2) and (3), we restrict the sample to matrimonies occurring before and after 1770—a date that marks the start of the first Industrial Revolution.[24] The estimated effects are very similar to those in the baseline specification.

Overall, this suggests that preferences of aristocrats over signing a settlement and over fertility persisted over time, even after the Industrial Revolution.

[23] This is a relative statement. It was not until the twentieth century that industrial wealth became more important than landownership. For example, from 1800 to the 1870s, 80–95 percent of millionaires were still landowners (Rubinstein 1977: 102).

[24] We chose 1870 to mark the start of the Industrial Revolution because in the 1770s the spinning jenny was patented (1770), water frames were installed in cotton mills (1771), the Boulton & Watt partnership was founded (1775), and the spinning mule was invented (1779).

This provides empirical support for the theory developed by Doepke and Zilibotti (2008), which claims that preferences (over leisure) of landowners were constant in time, eventually triggering their downfall as the economically dominant group.

5 A model of inheritance and fertility

So far, we have shown that settlements had a causal effect on the extensive margin of fertility. Here, we provide a theoretical framework that explains this relation. The model has two important results. First, exponential discounting is hard to reconcile with inheritance systems which place restrictions on heirs (e.g., settlements or trusts). Instead, we show that the degree of hyperbolicity across generations (i.e., dynastic preferences) can rationalize settlements and their effects on fertility. Second, we show that such inheritance systems can emerge endogenously in response to a family head's concerns over the dynasty's survival and his heir's optimal decisions.

5.1 Setup

We assume a three-period sequential-move game played by three generations, $i=\{1,2,3\}$, of the same dynasty. One can think of these as father, son, and grandson. Each generation decides his consumption, x_i, and fertility, $n_i=\{0,1\}$. We model fertility as a binary choice and assume there is no uncertainty regarding the production of an heir.[25] If a generation decides not to have children, we assume that the dynasty dies out after this generation. This can be interpreted as the dynasty's wealth passing to a distant relative whose utility is fully discounted.[26]

Each generation derives utility from his consumption and that of the following generation(s) in case the dynasty continues.[27] Formally, the utilities of generations 1, 2 and 3, respectively v_1, v_2, and v_3, are

$$v_1(x_1, x_2, x_3, n_1, n_2) = u(x_1) + n_1 \cdot \left[\beta\delta\ u(x_2) + n_2 \cdot \beta\delta^2\ u(x_3)\right], \tag{5}$$

$$v_2(x_2, x_3, n_2) = u(x_2) + n_2 \cdot \beta\delta\ u(x_3), \tag{6}$$

[25] Alternatively, Li and Pantano (2014) model fertility in a dynamic framework in order to account for sex selection. In our setting, sex selection was not prevalent: 49% of last births were girls, suggesting that families did not stop having children after conceiving an heir (see Table 1).

[26] In Appendix D, we remove the assumption that the dynasty dies out after generation 3 and show that our results are robust.

[27] We abstact from parental investments in shaping children's preferences. Doepke and Zilibotti (2008) provide a model of endogenous preferences (without fertility decisions) showing that landowner's preferences are constant over time, even after the Industrial Revolution. Hence, our theoretical results should be robust to allowing parents to shape their children's preferences.

$$v_3 = u(x_3). \tag{7}$$

We depart from the classic bequests' models by assuming a quasi-hyperbolic discount function towards future generations. This means that individuals are present biased but, at the same time, do not value their children's well-being significantly more than that of future generations, namely their grandsons. Formally, the discount function has two components: $\delta \in [0, 1]$ is the standard discount for future generations; $\beta \in [0, 1]$ discounts all the future consumptions compared to his own. Note that this additional discount factor captures dynastic preferences. Consider Figure 2: for low values of β, generation 1 has a strong dynastic preference, as he discounts generations 2 and 3 similarly. For high values of β, the discount function tends to the exponential discount function, implying that he values the consumption of generation 3 much less than that of generation 2. In Section 5.3 we will show that hyperbolic discounting is crucial to rationalize settlements, or more generally, inheritance systems that restrict heirs.

Each dynasty is endowed with wealth K (e.g., landholdings). This endowment is used to subsidize consumption of all generations. Therefore, the decisions of each generation depend on how the dynasty's wealth K is passed down from one generation to the next—that is, they depend on the inheritance system.

Dynasties are heterogeneous with respect to the inheritance system. Specifically, we consider two dynasties: In one, each generation decides the bequest of the next generation. Henceforth, we refer to this as the *no-settlement* inheritance regime (subscripted by $\neg s$). Alternatively, in another dynasty generation 1 decides the bequests of generations 2 and 3. We call this the *settlement* regime (subscripted by s). Note that this regime allows generation 1 (the father) to settle part of the dynasty's wealth for generation 3 (the grandson). More generally, it represents any inheritance system that restricts heirs' powers in the management and control of the dynasty's wealth.

Formally, we model these two inheritance regimes through the budget constraints faced by each generation. For the dynasty in the no-settlement regime, generation i decides the bequests of the next generation, k_{i+1}. The budget constraints of generations 1 and 2 are, respectively,

$$K = x_1 + k_2 \tag{8}$$

$$k_2 = x_2 + k_3. \tag{9}$$

For the dynasty in the settlement regime, i.e., generation 1 decides all bequests k_2 and k_3, the budget constraints of generations 1 and 2 are, respectively,

$$K = x_1 + k_2 + k_3 \tag{10}$$
$$k_2 = x_2. \tag{11}$$

Finally, the dynasty disappears after generation 3. Hence, this last generation will consume all the bequests he receives from previous generations: $k_3 = x_3$.

5.2 Equilibrium

This subsection defines and characterizes the model's equilibrium. Since we have a sequential-move game with perfect information and finite time, we use subgame perfect equilibrium (SPE) as the solution concept.

Definition 1 (SPE) *The SPE is a strategy profile $\{k_2, k_3, x_1, x_2, x_3, n_1, n_2\}$ for the dynasty in the no-settlement regime and a strategy profile $\{k_2', k_3', x_1', x_2', x_3', n_1', n_2'\}$ for the dynasty in the settlement regime, where:*

- *$\{k_2, x_1, n_1\}$ maximize v_1 subject to (8), $\{k_3, x_2, n_2\}$ maximize v_2 subject to (9), and x_3 maximize v_3 subject to $x_3 = k_3$,*
- *$\{k_2', k_3', x_1', n_1'\}$ maximize v_1 subject to (10), $\{x_2', n_2'\}$ maximize v_2 subject to (11), and x_3' maximize v_3 subject to $x_3' = k_3'$.*

We solve this model in three steps: First, we use backward induction to find optimal consumption and bequests for different fertility scenarios. Second, we define fertility gains by comparing the indirect utilities of having children and being childless. This allows us to show how settlements affect fertility incentives. Finally, we use these fertility gains to characterize the SPE. Hereafter, we assume log-utility for simplicity; i.e. $u(x_i) = \ln x_i$.

Consumption and bequests. Equilibrium consumption and bequests are identical for dynasties in the settlement and no-settlement regime when fertility is low. When n_1=0 generation 1 consumes all the dynasty's wealth, x_1=K, regardless of the inheritance regime. Similarly, when n_1=1 and n_2=0 the optimal consumption and bequests are given by x_1^*, x_2^*, and k_2^* in both regimes. Only for high fertility levels, i.e., when n_1=n_2=1, do consumption and bequests differ: $x_{1,\neg s}^{**}$, $x_{2,\neg s}^{**}$, $x_{3,\neg s}^{**}$, $k_{2,\neg s}^{**}$, and $k_{3,\neg s}^{**}$ in the no-settlement regime, and $x_{1,s}^{**}$, $x_{2,s}^{**}$, $x_{3,s}^{**}$, $k_{2,s}^{**}$, and $k_{3,s}^{**}$ in the settlement regime. Importantly, $x_{2,s}^{**} < x_{2,\neg s}^{**}$, $x_{3,s}^{**} > x_{3,\neg s}^{**}$, and $k_{3,s}^{**} > k_{3,\neg s}^{**}$. That is, in the settlement regime, generation 1 redistributes consumption from generation 2 to generation 3. He does so by settling a larger bequest for generation 3 than the one generation 2 would have passed down in the no-settlement regime. All optimal choices are characterized in detail in Appendix C.1, Lemmas 1 and 2.[28]

Fertility. Next, we use these optimal consumptions and bequests to define fertility gains. Specifically, we compare the indirect utilities of having children and being childless under the different inheritance (and fertility) regimes.

Definition 2 (Fertility gains) *Fertility gain is the difference of the indirect utility of having children and being childless. For generation 2, $f_{2,\neg s}$ and $f_{2,s}$ are the fertility gains in, respectively, the no-settlement and the settlement regimes:*

$$\begin{aligned} f_{2,\neg s}(k_2) &:= v_2\left(x_2{=}\frac{k_2}{1+\beta\delta}, x_3{=}\frac{\beta\delta k_2}{1+\beta\delta}, n_2{=}1\right) - v_2(x_2{=}k_2, x_3{=}0, n_2{=}0), \\ f_{2,s}\ (k_3) &:= v_2\left(x_2{=}k_2, x_3{=}k_3, n_2{=}1\right) - v_2(x_2{=}k_2, x_3{=}0, n_2{=}0). \end{aligned}$$

For generation 1, $f_1^{n_2=0}$ defines fertility gains when $n_2{=}0$ in both regimes:

$$f_1^{n_2=0}(K) := v_1(x_1^*, x_2^*, n_1{=}1) - v_1(x_1{=}K, n_1{=}0)$$

and $f_{1,\neg s}^{n_2=1}$ and $f_{1,s}^{n_2=1}$ are the fertility gains of generation 1 when $n_2{=}1$ in, respectively, the no-settlement and settlement inheritance regimes:

$$\begin{aligned} f_{1,\neg s}^{n_2=1}(K) &:= v_1(x_{1,\neg s}^{**}, x_{2,\neg s}^{**}, x_{3,\neg s}^{**}, n_1{=}1) - v_1(x_1{=}K, n_1{=}0), \\ f_{1,s}^{n_2=1}(K) &:= v_1(x_{1,s}^{**}\ , x_{2,s}^{**}\ , x_{3,s}^{**}\ , n_1{=}1) - v_1(x_1{=}K, n_1{=}0). \end{aligned}$$

The fertility gains of generation 2 illustrate the mechanism through which settlements can change fertility incentives. Consider the dynasty in the no-settlement regime. Generation 2's fertility gains, $f_{2,\neg s}(k_2)$, are increasing in his bequest, i.e., $\partial f_{2,\neg s}/\partial k_2{>}0$.[29] In other words, by passing down a larger bequest k_2, generation 1 can manipulate generation 2's incentives to have children. This differs in the dynasty subject to the settlement regime. Generation 2's fertility gains $f_{2,s}(k_3)$ no longer depend on the bequest he receives but on the bequest settled for generation 3, that is, k_3. If generation 1 settles a larger share of the dynasty wealth, generation 2 will be more likely to have children, i.e., $\partial f_{2,s}/\partial k_3{>}0$.[30]

[28] In short, $x_1^*{:=}\frac{K}{1+\beta\delta}$; $x_2^*{=}k_2^*{:=}\frac{\beta\delta K}{1+\beta\delta}$; $x_{1,\neg s}^{**}{:=}\frac{K}{1+\beta\delta+\beta\delta^2}$; $x_{2,\neg s}^{**}{:=}\frac{1+\delta}{1+\beta\delta}\ \frac{\beta\delta K}{1+\beta\delta+\beta\delta^2}$; $x_{3,\neg s}^{**}{=}k_{3,\neg s}^{**}{:=}\frac{\beta(1+\delta)}{1+\beta\delta}\ \frac{\beta\delta^2 K}{1+\beta\delta+\beta\delta^2}$; $k_{2,\neg s}^{**}{:=}K{-}x_{1,\neg s}^{**}$; $x_{1,s}^{**}{:=}\frac{K}{1+\beta\delta+\beta\delta^2}$; $x_{2,s}^{**}{=}k_{2,s}^{**}{:=}\frac{\beta\delta K}{1+\beta\delta+\beta\delta^2}$; $x_{3,s}^{**}{=}k_{3,s}^{**}{:=}\frac{\beta\delta^2 K}{1+\beta\delta+\beta\delta^2}$.

[29] Note that $f_{2,\neg s}(k_2){=}\ln\left(\frac{k_2}{1+\beta\delta}\right) + \beta\delta\ln\left(\frac{\beta\delta k_2}{1+\beta\delta}\right) - \ln(k_2)$, where $x_2{=}\frac{k_2}{1+\beta\delta}$ and $x_3{=}\frac{\beta\delta k_2}{1+\beta\delta}$ are the optimal consumption levels (when generation 2 has children), and k_2 is generation 2's optimal consumption when he is childless. It follows straightforwardly that $\partial f_{2,\neg s}/\partial k_2 > 0$.

[30] Note that $f_{2,s}(k_3){=}\beta\delta\ln(k_3)$, which is increasing in k_3.

Equilibrium. Proposition 1 characterizes the SPE. It identifies three possible equilibrium strategies for dynasties in the settlement and no-settlement regime: a high-fertility strategy in which generations 1 and 2 have children, a low-fertility strategy in which only generation 1 has children, and a no-fertility strategy in which generation 1 is childless.

Proposition 1 (SPE) *The SPE of the model is characterized by the equilibrium strategies of dynasties in the no-settlement and the settlement inheritance regimes. For the dynasty in the no-settlement regime (i.e. every generation decides next generation's bequest) the equilibrium strategy is:*

(*i*) *A high-fertility strategy* $\{k^{**}_{2,\neg s}, k^{**}_{3,\neg s}, x^{**}_{1,\neg s}, x^{**}_{2,\neg s}, x^{**}_{3,\neg s}, n_1{=}n_2{=}1\}$ *if:*

- (*a*) $f_{2,\neg s}(k^{**}_{2,\neg s}) \geq 0$*;* $f^{n_2=1}_{1,\neg s}(K) > 0$*; and*
- (*b*) $v_1(x^{**}_{1,\neg s}, x^{**}_{2,\neg s}, x^{**}_{3,\neg s}, n_1{=}n_2{=}1) > v_1(x^*_1, x^*_2, 0, n_1{=}1, n_2{=}0)$ *when* $f_{2,\neg s}(k^*_2) < 0$ *and* $f_{2,\neg s}(k^{**}_2) > 0$.

(*ii*) *A low-fertility strategy* $\{k^*_2, k_3{=}0, x^*_1, x^*_2, x_3{=}0, n_1{=}1, n_2{=}0\}$ *if:*

- (*a*) $f_{2,\neg s}(k^*_2) < 0$*;* $f^{n_2=0}_1(K) > 0$*; and*
- (*b*) $v_1(x^{**}_{1,\neg s}, x^{**}_{2,\neg s}, x^{**}_{3,\neg s}, n_1{=}n_2{=}1) \leq v_1(x^*_1, x^*_2, 0, n_1{=}1, n_2{=}0)$ *when* $f_{2,\neg s}(k^*_2) < 0$ *and* $f_{2,\neg s}(k^{**}_{2,\neg s}) > 0$.

(*iii*) *A no-fertility strategy* $\{k_2{=}k_3{=}0, x_1{=}K, x_2{=}x_3{=}0, n_1{=}n_2{=}0\}$ *if* $f^{n_2=0}_1(K) \leq 0$ *and* $f^{n_2=1}_{1,\neg s}(K) \leq 0$.

And for the dynasty in the settlement regime (i.e., generation 1 decides the bequests of the following two generations) the equilibrium strategy is:

(*i*) *A high-fertility strategy* $\{k^{**}_{2,s}, k^{**}_{3,s}, x^{**}_{1,s}, x^{**}_{2,s}, x^{**}_{3,s}, n_1{=}n_2{=}1\}$ *if:*

- (*a*) $f_{2,s}(k^{**}_{3,s}) \geq 0$*;* $f^{n_2=1}_{1,s}(K) > 0$*; and*
- (*b*) $v_1(x^{**}_{1,s}, x^{**}_{2,s}, x^{**}_{3,s}, n_1{=}n_2{=}1) > v_1(x^*_1, x^*_2, 0, n_1{=}1, n_2{=}0)$*,*

(*ii*) *A low-fertility strategy* $\{k^*_2, k_3{=}0, x^*_1, x^*_2, x_3{=}0, n_1{=}1, n_2{=}0\}$ *if:*

- (*a*) $f^{n_2=0}_1(K) > 0$*, and*
- (*b*) $v_1(x^{**}_{1,s}, x^{**}_{2,s}, x^{**}_{3,s}, n_1{=}n_2{=}1) \leq v_1(x^*_1, x^*_2, 0, n_1{=}1, n_2{=}0)$ *when* $f_{2s}(k^{**}_{3s}){>}0$*,*

(*iii*) *A no-fertility strategy* $\{k_2{=}k_3{=}0, x_1{=}K, x_2{=}x_3{=}0, n_1{=}n_2{=}0\}$ *if* $f^{n_2=0}_1(K) \leq 0$ *and* $f^{n_2=1}_{1,s}(K) \leq 0$.

Proof: See Appendix C.2. ■

For each possible equilibrium strategy, condition (a) guarantees that generations 1 and 2 optimize fertility decisions given k_2, k_3, x_1, x_2, x_3. Condition (b) ensures that generation 1 internalizes optimally that he can influence the fertility choices of generation 2. Specifically, for some parameter values generation 1 can choose between an equilibrium in which generation 2 is childless and another one in which generation 2 has children. If he prefers the equilibrium with high-fertility strategy, he gives a high bequest to generation 2, $k^{**}_{2,\neg s}$, in the no-settlement regime, and settles a high bequest to the third generation, $k^{**}_{3,s}$, in the settlement regime.

5.3 Results

Here we present the main results of our model. First, the model replicates the effect of settlements on fertility documented in the empirical analysis. Second, we show that under exponential discounting, the settlement and no-settlement regimes are equivalent. Instead, settlements increase fertility for discount functions with a stronger degree of hyperbolicity (or dynastic preference). Third, we show that settlements, or, more generally, inheritance systems that restrict heirs, can emerge endogenously as a result of a father's concerns over the dynasty's survival and his heir's optimal decisions.

Settlements and fertility. First, we show that settlements can increase fertility. As discussed above, generation 1 can affect the probability that the dynasty survives until generation 3 under both inheritance regimes. In the no-settlement regime, he does so by giving generation 2 a large bequest k_2. In the settlement regime, he does so by settling a larger share of the dynasty's wealth k_3 for the third generation (see Definition 2).

The second mechanism is more effective in moving the dynasty to a high-fertility equilibrium. For the sake of illustration, Figure 3 plots the equilibrium strategies (no-fertility, low-fertility, and high-fertility) for different discount factors β and δ and a given K. Panel (a) is for the dynasty in the no-settlement regime and panel (b) is for the dynasty in the settlement regime. The highlighted area in panel (c) is the parameter region where settlements (strictly) increase fertility; i.e., where generation 2 is childless in the no-settlement regime, but has children in the settlement regime. Proposition 2 generalizes this result.

Proposition 2 (The effect of settlements on fertility) *The set of parameters supporting a high-fertility equilibrium strategy for the dynasty in the settlement regime nests the corresponding set for the dynasty in the no-settlement regime.*

Proof: See Appendix C.3. ■

Intuitively, for any bequest profile $\{k_2, k_3\}$, generation 2 has a lower incentive to deviate to a low-fertility strategy if he is subject to a settlement. In the no-settlement regime, generation 2 can remain childless and appropriate all the bequest k_2—which otherwise would be split between his own consumption and that of generation 3. In contrast, in the settlement regime, generation 2 cannot appropriate any of the bequest k_3 that generation 1 settled. If generation 2 deviates to a low-fertility strategy, the dynasty dies out and k_3 is lost.[31] Hence, generation 1 can increase the fertility of generation 2 more effectively in the settlement regime (i.e., by settling a large bequest k_3) than in the no-settlement regime (i.e., by giving generation 2 a large bequest k_2).

Discount function. Here we show that the effect of settlements on fertility is driven by the intergenerational discount function. We begin by showing that under the standard assumption of exponential discounting both the settlement and no-settlement inheritance regimes are equivalent.

Proposition 3 (Exponential discounting) *Assume that the discount function is exponential, i.e., $\beta = 1$. The equilibrium strategies of dynasties in the settlement and no-settlement inheritance regime are identical for all δ and K.*

Proof: See Appendix C.4. ■

Intuitively, under exponential discounting preferences are time consistent *across generations*: Generation 1 values his consumption x_1 relative to generation 2's consumption x_2 in the same manner as generation 2 values his own consumption x_2 relative to generation 3's consumption x_3. In other words, generations 1 and 2 (father and son) agree on how to provide for generation 3 (grandson). In this scenario, a contract like a settlement—which restricts a son's powers to decide over the grandson's bequest—is innocuous. Exponential discounting, hence, is hard to reconcile with the existence of settlements and its effects on fertility.

Next, we show that the effect of settlements on fertility can be rationalized by introducing intergenerational hyperbolic discounting (i.e., dynastic preferences). To see this we first need to define a measure capturing the degree of hyperbolism of the discount function. Note that our discount function has two elements: the discount rate for future generations, δ, and the discount rate for all the future consumptions, β. On the one hand, for low values of β (and δ) individuals are present biased, and hence, are likely to pursue a low- or a no-fertility strategy.[32] On the other hand, for low values of β preferences are more hyperbolic. In other words, individuals have strong dynastic preferences, and hence, may prefer the dynasty not to die out. To disentangle the two effects of β, we consider combinations of β and δ with the same degree of present-biasedness; i.e., we keep $\beta \cdot \delta$ constant:[33]

[31] Alternatively, one can think of k_3 going to a distant relative whose utility is fully discounted.

[32] Specifically, if generation 1 is present biased he either consumes all dynasty wealth K and is childless or passes down a small share of it such that generation 2 chooses to remain childless.

[33] Formally, let $\beta{\cdot}\delta{=}\Gamma$. Generation 1 discounts the next two generations with Γ and $\frac{\Gamma^2}{\beta}$ re-

Definition 3 (Hyperbolic discounting) *A discount function defined by* β, δ *is more hyperbolic than a discount function defined by* β', δ' *if* $\beta \cdot \delta = \beta' \cdot \delta'$ *and* $\beta < \beta'$*.*

Once equipped with this definition, we evaluate the effects of hyperbolic discounting on fertility. Consider Figure 3. As before, isolines represent combinations of β and δ with the same degree of present-biasedness, i.e., $\beta \cdot \delta$ constant. Along an isoline, lower values of β capture more hyperbolic discount functions (i.e., dynastic preferences). First, note that more hyperbolic discount functions are associated with high-fertility strategies, independently of the inheritance regime (see isolines in panels (a) and (b)). Proposition 4 generalizes this result.

Proposition 4 (Hyperbolic discounting and fertility) *Under both the settlement and no-settlement inheritance regimes, the conditions for a high-fertility strategy are more likely to be satisfied for more hyperbolic discount functions.*

Proof: See Appendix C.5. ■

In other words, if generation 1 exhibits more hyperbolic discounting, i.e., dynastic preferences, he strongly prefers the dynasty not to die out after generation 2. This objective, however, is achieved more effectively in the settlement regime than in the no-settlement regime (Proposition 2). As a result, a parameter region exists where discounting is hyperbolic, and hence, generation 1 would like to keep the dynasty alive, but can *only* do so with a settlement. Panel (c) illustrates this region: when individuals exhibit exponential discounting ($\beta = 1$), are highly present-biased (low β and δ), or do not discount the future (high β and δ), both inheritance regimes produce identical fertility outcomes. Only when the dynasty exhibits hyperbolic discounting do outcomes differ across regimes. Specifically, lower values of β along a given isoline lead to the parameter region where the settlement regime is associated with high-fertility and the no-settlement regime with low-fertility strategies. Proposition 5 generalizes this result.

Proposition 5 (Settlements and hyperbolic discounting) *For more hyperbolic discount functions, fertility is weakly larger in the dynasty under the settlement regime than in the dynasty under the no-settlement regime.*

Proof: See Appendix C.6. ■

In sum, Propositions 3 to 5 show that the empirical effect of settlements on fertility can only be rationalized with discount functions that (1) value the dynasty's survival and (2) generate a time-inconsistency across generations—which

spectively, where $\beta \in [\Gamma, 1]$. Keeping Γ constant, a lower β is associated with a more similar discounting for the next two generations; that is, a more hyperbolic discount function.

is resolved by settlements. Exponential discounting, hence, is hard to reconcile with the effect of settlements on fertility. In contrast, our proposed hyperbolic discount function (or dynastic preference) satisfies both conditions.

Endogenous settlements. So far, we have shown both empirically and theoretically that settlements can increase fertility on the extensive margin. Here we show that, in turn, settlements can emerge endogenously as a result of an individual's concerns over the dynasty's survival and his heir's optimal decisions.

To do so, we endogenize the decision to sign a settlement. We assume a settlement is signed between generation 1 and 2 if both are better off in the settlement regime than in the no-settlement regime. Clearly, this decision is only binding in the parameter region where the two inheritance regimes produce different outcomes; i.e., where settlements increase fertility. Proposition 6 compares each generation's utility.

Proposition 6 (Welfare) *Consider the parameter region where a dynasty in the no-settlement regime follows a low-fertility strategy and a dynasty in the settlement regime follows a high-fertility strategy. All generations are better off in the settlement regime; i.e.,* $v_3(x^{**}_{3,s}) > v_3(x_3{=}0)$, $v_2(x^{**}_{2,s}, x^{**}_{3,s}, n_2{=}1) > v_2(x^*_2, x_3{=}0, n_2{=}0)$, *and* $v_1(x^{**}_{1,s}, x^{**}_{2,s}, x^{**}_{3,s}, n_1{=}1, n_2{=}1) > v_1(x^*_1, x^*_2, x_3{=}0, n_1{=}1, n_2{=}0)$.

Proof: See Appendix C.7. ■

Signing a settlement is welfare improving for each individual generation of a dynasty with hyperbolic discounting. Clearly, generation 1 always prefers to sign a settlement. Under this contract, he can solve the intergenerational time inconsistency and ensure that the dynasty will not die out after generation 2.

What is less obvious is why generation 2 agrees to sign a settlement. Under a settlement, he gives away his powers to decide generation 3's bequest and freely dispose of the dynasty's wealth. However, a settlement makes him better off *ex ante*. The reason is that by signing a settlement he credibly commits to have children, which ensures that generation 1 will pass down a larger share of the dynasty's wealth K to the following two generations; i.e., $k^{**}_{2,s} + k^{**}_{3,s} > k^*_2$.[34]

Finally, note that settlements will only emerge endogenously in the parameter region corresponding to more hyperbolic discount functions, i.e., stronger dynastic preferences. This suggests that settlements and trusts emerged among, respectively, aristocrats and the very wealthy because these exhibit stronger dynastic preferences than the general population.

[34]According to this model signing a settlement should occur before the heir's wedding, as this reduces the probability that the father dies before the settlement is not signed. The reason why the signing of settlements was not anticipated is that settlements also included family provisions for the the bride and the younger children of the couple.

6 Conclusion

From 1650 to 1882, British aristocrats did not freely dispose of their estates. Upon their marriage, they signed a settlement committing to pass down the family estate, unbroken, to the next generation. This paper shows thats such arrangements were crucial to change fertility incentives, reduce the high rates of childlessness in the British aristocracy, and ensure the survival of their dynasties. Using demographic data from about 1,500 peer heirs between 1650 and 1882, we find that signing a settlement increased the probability of having children by 83.5 percent, pushing childlessness rates close to the "natural" rate of 2.4 percent (Tietze 1957). To establish causality, we exploit that, because of tradition and institutional constraints, settlements had to be renewed by a father and his heir at the heir's wedding. Specifically, we exploit variation in the probability that a father lived until his heir's wedding coming from the heir's birth order. This generates as good as a random assignment into signing (renewing) a settlement. In addition, we run placebo tests on non-heirs and Scottish heirs. We find no effects for these populations who did not use settlements. This suggests that our estimation captures the effect of settlements and not of confounding factors correlated with fertility.

This paper also provides a novel theory of wealth transmission to pin down the mechanism through which settlements change fertility incentives. In our model, individuals are present-biased, but also exhibit dynastic preferences, i.e., they discount their offspring and future generations similarly. Under this type of hyperbolic discounting, a family head would like the dynasty to survive, and hence, passes down a large bequest. When an heir is subject to a settlement, he cannot appropriate the bequest settled for the next generation. He can only derive utility from it by continuing the family line. We show that the effect of settlements on fertility is increasing in the degree of hyperbolicity of the discount function, and disappears under exponential discounting. Furthermore, our model rationalizes why a father and his heir would agree to sign a settlement, even if this limits the latter's powers to freely dispose of all the dynasty's wealth. Specifically, a settlement increases the father's welfare by ensuring the survival of the dynasty. In turn, it makes the heir better off as it allows him to commit *ex ante* to have children—which guarantees that a larger share of the original dynasty's wealth will pass down in the form of bequests. This result shows that settlements can emerge endogenously in response to concerns over the dynasty's survival and the heirs optimal decisions.

These results have several implications: First, research on inheritance typically treats fertility as exogenous or ignores endogenous fertility choices on the extensive margin—i.e., to have children or not. In contrast, we show that inheritance systems can affect this margin of fertility and, in turn, concerns over childlessness can shape inheritance systems endogenously. Second, we argue that models of bequests assuming exponential discounting (Barro 1974) are inconsistent with a broad range of inheritance systems that restrict successors' powers to manage inherited wealth; like settlements (England), trusts, fee tails (United States), entails (Scotland), *majorat* (France), *mayorazgo* (Spain), or *ordynacja* (Poland). Third, our results imply that settlements contributed to the perpetuation of elite lineages, as suggested by Adam Smith. However, we argue that they did so not only by entailing the land or favoring primogeniture, but also through changing fertility incentives. This challenges the common wisdom that fertility and inequality to be negatively related.[35] This relation may be the opposite on the extensive margin of fertility. Finally, the historical episode we studied echoes with today's fertility concerns and inheritance practices among the richest. Specifically, British aristocrats faced high childlessness rates in the sixteenth century. Their response was to restrict their heir's powers to manage the dynasty's wealth with a settlement. Similarly, today's elite, i.e., individuals at the top of the income distribution, are facing high childlessness rates (Baudin, de la Croix, and Gobbi 2015) and are increasingly restricting their successors powers with trust funds (Wolff and Gittleman 2014). Whether fertility and inheritance systems are related in the same manner as in the past is a question for future research.

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7 Figures and Tables

Figure 1: Childlessness rates and average births of mothers, by marriage decade.

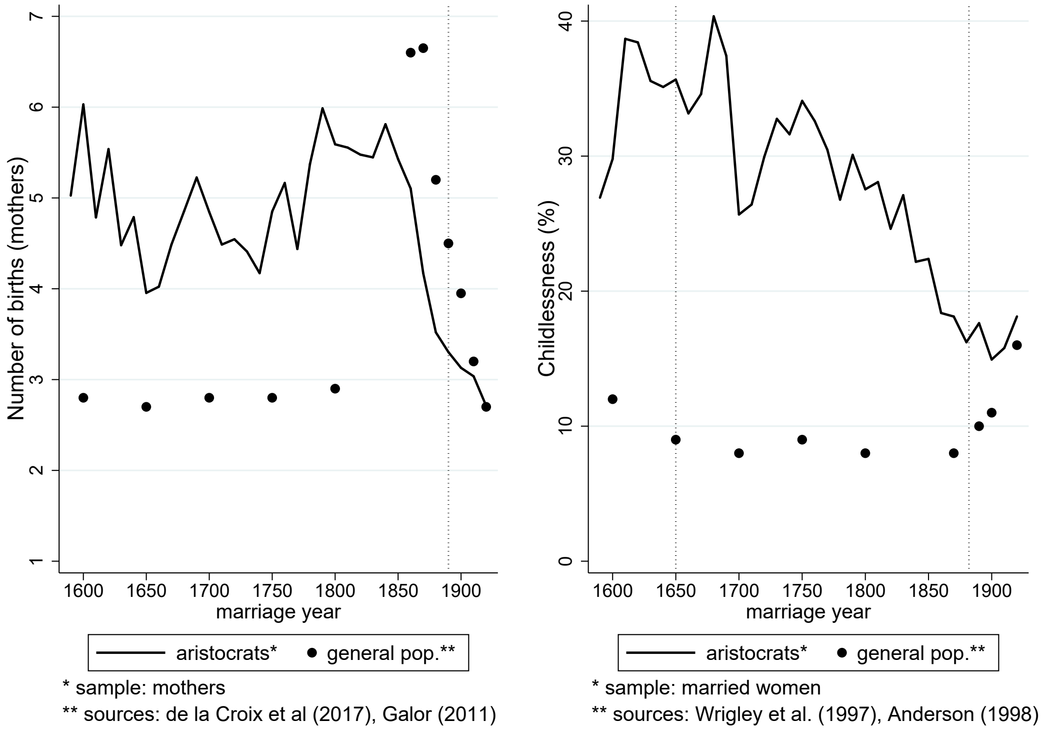


Figure 2: Quasi-hyperbolic discrete discount function

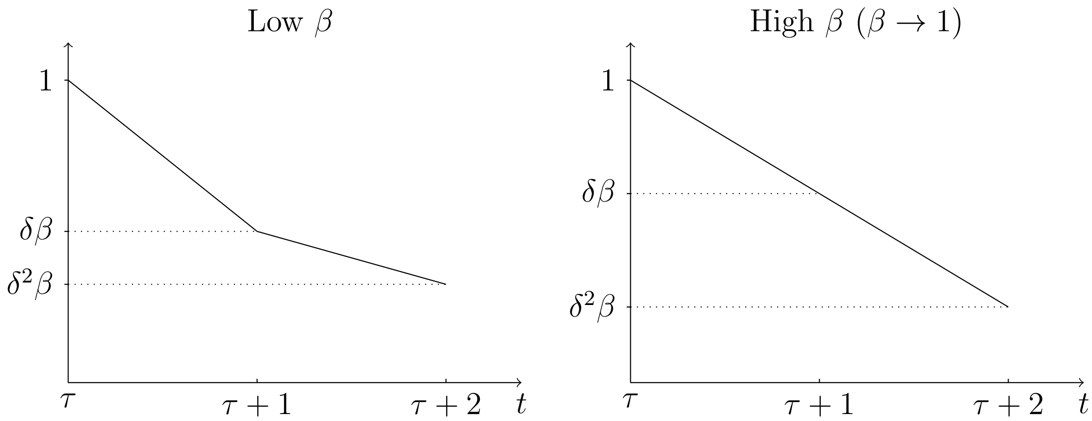
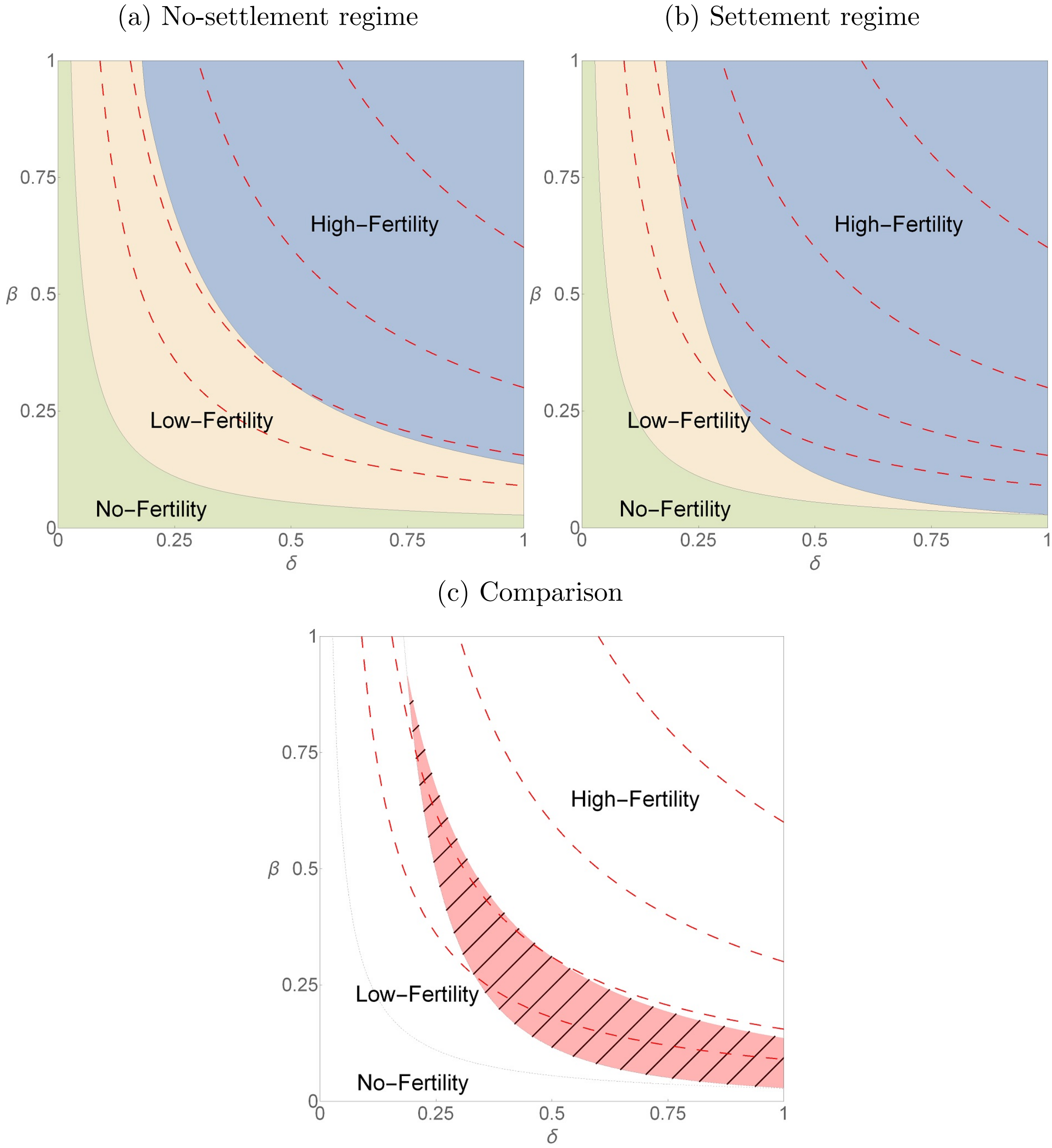


Figure 3: Discount factors and fertility



Region where settlements increase fertility (panel (c))

Isolines for $\beta \cdot \delta$ constant

Notes: Family wealth K is fixed to 100.

Table 1: Summary statistics for the Hollingsworth's dataset (1650–1882)

	mean	std. dev.	min	max	N
A. Fertility variables					
% childless	0.17	0.38	0	1	1,529
All live births	4.67	3.88	0	22	1,529
All live births (if > 0)	5.64	3.56	1	22	1,267
Stillbirths	0.24	0.73	0	9	276
B. Other demographic variables					
Age at first marriage (wife)	21.94	4.93	11	55	1,556
Age at first marriage (husband)	27.20	6.90	8	62	1,558
Age at death (wife)	58.37	20.22	16	100	1,553
Age at death (husband)	60.25	16.94	16	97	1,559
Age difference	-5.25	6.49	-35	23	1,556
Number of marriages	1.25	0.51	1	4	1,559
Last child is a girl	0.53	0.50	0	1	899
C. Socioeconomic status variables					
Baron heir	0.37	0.48	0	1	1,559
Duke heir	0.63	0.48	0	1	1,559
Wife is a commoner	0.58	0.49	0	1	1,559
English peerage	0.45	0.50	0	1	1,559
Scottish peerage	0.24	0.43	0	1	1,559
Irish peerage	0.31	0.46	0	1	1,559
Proxy for settlement [i.e., father died after wedding]	0.56	0.50	0	1	1,559

Notes: The sample are all matrimonies in 1650–1882 where the husband was heir to a peerage. Marriages to women below 12 are excluded (birth or marriage date was probably missreported).

Table 2: Baseline results

	(1)	(2)	(3)	(4)	(5)
		Childlessness			All live births (if > 0)
	OLS	OLS	OLS	OLS	poisson
Settlement	-0.050***	-0.052***	-0.036**	-0.079**	0.036
[i.e., father died after wedding]	(0.019)	(0.019)	(0.018)	(0.035)	(0.042)
Husband's siblings (#)	-0.001	-0.001	-0.001	-0.004	-0.010
	(0.002)	(0.002)	(0.002)	(0.005)	(0.006)
Duke heir		0.022	0.022	-0.041	0.042
		(0.019)	(0.018)	(0.049)	(0.076)
Baron heir		ref.	ref.	ref.	ref.
Wife's age at marriage			0.014***	0.014***	-0.024***
			(0.002)	(0.004)	(0.005)
Wife's age at death			0.000	-0.000	0.003***
			(0.000)	(0.001)	(0.001)
Husband's age at death			-0.003***	-0.004***	0.013***
			(0.001)	(0.001)	(0.002)
Still to live births (fam)			0.175	0.050	3.4
			(0.311)	(2.940)	(2.7)
Wife's social status	NO	YES	YES	YES	YES
Family FE	NO	NO	NO	YES	YES
Birth year FE	NO	NO	NO	YES	YES
Marr. quarter-century FE	NO	NO	NO	YES	YES
Observations	1,526	1,525	1,505	1,505	1,261
% correctly predicted	81.2	81.2	82.8	90.9	-

Notes: The sample are all matrimonies in 1650–1882 where the husband was heir to a peerage. In column (5), the sample is restricted to women who gave birth at least once. Standard errors clustered by family in parentheses; *** p<0.01, ** p<0.05, * p<0.1.

Table 3: Instrumental variables' results

Second stage	Dep. Var.: Childlessness	
	coef.	s.e.
Settlement [i.e., father died after wedding]	-0.146***	(0.036)
Controls	YES	
Family and birth year FE	YES	
Marr. quarter-century FE	YES	
Observations	1,505	
% correctly predicted	91.1	

First stage	Settlement [i.e., father died after wedding]	
	coef.	s.e.
Birth order of the heir		
1st	reference	
2nd	-0.037	(0.024)
3rd	-0.102***	(0.026)
4th	-0.119***	(0.033)
5th	-0.118***	(0.045)
6th	-0.150***	(0.055)
7th	-0.165**	(0.074)
8th	-0.117	(0.106)
9th	-0.154	(0.114)
10th	-0.042	(0.093)
11th	0.108	(0.235)
12th	-0.139	(0.115)
13th	0.222	(0.196)
15th	0.426***	(0.049)
Father age at death	0.021***	(0.001)
Controls	YES	
Marr. quarter-century FE	YES	
Observations	1,530	
% correctly predicted	74.8	
F-test	110.0	
Sargan-Hansen test	13.12	p-val=0.4

Notes: The sample are all matrimonies in 1650–1882 where the husband was heir to a peerage. Controls: number of siblings of the husband, wife's age at marriage, spouses' age at death, history of stillbirths in husband's family, spouses' social status; s.e. clustered by family; *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

Table 4: Placebo tests

	(1)	(2)	(3)	(4)
Dep. Variable: Childlessness				
	benchmark IV	non-heirs IV	England and Ireland IV	Scotland IV
Settlement	-0.146***	0.031	-0.159***	0.025
[i.e., father died after wedding]	(0.036)	(0.054)	(0.054)	(0.093)
Ho:	-	$\beta(1) = \beta(2)$	-	$\beta(3) = \beta(4)$
prob > chi2	-	0.006***	-	0.087*
Controls	YES	YES	YES	YES
Family FE	YES	YES	YES	YES
Birth year FE	YES	YES	YES	YES
M. quarter-century FE	YES	YES	YES	YES
Father-in-law status	-	YES	-	-
Observations	1,506	1,442	1,139	366
% correctly predicted	91	54	79	40
F-stat from first stage	110	90	85	51

Notes: The sample are all matrimonies in 1650–1882 where the husband is heir to a peerage (col. 1), the husband is not a heir and the wife is a peers' daughter (col. 2), the husband is heir to an English or Irish peerage (col. 3), and the husband is heir to a Scottish peerage (col. 4). Controls are: number of siblings of the husband, wife's age at marriage, spouses' age at death, history of stillbirths in the husband's family, and spouses' social status. Standard errors clustered by family in parentheses; *** p<0.01, ** p<0.05, * p<0.1.

Table 5: Settlements signed at heir's majority

	(1)	(2)	(3)	(4)	(5)	(6)
				England		
	heirs	heirs	non-heirs	and Ireland	Scotland	heirs
Panel A: Second stage						All live births
			Childlessness			(if > 0)
	OLS	IV	IV	IV	IV	poisson
Settlement	-0.078***	-0.149***	0.031	-0.180***	0.033	0.018
[i.e., father died	(0.030)	(0.038)	(0.054)	(0.055)	(0.053)	(0.040)
after heir's majority]						
Ho:	-	-	$\beta(2) = \beta(3)$	-	$\beta(4) = \beta(5)$	-
prob > chi2	-	-	0.008***	-	0.005***	-
Controls	YES	YES	YES	YES	YES	YES
Family FE	YES	YES	YES	YES	YES	YES
Birth year FE	YES	YES	YES	YES	YES	YES
M. quarter-century FE	YES	YES	YES	YES	YES	YES
Observations	1,699	1,699	1,807	1,264	434	1,415
% correctly predicted	90	90	58	77	33	-
Panel B: First stage	Dep. Variable: Settlement [i.e., father died after heir's majority]					
Birth order of the heir						
1st	-	reference	reference	reference	reference	-
2nd	-	-0.040**	-0.068**	-0.033	-0.070*	-
		(0.020)	(0.028)	(0.023)	(0.039)	-
3rd	-	-0.089***	-0.089**	-0.076**	-0.142***	-
		(0.025)	(0.038)	(0.030)	(0.048)	-
4th	-	-0.113***	-0.130***	-0.085***	-0.215***	-
		(0.026)	(0.040)	(0.028)	(0.063)	-
5th to 15th not reported						
Controls	-	YES	YES	YES	YES	-
M. quarter-century FE	-	YES	YES	YES	YES	-
Observations	-	1,699	1,807	1,264	434	-
F-stat	-	105.8	101.7	88	52.8	-

Notes: The sample are all matrimonies in 1650–1882 where the husband is heir to a peerage (cols. 1, 2, and 6), the husband is not a heir and the wife is a peers' daughter (col. 3), the husband is heir to an English or Irish peerage (col. 4), and the husband is heir to a Scottish peerage (col. 5). In col. (6), the sample is restricted to women who gave birth at least once. Controls are: number of siblings of the husband, wife's age at marriage, spouses' age at death, history of stillbirths in the husband's family, and spouses' social status. First-stage also includes father's age at death as a covariate. Standard errors clustered by family in parentheses; *** p<0.01, ** p<0.05, * p<0.1.

Table 6: IV using the gender of the first birth

	(1)	(2)	(3)	(4)	(5)
				England	
	heirs	heirs	non-heirs	and Ireland	Scotland
Panel A: Second stage	Dep. Variable: Childlessness				
Settlement	-0.146***	-0.146***	0.011	-0.176***	0.027
[i.e., father died after wedding]	(0.036)	(0.035)	(0.058)	(0.051)	(0.079)
Ho:	-		$\beta(2) = \beta(3)$	-	$\beta(3) = \beta(4)$
prob > chi2	-		0.022**		0.029**
Controls	YES	YES	YES	YES	YES
Family FE	YES	YES	YES	YES	YES
Birth year FE	YES	YES	YES	YES	YES
M. quarter-century FE	YES	YES	YES	YES	YES
Observations	1,506	1,506	1,442	1,139	366
% correctly predicted	91	91	54	79	39
Panel B: First stage	Dep. Variable: Settlement [i.e., father died after wedding]				
Gender of first birth:					
son	-	reference	reference	reference	reference
daughter	-	-0.079***	-0.072***	-0.057**	-0.138***
		(0.020)	(0.019)	(0.022)	(0.041)
Instrument	birth order	daughters	daughters	daughters	daughters
Controls	YES	YES	YES	YES	YES
M. quarter-century FE	YES	YES	YES	YES	YES
Observations	1,506	1,506	1,442	1,139	366
F-stat	110	160	80	122	73

Notes: The sample are all matrimonies in 1650–1882 where the husband is heir to a peerage (cols. 1 & 2), the husband is not a heir and the wife is a peers' daughter (col. 3), the husband is heir to an English or Irish peerage (col. 4), and the husband is heir to a Scottish peerage (col. 5). Controls are: number of siblings of the husband, wife's age at marriage, spouses' age at death, history of stillbirths in the husband's family, and spouses' social status. First-stage also includes father's age at death as a covariate. Standard errors clustered by family in parentheses; *** p<0.01, ** p<0.05, * p<0.1.

Table 7: IV estimates before and after the Industrial Revolution

	(1)	(2)	(3)
Dep. Variable: Childlessness			
	benchmark (1650–1882)	before IR (1650–1769)	after IR (1770–1882)
	IV	IV	IV
Settlement	-0.148***	-0.140**	-0.147**
[i.e., father died after wedding]	(0.036)	(0.059)	(0.064)
Controls	YES	YES	YES
Family FE	YES	YES	YES
Birth year FE	YES	YES	YES
Marriage decade FE	YES	YES	YES
Observations	1,530	708	823
% correctly predicted	91	94	94
Instrument	birth order	birth order	birth order
F-stat from first-stage	111	60	87

Notes: The sample are all matrimonies in the indicated years, where the husband was heir to a peerage. Controls are: number of siblings of the husband, wife's age at marriage, spouses' age at death, history of stillbirths in the husband's family, and spouses' social status. First-stage results not reported. Standard errors clustered by family in parentheses; *** p<0.01, ** p<0.05, * p<0.1.

Online Appendix

Appendix A. Data appendix

This appendix describes in detail the process of matching parents to offspring in the Hollingsworth (2001) dataset.

To guide the reader, we first describe how the entries in the Hollingsworth (2001) dataset look like. Figure A.I shows the entry for James Hamilton, first Earl Abercorn. Each entry is identified by a reference number, in this case, zero. The entry reports James Hamilton's full name, surname, the date of birth, marriage, and death, as well as a variable indicating its accuracy. Importantly for our matching algorithm, the entry also lists the name and the date of birth of the children born to his marriage. In this case, James Hamilton had 9 children, two of which eventually inherited titles (James, 2nd Earl Abercorn and Claude, 2nd Baron Strabane).

Unfortunately, the entries from the Hollingsworth (2001) dataset are not linked across generations. In other words, there is no reference number that links this entry of James Hamilton, first Earl Abercorn, to the entry of his son James Hamilton, 2nd Earl Abercorn. To resolve this issue, we manually matched each entry in the database to their father's entry. For individuals whose father could not be found in the database we tried to match them with their mothers.

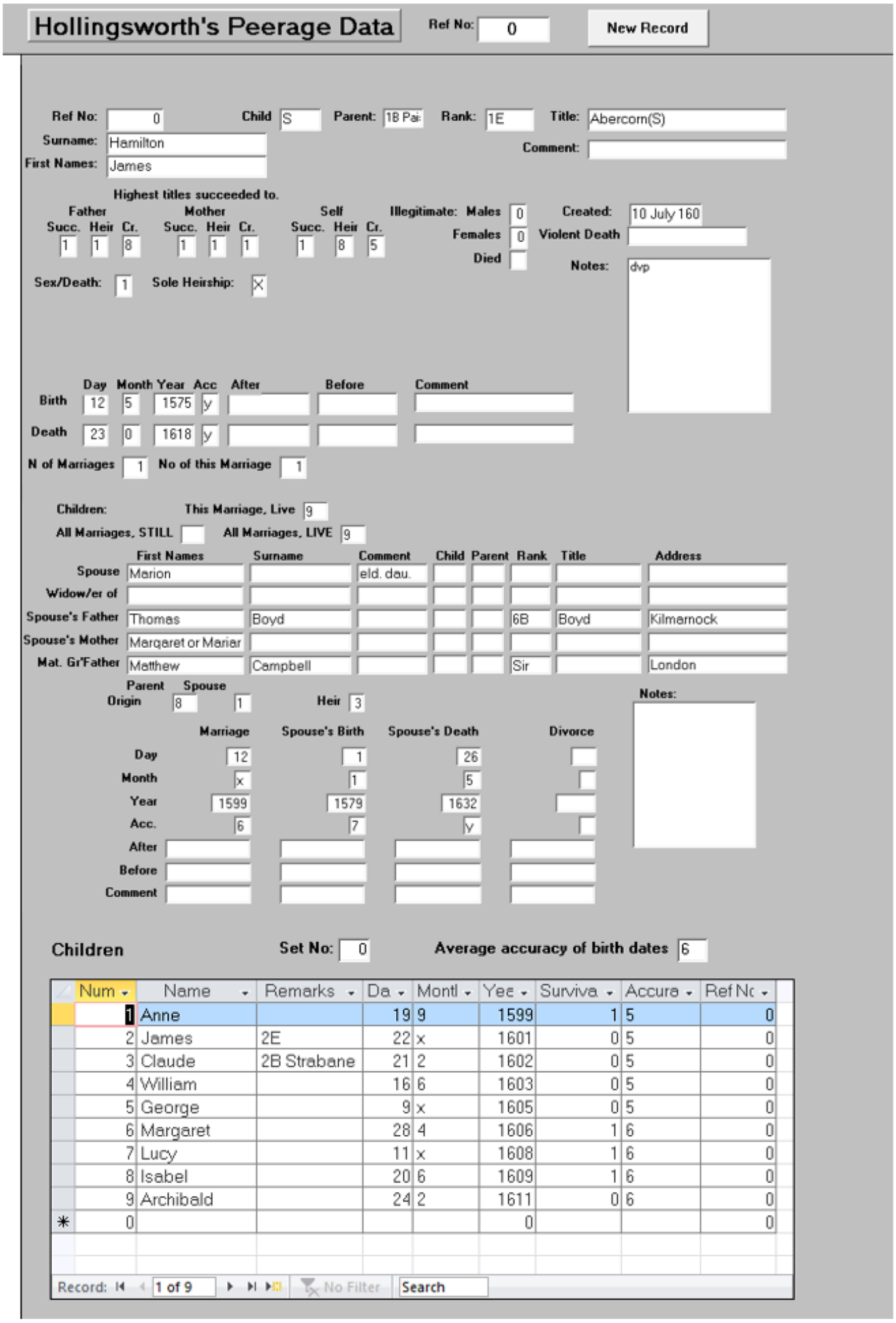
In detail, we first match non-heirs (i.e., peers' daughters and younger sons) to their parents. To do so, we exploit a particularity of the Hollingsworth (2001) database. An entry corresponding to a peer or a peer heir has a reference number which is typically a multiple of 20 or 50. The reference number for his daughters and younger sons (if any) are consecutive numbers of this (i.e., the father's) reference number. Thus, we match an entry C (children) to entry P (parent) if entry P has a reference number that is a multiple of 20 or 50 and entry C has a consecutive reference number. Using this procedure, we match 12,593 peers' daughters and 9,240 peers' younger sons to their parents.

The matching of heirs is less trivial. It involves four iterations. In the first iteration, we match entries C and P if entry P corresponds to a male and the information in entry C corresponds to what entry P reports about P's children. Specifically, we match entries C and P if the C's surname, name, date of birth, and accuracy coincides with P's surname and the name, date of birth, and accuracy of any of the children listed in entry P. We then restrict the sample to unmatched individuals, and repeat the procedure considering female P entries only. This concludes iteration 1. For the remaining unmatched individuals, we consider a similar matching procedure based on birth date and accuracy (iteration 2), first name and birth date (iteration 3), and unique birth dates—that is, restricting the sample to individuals born on a date where no other peer or peer's offspring was born (iteration 4). At each iteration, we check double matches manually using information from thepeerage.com, an online genealogical survey of the peerage of Britain. Finally, we try to match the remaining unmatched heirs to their parents using information from thepeerage.com. Using this iterative procedure, we match 4,666 peers' heirs to their parents.

The validity of the matching is essential to the credibility of the paper. For this reason, we perform several additional manual checks. First, we use thepeerage.com to check manually if individuals matched to their mother do not have siblings who were matched to their father. If this is the case, we re-match those to their fathers. Second, we calculate the distance between father's and children's surnames for individuals matched in iterations 2 to 4. To do so, we use the Levenshtein distance algorithm, which measures the minimum number of single-character edits required to change one surname into the other. We then use thepeerage.com to check manually all the matches with a Levenshtein distance above one.

Overall, we match 98.25 percent of the 26,499 entries in the dataset to their parents. Only 2.22 percent of them are matches to the mother.

Figure A.I: James Hamilton, 1st Earl of Abercorn, Hollingsworth database.



Hollingsworth's Peerage Data

Ref No: 0

New Record

Ref No: 0 | Child: S | Parent: 1B Pai | Rank: 1E | Title: Abercorn(S)

Surname: Hamilton

First Names: James

Comment:

Highest titles succeeded to.

	Father Succ.	Father Heir	Father Cr.	Mother Succ.	Mother Heir	Mother Cr.	Self Succ.	Self Heir	Self Cr.
	1	1	8	1	1	1	1	8	5

Illegitimate: Males 0 | Females 0 | Died

Created: 10 July 160

Violent Death

Notes: dvp

Sex/Death: 1 | Sole Heirship: X

	Day	Month	Year	Acc	After	Before	Comment
Birth	12	5	1575	y			
Death	23	0	1618	y			

N of Marriages 1 | No of this Marriage 1

Children: This Marriage, Live 9

All Marriages, STILL | All Marriages, LIVE 9

	First Names	Surname	Comment	Child	Parent	Rank	Title	Address
Spouse	Marion		eld. dau.					
Widow/er of								
Spouse's Father	Thomas	Boyd				6B	Boyd	Kilmarnock
Spouse's Mother	Margaret or Mariar							
Mat. Gr'Father	Matthew	Campbell				Sir		London

Origin: Parent 8 | Spouse 1 | Heir 3

Notes:

	Marriage	Spouse's Birth	Spouse's Death	Divorce
Day	12	1	26	
Month	x	1	5	
Year	1599	1579	1632	
Acc.	6	7	y	
After				
Before				
Comment				

Children | Set No: 0 | Average accuracy of birth dates 6

Num	Name	Remarks	Da	Montl	Yea	Surviva	Accura	Ref Nc
1	Anne		19	9	1599	1	5	0
2	James	2E	22	x	1601	0	5	0
3	Claude	2B Strabane	21	2	1602	0	5	0
4	William		16	6	1603	0	5	0
5	George		9	x	1605	0	5	0
6	Margaret		28	4	1606	1	6	0
7	Lucy		11	x	1608	1	6	0
8	Isabel		20	6	1609	1	6	0
9	Archibald		24	2	1611	0	6	0
0					0			0

Record: 1 of 9 | No Filter | Search

Appendix B. Additional figures and tables

Childlessness

	1650-99	1700-49	1750-99	1800-49	1850-99
Lévy and Henry (1960)[a]	9%	21%	35%	-	-
Ducs et pairs de France	(N=34)	(N=24)	(N=20)		
Pedlow (1982)[b]	5%	14%	9%	8%	8%
Nobility of Hesse-Kassel	(N=39)	(N=51)	(N=56)	(N=121)	(N=84)
This study:					
Peers' daughters[b]	40%	30%	32%	25%	18%
	(N=603)	(N=493)	(N=603)	(N=972)	(N=1,278)
Peers and peers' sons[b]	22%	26%	22%	20%	20%
	(N=492)	(N=493)	(N=627)	(N=1,057)	(N=1,391)

Notes: The sample are: *a*) women marrying before 20 years old whose marriage remained intact because neither spouse died before 45 years old; *b*) marriages that remained intact at least until the wife reached age 45.

Table B.i: Comparison with other nobilities

Dep. Variable: Childlessness

	(1)	(2)	(3)	(4)	(5)
	IV triangular	IV classic			
	heirs	heirs	non-heirs	England and Ireland	Scotland
Settlement	-0.144***	-0.145***	0.026	-0.165***	-0.008
[i.e., father died after wedding]	(0.036)	(0.035)	(0.060)	(0.050)	(0.077)
Controls	YES	YES	YES	YES	YES
Family FE	YES	YES	YES	YES	YES
Birth year FE	YES	YES	YES	YES	YES
M. quarter-century FE	YES	YES	YES	YES	YES
Father-in-law status	-	-	YES	-	-
Observations	1,531	1,504	1,258	1,139	365
% correctly predicted	91.0	90.9	55.8	55.8	59.8
F-stat from first-stage	23.0	27.5	23.1	15.8	3.3

Notes: Column 1 presents the results from the benchmark IV triangular model described in Section 3.3. Columns 2 to 5 present the results from a classic IV model including all covariates in the first stage. The sample are all marriages in 1650–1882 where the husband is heir to a peerage (cols. 1 & 2), the husband is not a heir and the wife is a peers' daughter (col. 3), the husband is heir to an English or Irish peerage (col. 4), and the husband is heir to a Scottish peerage (col. 5). Controls: number of husband's siblings, wife's age at marriage, spouses' age at death, history of stillbirths in husband's family, and wife's social status. First-stage results not reported; Standard errors clustered by family in parentheses; *** p<0.01, ** p<0.05, * p<0.1.

Table B.ii: Instrumental variables' model with all covariates in first-stage (1650-1882)

Appendix C. Proofs

This appendix proves Propositions 1 to 6 of Section 5. We begin by deriving the optimal consumption and bequests, conditional on fertility decisions.

C. 1 Optimal decisions regarding consumption and bequests

Lemmas 1 and 2 summarize the optimal decisions regarding consumption and bequests conditional on fertility choices for dynasties in the no-settlement regime and for dynasties in the settlement regime respectively.

Lemma 1 (Consumption and bequests in the no-settlement regime) *Suppose each generation decides over the bequests for the next generation. In any SPE:*

(*a*) *If n_1=0, generation 1 consumes all the dynasty wealth, $x_1 = K$.*

(*b*) *If n_1=1 and n_2=0, generations 1 and 2 consume $x_1^*{:=}\dfrac{K}{1+\beta\delta}$ and $x_2^*{:=}\dfrac{\beta\delta K}{1+\beta\delta}$ respectively, and generation 1 gives a bequest $k_2^* := x_2^*$.*

(*c*) *If n_1=1 and n_2=1, generations 1, 2, and 3 consume $x_{1,\neg s}^{**} := \dfrac{K}{1+\beta\delta+\beta\delta^2}$, $x_{2,\neg s}^{**} := \dfrac{1+\delta}{1+\beta\delta}\,\dfrac{\beta\delta K}{1+\beta\delta+\beta\delta^2}$, and $x_{3,\neg s}^{**} := \dfrac{\beta(1+\delta)}{1+\beta\delta}\,\dfrac{\beta\delta^2 K}{1+\beta\delta+\beta\delta^2}$ respectively, and generations 1 and 2 give a bequest $k_{2,\neg s}^{**}{:=}K - x_{1,\neg s}^{**}$ and $k_{3,\neg s}^{**}{:=}x_{3,\neg s}^{**}$ respectively.*

Proof: We solve for the optimal levels of consumption and bequests by backward induction. Generation 3 chooses the level of consumption that maximizes (7) subject to $x_3 = k_3$, where k_3 follows from the choices of generation 2.

Generation 2 chooses consumption, x_2, and bequests, k_3, to maximize (6) subject to (9), given the level of bequests chosen by generation 1, k_2. The optimal choices depend on whether generation 2 has children or not. If $n_2 = 0$, the optimal solutions are $x_2 = x_2^* := k_2$ and $k_3 = 0$. If $n_2 = 1$, the optimal solutions are

$$x_2 = x_{2,\neg s}^{**} := \frac{k_2}{1+\beta\delta}, \quad \text{and} \quad k_3 = x_{3,\neg s}^{**} := \frac{\beta\delta k_2}{1+\beta\delta}.$$

Generation 1 chooses consumption, x_1, and the bequests, k_2, to maximize (5) subject to (8). If $n_1 = 0$, the optimal solutions are $x_1 = K$ and $k_2 = 0$. If $n_2 = 0$ and $n_1 = 1$, the optimal solutions are

$$x_1 = x_1^* := \frac{K}{1+\beta\delta}, \quad \text{and} \quad k_2 = k_2^* := \frac{\beta\delta K}{1+\beta\delta}.$$

If $n_2 = 1$ and $n_1 = 1$, the optimal solutions are

$$x_1 = x^{**}_{1,\neg s} := \frac{K}{1+\beta\delta+\beta\delta^2}, \text{ and } k_2 = k^{**}_{2,\neg s} := K - \frac{K}{1+\beta\delta+\beta\delta^2}.$$

Replacing k^*_2 in x^*_2, and $k^{**}_{2,\neg s}$ in $x^{**}_{2,\neg s}$ and $x^{**}_{3,\neg s}$, Lemma 1 summarizes the optimal conditions detailed above. ■

Lemma 2 (Consumption and bequests in the settlement regime) *Suppose generation 1 decides over the bequests for the following two generations. In any SPE:*

(i) *If n_1=0, generation 1 consumes all the dynasty wealth, $x_1 = K$.*

(ii) *If n_1=1 and n_2=0, generations 1 and 2 consume x^*_1 and x^*_2 and generation 1 gives a bequest $k_2 = x^*_2$ as in the no-settlement inheritance regime.*

(iii) *If n_1=1 and n_2=1, generation 1 consumes $x^{**}_{1,s} := \frac{K}{1+\beta\delta+\beta\delta^2}$, generation 2 consumes $x^{**}_{2,s} := \frac{\beta\delta K}{1+\beta\delta+\beta\delta^2}$, generation 3 consumes $x^{**}_{3,s} := \frac{\beta\delta^2 K}{1+\beta\delta+\beta\delta^2}$, and generation 1 chooses $k^{**}_{2,s} := x^{**}_{2,s}$ and $k^{**}_{3,s} := x^{**}_{3,s}$ as bequests.*

Proof: We solve for the optimal levels of consumption and bequests by backward induction. Generation 3 chooses the level of consumption that maximizes (7) subject to $x_3 = k_3$, where k_3 is given by the choices of generation 1.

Generation 2 chooses the level of consumption that maximizes (6) subject to $x_2 = k_2$, where k_3 is given by the choices of generation 1.

Generation 1 chooses consumption, x_1, and bequests, k_2 and k_3 to maximize (5) subject to (10). If $n_1 = 0$, the optimal solutions are $x_1 = K$ and $k_2 = k_3 = 0$. If $n_2 = 0$ and $n_1 = 1$, the optimal solutions are

$$x_1 = x^*_1 := \frac{K}{1+\beta\delta}, \quad k_2 = k^*_2 := \frac{\beta\delta K}{1+\beta\delta}, \quad \text{and} \quad k_3 = k^*_3 := 0.$$

If $n_2 = 1$ and $n_1 = 1$, the optimal solutions are

$$x_1 = x^{**}_{1,s} := \frac{K}{1+\beta\delta+\beta\delta^2}, \quad k_2 = k^{**}_{2,s} := \frac{\beta\delta K}{1+\beta\delta+\beta\delta^2},$$

$$\text{and} \quad k_3 = k^{**}_{3,s} := \frac{\beta\delta^2 K}{1+\beta\delta+\beta\delta^2}.$$

Replacing k_2^* and k_3^* in x_2^*, $k_{2,s}^{**}$ in $x_{2,s}^{**}$, and $k_{3,s}^{**}$ in $x_{3,s}^{**}$, Lemma 2 summarizes the optimal conditions detailed above. ■

C. 2 Proof of Proposition 1

Since our model is a sequential move game with perfect information and finite time, we use the sub-game perfect equilibrium as the solution concept. For each inheritance regime, we solve the model by backward induction and compare indirect utilities.

Dynasties in the no-settlement regime. From Definition 2, the functions $f_1^{n_2=0}$ and $f_{1,\neg s}^{n_2=1}$ compare generation 1's indirect utilities of having children and being childless at the optimal levels of x_1, x_2, and x_3 (Lemma 1). Function $f_{2,\neg s}$ compares the indirect utilities of generation 2 of having children and being childless at the optimal level of k_2 (Lemma 1). The sign of these functions gives the equilibrium strategy for dynasties in the no-settlement regime.

Dynasties in the settlement regime. From Definition 2, the functions $f_1^{n_2=0}$ and $f_{1,s}^{n_2=1}$ compare generation 1's indirect utilities of having children and being childless at the optimal levels of x_1, x_2, and x_3 (Lemma 2). Function $f_{2,s}$ compares the indirect utilities of generation 2 of having children and being childless at the optimal level of k_3 (Lemma 2). The sign of these functions gives the equilibrium strategy for dynasties in the settlement regime.

C. 3 Proof of Proposition 2

We need to show that $f_{2,s}(k_{3,s}^{**}) - f_{2,\neg s}(k_{2,\neg s}^{**}) > 0$, $f_{1,s}^{n_2=1}(K) - f_{1,\neg s}^{n_2=1}(K) > 0$, and $v_1(x_{1,s}^{**}, x_{2,s}^{**}, x_{3,s}^{**}, n_1{=}1, n_2{=}1) - v_1(x_{1,\neg s}^{**}, x_{2,\neg s}^{**}, x_{3,\neg s}^{**}, n_1{=}1, n_2{=}1) > 0$ for all β and δ in $[0,1]$. First, note that:

$$f_{2,s}(k_{3,s}^{**}) - f_{2,\neg s}(k_{2,\neg s}^{**}) = \beta \ln \frac{1+\beta\delta}{\beta(1+\delta)} - \ln \frac{1}{1+\beta\delta}$$

where $\ln \dfrac{1+\beta\delta}{\beta(1+\delta)} \geq 0$ and $\ln \dfrac{1}{1+\beta\delta} \leq 0$. Hence, $f_{2,s}(k_{3,s}^{**}) - f_{2,\neg s}(k_{2,\neg s}^{**}) \geq 0$.

Second, note that:

$$f_{1,s}^{n_2=1}(K) - f_{1,\neg s}^{n_2=1}(K) = \beta\delta\mathcal{A}(\beta,\delta)$$

where $\mathcal{A}(\beta,\delta) := \ln\dfrac{1+\beta\delta}{1+\delta} + \delta\ln\dfrac{1+\beta\delta}{\beta(1+\delta)}$. The partial derivatives of $\mathcal{A}(\beta,\delta)$ are:

$$\frac{\partial\mathcal{A}(\beta,\delta)}{\partial\beta} = -\frac{\delta(1-\beta)}{\beta(1+\beta\delta)} \leq 0 \quad\text{and}\quad \frac{\partial\mathcal{A}(\beta,\delta)}{\partial\delta} = -\frac{1-\beta}{1+\beta\delta} + \ln\frac{1+\beta\delta}{\beta(1+\delta)} \geq 0.$$

To see why the second derivative is (weakly) positive, note that

$$\frac{\partial^2\mathcal{A}(\beta,\delta)}{\partial\delta\partial\beta} = -\frac{1-\beta}{\beta(1+\beta\delta)^2} \leq 0 \quad\text{and}\quad \frac{\partial^2\mathcal{A}(\beta,\delta)}{\partial\delta^2} = -\frac{1-\beta}{(1+\delta)(1+\beta\delta)^2} \leq 0.$$

This implies that the $\underset{\beta,\delta\in[0,1]}{\operatorname{argmin}}\dfrac{\partial\mathcal{A}(\beta,\delta)}{\partial\delta} = \dfrac{\partial\mathcal{A}(\beta,\delta)}{\partial\delta}(1,1) = 0$.
In addition, $\lim_{\beta\to 0}\dfrac{\partial\mathcal{A}(\beta,\delta)}{\partial\delta} = +\infty$, which implies that $\dfrac{\partial\mathcal{A}(\beta,\delta)}{\partial\delta} \geq 0$.

Given that $\dfrac{\partial\mathcal{A}(\beta,\delta)}{\partial\beta} \leq 0$ and $\dfrac{\partial\mathcal{A}(\beta,\delta)}{\partial\delta} \geq 0$, the $\underset{\beta,\delta\in[0,1]}{\operatorname{argmin}}\mathcal{A}(\beta,\delta) = \mathcal{A}(1,0) = 0$. In addition, $\lim\mathcal{A}(\beta,\delta)_{\beta\to 0} = +\infty$, which implies that $\mathcal{A}(\beta,\delta) \geq 0$ for all β and δ in $[0,1]$.

Third, note that:

$$f_{1,s}^{n_2=1}(K) - f_{1,\neg s}^{n_2=1}(K) = v_1(x_{1,s}^{**}, x_{2,s}^{**}, x_{3,s}^{**}, n_1{=}1, n_2{=}1) - v_1(x_{1,\neg s}^{**}, x_{2,\neg s}^{**}, x_{3,\neg s}^{**}, n_1{=}1, n_2{=}1).$$

This concludes the proof.

C. 4 Proof of Proposition 3

Assume $\beta = 1$. Note that, from Lemmas 1 and 2, $x_{1,\neg s}^{**} = x_{1,s}^{**}$; $x_{2,\neg s}^{**} = x_{2,s}^{**}$; and $x_{3,\neg s}^{**} = x_{3,s}^{**}$. Then, $f_{1,\neg s}^{n_2=1}(K) = f_{1,s}^{n_2=1}(K)$ and $v_1(x_{1,s}^{**}, x_{2,s}^{**}, x_{3,s}^{**}, n_1{=}1, n_2{=}1) = v_1(x_{1,\neg s}^{**}, x_{2,\neg s}^{**}, x_{3,\neg s}^{**}, n_1{=}1, n_2{=}1)$. We can write the difference in indirect utilities of generation 1 in the high fertility and low fertility equilibrium strategies as follows

$$\begin{aligned} &v_1(x_{1,\neg s}^{**}, x_{2,\neg s}^{**}, x_{3,\neg s}^{**}, n_1{=}1, n_2{=}1) - v_1(x_1^{*}, x_2^{*}, x_3{=}0, n_1{=}1, n_2{=}0) \\ &\qquad = (1+\delta)\ln\left(\frac{1+\delta}{1+\delta+\delta^2}\right) + \delta^2\ln\left(\frac{\delta^2 K}{1+\delta+\delta^2}\right). \end{aligned} \tag{12}$$

We need to show that, for any K and δ, the equilibrium strategies are identical for dynasties in the settlement and in the no-settlement inheritance regimes.

First, let the dynasty in the settlement regime follow a high-fertility equilibrium strategy. That is, $f_{2,s}(k_{3,s}^{**}) \geq 0$, $f_{1,s}^{n_2=1}(K) > 0$, and $v_1(x_{1,s}^{**}, x_{2,s}^{**}, x_{3,s}^{**}, n_1{=}1, n_2{=}1) > v_1(x_1^{*}, x_2^{*}, x_3{=}0, n_1{=}1, n_2{=}0)$ hold. Then, the dynasty in the no-settlement regime

- cannot follow a no-fertility strategy because $f_{1,s}^{n_2=1}(K) = f_{1,\neg s}^{n_2=1}(K) > 0$.
- cannot follow a low-fertility strategy. We prove this by contradiction. In the low-fertility equilibrium strategy, it must be that $f_{2,\neg s}(k_{2,\neg s}^{**}) < 0$ (otherwise the conditions for the high-fertility equilibrium strategy would be met). We have that

$$f_{2,\neg s}(k_{2,\neg s}^{**}) < 0 \iff \delta \ln\left(\frac{\delta^2 K}{1+\delta+\delta^2}\right) < \ln(1+\delta).$$

But then, from Equation (12),

$$v_1(x_{1,\neg s}^{**}, x_{2,\neg s}^{**}, x_{3,\neg s}^{**}, n_1{=}1, n_2{=}1) - v_1(x_1^*, x_2^*, x_3{=}0, n_1{=}1, n_2{=}0) < (1+\delta)\ln\left(\frac{1+\delta}{1+\delta+\delta^2}\right) + \delta \ln(1+\delta) \le 0.$$

Now, let the dynasty in the settlement regime follow a low-fertility equilibrium strategy. That is, $f_1^{n_2=0}(K) > 0$ and $v_1(x_{1,s}^{**}, x_{2,s}^{**}, x_{3,s}^{**}, n_1{=}1, n_2{=}1) \le v_1(x_1^*, x_2^*, x_3{=}0, n_1{=}1, n_2{=}0)$ when $f_{2,s}(k_{3,s}^{**}) > 0$. Then, the dynasty in the no-settlement regime

- cannot follow a no-fertility strategy because $f_1^{n_2=0}(K) > 0$.
- cannot follow a high-fertility strategy from Proposition 2.

Finally, let the dynasty in the settlement regime follow a no-fertility equilibrium strategy. Then, the dynasty in the no-settlement regime also follows a no-fertility equilibrium strategy as $f_1^{n_2=0}(K) \le 0$ and $f_{1,s}^{n_2=1}(K) = f_{1,\neg s}^{n_2=1}(K) \le 0$.

C. 5 Proof of Proposition 4

Let $\Gamma := \beta \cdot \delta$. The conditions for a high fertility equilibrium strategy in the no-settlement regime can be written as:

$$f_{2,\neg s}(k_{2,\neg s}^{**}) \ge 0 \iff \mathcal{C}_{1,\neg s}(\beta) := \ln \frac{\beta\Gamma(1+\Gamma)K}{(1+\Gamma)(\beta+\beta\Gamma+\Gamma^2)} + \Gamma \ln \frac{\beta\Gamma^2(1+\Gamma)K}{(1+\Gamma)(\beta+\beta\Gamma+\Gamma^2)} - \ln \frac{\beta\Gamma(1+\Gamma)K}{\beta+\beta\Gamma+\Gamma^2} \ge 0, \quad (13)$$

$$f_{1,\neg s}^{n_2=1}(K) > 0 \iff \mathcal{C}_{2,\neg s}(\beta) := \ln \frac{\beta K}{\beta + \beta\Gamma + \Gamma^2} + \Gamma \ln \frac{\beta+\Gamma}{1+\Gamma} \frac{\Gamma K}{\beta + \beta\Gamma + \Gamma^2} + \frac{\Gamma^2}{\beta} \ln \frac{\beta+\Gamma}{1+\Gamma} \frac{\Gamma^2 K}{\beta + \beta\Gamma + \Gamma^2} - \ln K > 0 \quad (14)$$

and

$$v_1(x^{**}_{1,\neg s}, x^{**}_{2,\neg s}, x^{**}_{3,\neg s}, n_1{=}1, n_2{=}1) > v_1(x^*_1, x^*_2, x_3{=}0, n_1{=}1, n_2{=}0) \iff$$
$$\mathcal{C}_{3,\neg s}(\beta) := \ln \frac{\beta K}{\beta + \beta\Gamma + \Gamma^2} + \Gamma \ln \frac{\beta+\Gamma}{1+\Gamma} \frac{\Gamma K}{\beta + \beta\Gamma + \Gamma^2} + \frac{\Gamma^2}{\beta} \ln \frac{\beta+\Gamma}{1+\Gamma} \frac{\Gamma^2 K}{\beta + \beta\Gamma + \Gamma^2} - \ln \frac{K}{1+\Gamma} - \Gamma \ln \frac{\Gamma K}{1+\Gamma} > 0. \quad (15)$$

For a constant Γ, conditions (13)-(15) only depend on β. We then need to show that $\frac{\partial \mathcal{C}_{1,\neg s}(\beta)}{\partial \beta} < 0$, $\frac{\partial \mathcal{C}_{2,\neg s}(\beta)}{\partial \beta} < 0$, and $\frac{\partial \mathcal{C}_{3,\neg s}(\beta)}{\partial \beta} < 0$. Computing the derivatives, we have:

$$\frac{\partial \mathcal{C}_{1,\neg s}(\beta)}{\partial \beta} = -\frac{\Gamma^2}{(\beta+\Gamma)(\beta + \beta\Gamma + \Gamma^2)} < 0,$$

and

$$\frac{\partial \mathcal{C}_{2,\neg s}(\beta)}{\partial \beta} = \frac{\partial \mathcal{C}_{3,\neg s}(\beta)}{\partial \beta} = -\left(\frac{\Gamma}{\beta}\right)^2 \ln \frac{\beta+\Gamma}{1+\Gamma} \frac{\Gamma^2 K}{\beta + \beta\Gamma + \Gamma^2} = -\left(\frac{\Gamma}{\beta}\right)^2 \ln x^{**}_{3,\neg s} < 0,$$

since n_2 would be nil otherwise.

The conditions for a high fertility equilibrium strategy in the settlement regime can be written as:

$$f_{2,s}(k^{**}_{3,s}) \geq 0 \iff \mathcal{C}_{1,s}(\beta) := \Gamma \ln \frac{\Gamma^2 K}{\beta(1+\Gamma) + \Gamma^2} \geq 0, \quad (16)$$

$$f_{1,s}^{n_2=1}(K) > 0 \iff \mathcal{C}_{2,s}(\beta) := \ln \frac{\beta K}{\beta(1+\Gamma) + \Gamma^2} + \Gamma \ln \frac{\beta\Gamma K}{\beta(1+\Gamma) + \Gamma^2} + \frac{\Gamma^2}{\beta} \ln \frac{\Gamma^2 K}{\beta(1+\Gamma) + \Gamma^2} - \ln K > 0 \quad (17)$$

and

$$v_1(x^{**}_{1,s}, x^{**}_{2,s}, x^{**}_{3,s}, n_1{=}1, n_2{=}1) > v_1(x^*_1, x^*_2, x_3{=}0, n_1{=}1, n_2{=}0) \iff$$
$$\mathcal{C}_{3,s}(\beta) := \ln \frac{\beta K}{\beta(1+\Gamma)+\Gamma^2} + \Gamma \ln \frac{\beta \Gamma K}{\beta(1+\Gamma)+\Gamma^2} + \frac{\Gamma^2}{\beta} \ln \frac{\Gamma^2 K}{\beta(1+\Gamma)+\Gamma^2} - \ln \frac{K}{1+\Gamma} - \Gamma \ln \frac{\Gamma K}{1+\Gamma} > 0. \quad (18)$$

Keeping Γ constant, conditions (16)-(18) only depend on β. We then need to show that $\frac{\partial \mathcal{C}_{1,s}(\beta)}{\partial \beta} < 0$, $\frac{\partial \mathcal{C}_{2,s}(\beta)}{\partial \beta} < 0$, and $\frac{\partial \mathcal{C}_{3,s}(\beta)}{\partial \beta} < 0$. Computing the derivatives, we then have:

$$\frac{\partial \mathcal{C}_{1,s}(\beta)}{\partial \beta} = -\frac{\Gamma(1+\Gamma)}{\beta + \Gamma(\Gamma+\beta)} < 0,$$

and

$$\frac{\partial \mathcal{C}_{2,s}(\beta)}{\partial \beta} = \frac{\partial \mathcal{C}_{3,s}(\beta)}{\partial \beta} = -\left(\frac{\Gamma}{\beta}\right)^2 \ln \frac{\Gamma^2 K}{\beta(1+\Gamma)+\Gamma^2} = -\left(\frac{\Gamma}{\beta}\right)^2 \ln x^{**}_{3,\neg s} < 0.$$

C. 6 Proof of Proposition 5

For any fixed value of $\Gamma := \beta \cdot \delta$, we need to show that:

$$\frac{\partial(\mathcal{C}_{1,s} - \mathcal{C}_{1,\neg s})}{\partial \beta} < 0, \quad \frac{\partial(\mathcal{C}_{2,s} - \mathcal{C}_{2,\neg s})}{\partial \beta} < 0, \quad \text{and} \quad \frac{\partial(\mathcal{C}_{3,s} - \mathcal{C}_{3,\neg s})}{\partial \beta} < 0,$$

where $\mathcal{C}_{1,\neg s}$, $\mathcal{C}_{2,\neg s}$ and $\mathcal{C}_{3,\neg s}$ are the conditions for a high fertility equilibrium strategy in the no-settlement regime, defined in (13)-(15), and $\mathcal{C}_{1,s}$, $\mathcal{C}_{2,s}$ and $\mathcal{C}_{3,s}$ are the conditions for a high fertility equilibrium strategy in the settlement regime, defined in (16)-(18). Computing the three derivatives we have,

$$\frac{\partial(\mathcal{C}_{1,s} - \mathcal{C}_{1,\neg s})}{\partial \beta} = -\frac{\Gamma}{\Gamma+\beta} < 0$$

and

$$\frac{\partial(\mathcal{C}_{2,s} - \mathcal{C}_{2,\neg s})}{\partial \beta} = \frac{\partial(\mathcal{C}_{3,s} - \mathcal{C}_{3,\neg s})}{\partial \beta} = \left(\frac{\Gamma}{\beta}\right)^2 \ln \frac{\beta+\Gamma}{1+\Gamma} < 0.$$

C. 7 Proof of Proposition 6

Generation 1 is better off in the settlement regime than in the no-settlement regime as the condition

$$v_1(x^{**}_{1,s}, x^{**}_{2,s}, x^{**}_{3,s}, n_1 = 1, n_2 = 1) > v_1(x^*_1, x^*_2, x_3 = 0, n_1 = 1, n_2 = 0) \quad (19)$$

defines the region characterized in Proposition 2. Note that condition (19) can be rewritten as:

$$\frac{1+\beta\delta}{\delta}\ln\left(\frac{1+\beta\delta}{1+\beta\delta+\beta\delta^2}\right)+\beta\delta\ln\left(\frac{\beta\delta^2 K}{1+\beta\delta+\beta\delta^2}\right)>0. \tag{20}$$

Generation 2 is better off in in the settlement regime than in the no-settlement regime, and in the region characterized in Proposition 2 if and only if

$$v_2(x_{2,s}^{**}, x_{3,s}^{**}, n_2=1) > v_2(x_2^{*}, x_3=0, n_2=0)$$

which holds if and only if

$$\ln\left(\frac{1+\beta\delta}{1+\beta\delta+\beta\delta^2}\right)+\beta\delta\ln\left(\frac{\beta\delta^2 K}{1+\beta\delta+\beta\delta^2}\right)>0. \tag{21}$$

Inequality (20) implies that inequality (21) is satisfied.

Appendix D. Extension with more than three generations

In this appendix, we relax the assumption that the dynasty dies after generation 3. We assume the opposite scenario in which the dynasty does not die at all if generation 2 chooses to have positive fertility. Assuming log-utility as in Section 5.2, the utility of generation 1 provided in Equation (5) can therefore be rewritten as follows

$$v_1(x_1, x_2, k_3, n_1, n_2) = \ln(x_1) + n_1 \cdot \left[\beta\delta \ \ln(x_2) + n_2 \cdot \beta\delta^2 \ V(k_3)\right], \tag{22}$$

where,

$$V(k_3) := \ln(c(k_3)) + \delta \ln(c(k_3)) + \dots + \delta^{n-3} \ln(c(k_3)) + \dots$$

and $c(k_3)$ determines the consumption of generations $i = \{3, 4, 5, \dots\}$ as a function of the bequest k_3.

We assume that $c(k_3) = x_3 = \alpha k_3$. That is, we assume that, in the long run, the (residual) family wealth k_3 generates a return of $(1 + \alpha)k_3$. Every future generation $i = \{3, 4, 5, \dots\}$ then consumes αk_3 and passes down k_3 as a bequest for the next generation. Under this assumption, $V(k_3)$ can be rewritten as:

$$V(k_3) = \left(\frac{1}{1-\delta}\right) \ln(\alpha k_3) \ .$$

The utility functions of generations 1 and 2 can then be written as, respectively:

$$v_1(x_1, x_2, x_3, n_1, n_2) = \ln(x_1) + n_1 \cdot \left[\beta\delta \ \ln(x_2) + n_2 \cdot \beta\delta^2 \ \left(\frac{1}{1-\delta}\right) \ln(x_3)\right], \tag{23}$$

and

$$v_2(x_2, x_3, n_2) = \ln(x_2) + n_2 \cdot \beta\delta \ \left(\frac{1}{1-\delta}\right) \ln(x_3). \tag{24}$$

As in Appendix C.1, we begin by deriving the optimal consumption and bequests, conditional on fertility decisions. Lemmas I and II summarize these optimal decisions for dynasties in the no-settlement regime and for dynasties in the settlement regime respectively.

Lemma I (Consumption and bequests in the no-settlement regime)
Suppose each generation decides over the bequests for the next generation. In any SPE:

(a) *If $n_1 = 0$, generation 1 consumes all the dynasty wealth, $x_1 = K$.*

(b) *If $n_1 = 1$ and $n_2 = 0$, generation 1 consumes x_1^*, generation 2 consumes x_2^*, and generation 1 gives a bequest k_2^*, where x_1^*, x_2^*, k_2^* are defined in Lemma 1.*

(c) *If* $n_1 = 1$ *and* $n_2 = 1$, *generation 1 consumes* $x'_{1,\neg s} := \dfrac{(1-\delta)K}{1-(1-\beta)\delta}$, *generation 2 consumes* $x'_{2,\neg s} := \dfrac{\beta\delta(1-\delta)K}{(1-(1-\beta)\delta)^2}$, *and all future generations consume* $x'_{3,\neg s} := \dfrac{\alpha(\beta\delta)^2 K}{(1-(1-\beta)\delta)^2}$, *and generations 1 and 2 give a bequest* $k'_{2,\neg s} {:=} K - x'_{1,\neg s}$ *and* $k'_{3,\neg s} {:=} \dfrac{x'_{3,\neg s}}{\alpha}$ *respectively.*

Proof: The proof follows that of Lemma 1. ■

Lemma II (Consumption and bequests in the settlement regime) *Suppose generation 1 decides over the bequests for the following two generations. In any SPE:*

(i) *If* $n_1 = 0$, *generation 1 consumes all the dynasty wealth,* $x_1 = K$.

(ii) *If* $n_1 = 1$ *and* $n_2 = 0$, *generations 1 and 2 consume* x_1^* *and* x_2^* *and generation 1 gives a bequest* $k_2 = x_2^*$ *as in the no-settlement inheritance regime.*

(iii) *If* $n_1 = 1$ *and* $n_2 = 1$, *generation 1 consumes* $x'_{1,s} := \dfrac{(1-\delta)K}{1-(1-\beta)\delta}$, *generation 2 consumes* $x'_{2,s} := \dfrac{\beta\delta(1-\delta)K}{1-(1-\beta)\delta}$, *generation 3 consumes* $x'_{3,s} := \dfrac{\alpha\beta\delta^2 K}{1-(1-\beta)\delta}$, *and generation 1 chooses* $k'_{2,s} := x'_{2,s}$ *and* $k'_{3,s} := \dfrac{x'_{3,s}}{\alpha}$ *as bequests.*

Proof: The proof follows that of Lemma 2. ■

Definition I provides the fertility gains that we obtain by comparing the indirect utilities of having children and being childless under the different inheritance (and fertility) regimes.

Definition I (Fertility gains) *For generation 2,* $F_{2,\neg s}$ *and* $F_{2,s}$ *are the fertility gains in, respectively, the no-settlement and the settlement regimes:*

$$F_{2,\neg s}(k_2) := v_2\left(x_2{=}\frac{(1-\delta)k_2}{1-\delta+\beta\delta}, x_3{=}\frac{\alpha\beta\delta k_2}{1-\delta+\beta\delta}, n_2{=}1\right) - v_2(x_2{=}k_2, x_3{=}0, n_2{=}0),$$

$$F_{2,s}\ (k_3) := v_2\left(x_2{=}k_2, x_3{=}\alpha k_3, n_2{=}1\right) - v_2(x_2{=}k_2, x_3{=}0, n_2{=}0).$$

For generation 1, $f_1^{n_2=0}$ *defines fertility gains when* $n_2{=}0$ *in both regimes:*

$$f_1^{n_2=0}(K) := v_1(x_1^*, x_2^*, n_1{=}1) - v_1(x_1{=}K, n_1{=}0)$$

and $F^{n_2=1}_{1,\neg s}$ *and* $F^{n_2=1}_{1,s}$ *are the fertility gains of generation 1 when* n_2=1 *in, respectively, the no-settlement and settlement inheritance regimes:*

$$F^{n_2=1}_{1,\neg s}(K) := v_1(x'_{1,\neg s}, x'_{2,\neg s}, x'_{3,\neg s}, n_1{=}1) - v_1(x_1{=}K, n_1{=}0),$$
$$F^{n_2=1}_{1,s}(K) := v_1(x'_{1,s}\ , x'_{2,s}\ , x'_{3,s}\ , n_1{=}1) - v_1(x_1{=}K, n_1{=}0).$$

Propositions I and VI generalize Propositions 1 and 6 under the assumption that the dynasty does not die if generation 2 is not childless. Propositions 2 to 5 write exactly the same as in the benchmark model. The proofs are given below.

Proposition I (SPE) *The SPE of the model is characterized by the equilibrium strategies of dynasties in the no-settlement and the settlement inheritance regimes. For the dynasty in the no-settlement regime (i.e. every generation decides next generation's bequest) the equilibrium strategy is:*

(*i*) *A high-fertility strategy* $\{k'_{2,\neg s}, k'_{3,\neg s}, x'_{1,\neg s}, x'_{2,\neg s}, x'_{3,\neg s}, n_1{=}n_2{=}1\}$ *if:*

   (*a*) $F_{2,\neg s}(k'_{2,\neg s}) \geq 0$; $F^{n_2=1}_{1,\neg s}(K) > 0$; *and*

   (*b*) $v_1(x'_{1,\neg s}, x'_{2,\neg s}, x'_{3,\neg s}, n_1{=}n_2{=}1) > v_1(x^*_1, x^*_2, 0, n_1{=}1, n_2{=}0)$ *when* $F_{2,\neg s}(k^*_2) < 0$ *and* $F_{2,\neg s}(k'_2) > 0$.

(*ii*) *A low-fertility strategy* $\{k^*_2, k_3{=}0, x^*_1, x^*_2, x_3{=}0, n_1{=}1, n_2{=}0\}$ *if:*

   (*a*) $F_{2,\neg s}(k^*_2) < 0$; $f^{n_2=0}_1(K) > 0$; *and*

   (*b*) $v_1(x'_{1,\neg s}, x'_{2,\neg s}, x'_{3,\neg s}, n_1{=}n_2{=}1) \leq v_1(x^*_1, x^*_2, 0, n_1{=}1, n_2{=}0)$ *when* $F_{2,\neg s}(k^*_2) < 0$ *and* $F_{2,\neg s}(k'_{2,\neg s}) > 0$.

(*iii*) *A no-fertility strategy* $\{k_2{=}k_3{=}0, x_1{=}K, x_2{=}x_3{=}0, n_1{=}n_2{=}0\}$ *if* $f^{n_2=0}_1(K) \leq 0$ *and* $F^{n_2=1}_{1,\neg s}(K) \leq 0$.

And for the dynasty in the settlement regime (i.e., generation 1 decides the bequests of the following two generations) the equilibrium strategy is:

(*i*) *A high-fertility strategy* $\{k'_{2,s}, k'_{3,s}, x'_{1,s}, x'_{2,s}, x'_{3,s}, n_1{=}n_2{=}1\}$ *if:*

   (*a*) $F_{2,s}(k'_{3,s}) \geq 0$; $F^{n_2=1}_{1,s}(K) > 0$; *and*

   (*b*) $v_1(x'_{1,s}, x'_{2,s}, x'_{3,s}, n_1{=}n_2{=}1) > v_1(x^*_1, x^*_2, 0, n_1{=}1, n_2{=}0)$,

(*ii*) *A low-fertility strategy* $\{k^*_2, k_3{=}0, x^*_1, x^*_2, x_3{=}0, n_1{=}1, n_2{=}0\}$ *if:*

   (*a*) $f^{n_2=0}_1(K) > 0$, *and*

   (*b*) $v_1(x'_{1,s}, x'_{2,s}, x'_{3,s}, n_1{=}n_2{=}1) \leq v_1(x^*_1, x^*_2, 0, n_1{=}1, n_2{=}0)$ *when* $F_{2,s}(k'_{3s}){>}0$,

(*iii*) *A no-fertility strategy* $\{k_2{=}k_3{=}0, x_1{=}K, x_2{=}x_3{=}0, n_1{=}n_2{=}0\}$ *if* $f^{n_2=0}_1(K) \leq 0$ *and* $F^{n_2=1}_{1,s}(K) \leq 0$.

Proof: The proof follows that of Proposition 1 given in Appendix C.2. ■

Proof of Proposition 2: We need to show that $F_{2,s}(k'_{3,s}) - F_{2,\neg s}(k'_{2,\neg s}) > 0$, $F^{n_2=1}_{1,s}(K) - F^{n_2=1}_{1,s}(K) > 0$, and $v_1(x'_{1,s}, x'_{2,s}, x'_{3,s}, n_1{=}1, n_2{=}1) - v_1(x'_{1,\neg s}, x'_{2,\neg s}, x'_{3,\neg s}, n_1{=}1, n_2{=}1) > 0$ for all β and δ in $[0, 1]$. First, note that:

$$F_{2,s}(k'_{3,s}) - F_{2,\neg s}(k'_{2,\neg s}) = \ln \frac{1-(1-\beta)\delta}{1-\delta} - \frac{\beta\delta}{1-\delta} \ln \frac{\beta}{1-(1-\beta)\delta}$$

where $\ln \frac{1-(1-\beta)\delta}{1-\delta} \geq 0$ and $\ln \frac{\beta}{1-(1-\beta)\delta} \leq 0$. Hence, $F_{2,s}(k'_{3,s}) - F_{2,\neg s}(k'_{2,\neg s}) \geq 0$.

Second, note that:

$$F^{n_2=1}_{1,s}(K) - F^{n_2=1}_{1,\neg s}(K) = \beta\delta\mathcal{A}'(\beta, \delta)$$

where $\mathcal{A}'(\beta, \delta) := \ln(1-(1-\beta)\delta) - \frac{\delta}{1-\delta} \ln \frac{\beta}{1-(1-\beta)\delta}$. The partial derivatives of $\mathcal{A}'(\beta, \delta)$ are:

$$\frac{\partial \mathcal{A}'(\beta, \delta)}{\partial \beta} = -\frac{(1-\beta)\delta}{\beta(1-(1-\beta)\delta)} \leq 0$$

and

$$\frac{\partial \mathcal{A}(\beta, \delta)}{\partial \delta} = \frac{1}{(1-\delta)^2} \left(\frac{\beta}{1-(1-\beta)\delta} - \ln\left(\frac{\beta}{1-(1-\beta)\delta} \right) - 1 \right) \geq 0.$$

To see why the second derivative is (weakly) positive, note that $a - \ln a - 1 \geq 0, \ \forall a > 0$.

Note that $\mathcal{A}'(1, \delta) = 0$ and $\lim \mathcal{A}'(\beta, \delta)_{\delta \to 0} = 0$. This implies that $\mathcal{A}'(\beta, \delta) \geq 0$ for all β and δ in $[0, 1]$.

Third, note that:

$$F^{n_2=1}_{1,s}(K) - F^{n_2=1}_{1,\neg s}(K) = v_1(x'_{1,s}, x'_{2,s}, x'_{3,s}, n_1{=}1, n_2{=}1) - v_1(x'_{1,\neg s}, x'_{2,\neg s}, x'_{3,\neg s}, n_1{=}1, n_2{=}1).$$

This concludes the proof. ■

Proof of Proposition 3: Assume $\beta = 1$. Note that, from Lemmas I and II, $x'_{1,\neg s} = x'_{1,s}$; $x'_{2,\neg s} = x'_{2,s}$; and $x'_{3,\neg s} = x'_{3,s}$. Then, $F^{n_2=1}_{1,\neg s}(K) = F^{n_2=1}_{1,s}(K)$ and $v_1(x'_{1,s}, x'_{2,s}, x'_{3,s}, n_1{=}1, n_2{=}1) = v_1(x'_{1,\neg s}, x'_{2,\neg s}, x'_{3,\neg s}, n_1{=}1, n_2{=}1)$. We can write the difference in indirect utilities of generation 1 in the high fertility and low fertility equilibrium strategies as follows

$$\begin{aligned} v_1(x'_{1,\neg s}, x'_{2,\neg s}, x'_{3,\neg s}, n_1{=}1, n_2{=}1) - v_1(x^*_1, x^*_2, x_3{=}0, n_1{=}1, n_2{=}0) \\ = (1+\delta)\ln\left(\frac{1-\delta}{1+\delta}\right) + \frac{\delta^2}{1-\delta}\ln\left(\alpha\delta^2 K\right). \quad (25) \end{aligned}$$

We need to show that, for any K and δ, the equilibrium strategies are identical for dynasties in the settlement and in the no-settlement inheritance regimes.

First, let the dynasty in the settlement regime follow a high-fertility equilibrium strategy. That is, $F_{2,s}(k'_{3,s}) \geq 0$, $F_{1,s}^{n_2=1}(K) > 0$, and $v_1(x'_{1,s}, x'_{2,s}, x'_{3,s}, n_1{=}1, n_2{=}1) > v_1(x_1^*, x_2^*, x_3{=}0, n_1{=}1, n_2{=}0)$ hold. Then, the dynasty in the no-settlement regime

- cannot follow a no-fertility strategy because $F_{1,s}^{n_2=1}(K) = F_{1,\neg s}^{n_2=1}(K) > 0$.
- cannot follow a low-fertility strategy. We prove this by contradiction. In the low-fertility equilibrium strategy, it must be that $F_{2,\neg s}(k'_{2,\neg s}) < 0$ (otherwise the conditions for the high-fertility equilibrium strategy would be met). We have that

$$F_{2,\neg s}(k'_{2,\neg s}) < 0 \iff \frac{\delta}{1-\delta} \ln\left(\alpha \delta^2 K\right) < \ln\left(\frac{1}{1-\delta}\right).$$

But then, from Equation (25),

$$\begin{aligned} v_1(x'_{1,\neg s}, x'_{2,\neg s}, x'_{3,\neg s}, n_1{=}1, n_2{=}1) - v_1(x_1^*, x_2^*, x_3{=}0, n_1{=}1, n_2{=}0) \\ < (1+\delta)\ln\left(\frac{1-\delta}{1+\delta}\right) + \delta \ln\left(\frac{1}{1-\delta}\right) \leq 0. \end{aligned}$$

Now, let the dynasty in the settlement regime follow a low-fertility equilibrium strategy. That is, $f_1^{n_2=0}(K) > 0$ and $v_1(x'_{1,s}, x'_{2,s}, x'_{3,s}, n_1{=}1, n_2{=}1) \leq v_1(x_1^*, x_2^*, x_3{=}0, n_1{=}1, n_2{=}0)$ when $F_{2,s}(k'_{3,s}) > 0$. Then, the dynasty in the no-settlement regime

- cannot follow a no-fertility strategy because $f_1^{n_2=0}(K) > 0$.
- cannot follow a high-fertility strategy from Proposition 2.

Finally, let the dynasty in the settlement regime follow a no-fertility equilibrium strategy. Then, the dynasty in the no-settlement regime also follows a no-fertility equilibrium strategy as $f_1^{n_2=0}(K) \leq 0$ and $F_{1,s}^{n_2=1}(K) = F_{1,\neg s}^{n_2=1}(K) \leq 0$.

■

Proof of Proposition 4: Let $\Gamma := \beta \cdot \delta$. The conditions for a high fertility equilibrium strategy in the no-settlement regime can be written as:

$$\begin{aligned} F_{2,\neg s}(k'_{2,\neg s}) \geq 0 \iff \mathcal{C}'_{1,\neg s}(\beta) := \ln \frac{\Gamma K\left(1-\frac{X}{\beta}\right)}{\left(1-\frac{\Gamma}{\beta}+\Gamma\right)^2} + \frac{\Gamma}{1-\frac{\Gamma}{\beta}} \ln \frac{\alpha K \Gamma^2}{\left(1-\frac{\Gamma}{\beta}+\Gamma\right)^2} \\ - \ln \frac{\Gamma K}{1-\frac{\Gamma}{\beta}+\Gamma} \geq 0, \end{aligned}$$

$$F_{1,\neg s}^{n_2=1}(K) > 0 \iff \mathcal{C}'_{2,\neg s}(\beta) := \ln \frac{\left(1-\frac{\Gamma}{\beta}\right)K}{1+\Gamma-\frac{\Gamma}{\beta}} + \Gamma \ln \frac{\Gamma K\left(1-\frac{\Gamma}{\beta}\right)}{\left(1+\Gamma-\frac{\Gamma}{\beta}\right)^2} + \frac{\frac{\Gamma^2}{\beta}\ln\frac{\alpha\Gamma^2 K}{\left(1+\Gamma-\frac{\Gamma}{\beta}\right)^2}}{1-\frac{\Gamma}{\beta}} - \ln K > 0$$

and

$$v_1(x'_{1,\neg s}, x'_{2,\neg s}, x'_{3,\neg s}, n_1{=}n_2{=}1) > v_1(x_1^*, x_2^*, x_3{=}0, n_1{=}1, n_2{=}0) \iff$$
$$\mathcal{C}'_{3,\neg s}(\beta) := \ln \frac{\left(1-\frac{\Gamma}{\beta}\right)K}{1+\Gamma-\frac{\Gamma}{\beta}} + \Gamma \ln \frac{\Gamma K\left(1-\frac{\Gamma}{\beta}\right)}{\left(1+\Gamma-\frac{\Gamma}{\beta}\right)^2} + \frac{\frac{\Gamma^2}{\beta}\ln\frac{\alpha\Gamma^2 K}{\left(1+\Gamma-\frac{\Gamma}{\beta}\right)^2}}{1-\frac{\Gamma}{\beta}} - \ln\frac{K}{1+\Gamma} - \ln\frac{\Gamma K}{1+\Gamma} > 0.$$

As in Appendix C.5, we then need to show that $\frac{\partial \mathcal{C}'_{1,\neg s}(\beta)}{\partial\beta} < 0$, $\frac{\partial \mathcal{C}'_{2,\neg s}(\beta)}{\partial\beta} < 0$, and $\frac{\partial \mathcal{C}'_{3,\neg s}(\beta)}{\partial\beta} < 0$. Computing the derivatives, we have:

$$\frac{\partial \mathcal{C}'_{1,\neg s}(\beta)}{\partial\beta} = -\frac{\Gamma^2\left(\beta-\Gamma+(\beta+\beta\Gamma-\Gamma)\ln\frac{\alpha\Gamma^2K}{\left(1-\frac{\Gamma}{\beta}+\Gamma\right)^2}\right)}{(\Gamma-\beta)^2(\beta+\beta\Gamma-\Gamma)} < 0,$$

and

$$\frac{\partial \mathcal{C}'_{2,\neg s}(\beta)}{\partial\beta} = \frac{\partial \mathcal{C}'_{3,\neg s}(\beta)}{\partial\beta} = -\left(\frac{\Gamma}{\Gamma-\beta}\right)^2\left(\frac{\Gamma(\beta-\Gamma)(1-\beta)}{\beta(\beta+\Gamma\beta-\Gamma)} + \ln\frac{\alpha\Gamma^2K}{\left(1+\Gamma-\frac{\Gamma}{\beta}\right)^2}\right) < 0,$$

as $\ln\frac{\alpha\Gamma^2K}{\left(1-\frac{\Gamma}{\beta}+\Gamma\right)^2} = \ln(x'_{3,\neg s})$ must be positive in order to satisfy $F_{2,\neg s}(k'_{2,\neg s}) \geq 0$.

The conditions for a high fertility equilibrium strategy in the settlement regime can be written as:

$$F_{2,s}(k'_{3,s}) \geq 0 \iff \mathcal{C}'_{1,s}(\beta) := \Gamma\ln\frac{\alpha\frac{\Gamma^2}{\beta}K}{1+\Gamma-\frac{\Gamma}{\beta}} \geq 0,$$

$$F_{1,s}^{n_2=1}(K) > 0 \iff \mathcal{C}'_{2,s}(\beta) := \ln \frac{\left(1-\frac{\Gamma}{\beta}\right)K}{1+\Gamma-\frac{\Gamma}{\beta}} + \Gamma \ln \frac{\Gamma\left(1-\frac{\Gamma}{\beta}\right)K}{1+\Gamma-\frac{\Gamma}{\beta}} + \frac{\Gamma^2}{\beta}\ln\frac{\alpha\frac{\Gamma^2}{\beta}K}{1+\Gamma-\frac{\Gamma}{\beta}} - \ln K > 0$$

and

$$v_1(x'_{1,s}, x'_{2,s}, x'_{3,s}, n_1{=}1, n_2{=}1) > v_1(x_1^*, x_2^*, x_3{=}0, n_1{=}1, n_2{=}0) \iff$$
$$\mathcal{C}'_{3,s}(\beta) := \ln \frac{\left(1-\frac{\Gamma}{\beta}\right)K}{1+\Gamma-\frac{\Gamma}{\beta}} + \Gamma \ln \frac{\Gamma\left(1-\frac{\Gamma}{\beta}\right)K}{1+\Gamma-\frac{\Gamma}{\beta}} + \frac{\Gamma^2}{\beta}\ln\frac{\alpha\frac{\Gamma^2}{\beta}K}{1+\Gamma-\frac{\Gamma}{\beta}} - \ln\frac{K}{1+\Gamma} - \Gamma\ln\frac{\Gamma K}{1+\Gamma} > 0.$$

We then need to show that $\frac{\partial \mathcal{C}'_{1,s}(\beta)}{\partial\beta} < 0$, $\frac{\partial \mathcal{C}'_{2,s}(\beta)}{\partial\beta} < 0$, and $\frac{\partial \mathcal{C}'_{3,s}(\beta)}{\partial\beta} < 0$. Computing the derivatives, we then have:

$$\frac{\partial \mathcal{C}'_{1,s}(\beta)}{\partial\beta} = -\frac{\Gamma}{(\Gamma-\beta)^2}\left(\frac{(1+\Gamma)(\beta-\Gamma)\beta}{\beta+\beta\Gamma-\Gamma} + \Gamma\ln\frac{\alpha\Gamma^2K}{\beta+\beta\Gamma-\Gamma}\right) < 0,$$

and

$$\frac{\partial \mathcal{C}'_{2,s}(\beta)}{\partial\beta} = \frac{\partial \mathcal{C}'_{3,s}(\beta)}{\partial\beta} = -\left(\frac{\Gamma}{\Gamma-\beta}\right)^2 \ln\frac{\alpha\Gamma^2K}{\beta+\beta\Gamma-\Gamma} < 0.$$

■

Proof of Proposition 5: The proof follows Appendix C.6. For any fixed value of $\Gamma := \beta\cdot\delta$, we need to show that:

$$\frac{\partial(\mathcal{C}'_{1,s}-\mathcal{C}'_{1,\neg s})}{\partial\beta} < 0, \quad \frac{\partial(\mathcal{C}'_{2,s}-\mathcal{C}'_{2,\neg s})}{\partial\beta} < 0, \quad \text{and} \quad \frac{\partial(\mathcal{C}'_{3,s}-\mathcal{C}'_{3,\neg s})}{\partial\beta} < 0,$$

Computing the first derivative we have,

$$\frac{\partial(\mathcal{C}'_{1,s}-\mathcal{C}'_{1,\neg s})}{\partial\beta} = \frac{\Gamma}{(\Gamma-\beta)^2}\left(\Gamma-\beta+\Gamma\ln\frac{\beta^2}{\beta+\beta\Gamma-\Gamma}\right) < 0$$

since $\Gamma-\beta<0$ and $\beta^2 < \beta+\beta\Gamma-\Gamma$. Computing the second and third derivatives we have,

$$\frac{\partial(\mathcal{C}'_{2,s}-\mathcal{C}'_{2,\neg s})}{\partial\beta} = \frac{\partial(\mathcal{C}'_{3,s}-\mathcal{C}'_{3,\neg s})}{\partial\beta} = \left(\frac{\Gamma}{\Gamma-\beta}\right)^2\left(\frac{\Gamma(\beta-\Gamma)(1-\beta)}{\beta(\beta+\beta\Gamma-\Gamma)} + \ln\frac{\beta^2}{\beta+\beta\Gamma-\Gamma}\right).$$

To analyze the sign of these derivatives, we first define

$$\mathcal{D}(\Gamma) := \Gamma(\beta - \Gamma)(1-\beta) + \beta(\beta + \beta\Gamma - \Gamma)\ln \frac{\beta^2}{\beta + \beta\Gamma - \Gamma}$$

where we can check that

$$\frac{\partial \mathcal{D}(\Gamma)}{\partial \Gamma} = -(1-\beta)\left(2(\Gamma - \beta) + \beta \ln \frac{\beta^2}{\beta + \beta\Gamma - \Gamma}\right) > 0$$

and that $\mathcal{D}(0) < 0$ and $\mathcal{D}(1) = 0$ for any $\beta \in (0,1)$. Hence, $\frac{\partial(\mathcal{C}'_{2,s} - \mathcal{C}'_{2,\neg s})}{\partial \beta} = \frac{\partial(\mathcal{C}'_{3,s} - \mathcal{C}'_{3,\neg s})}{\partial \beta} < 0$. ■

Proposition VI (Welfare) *Consider the parameter region where a dynasty in the no-settlement regime follows a low-fertility strategy and a dynasty in the settlement regime follows a high-fertility strategy. All generations are better off in the settlement regime; i.e.,* $v_2(x'_{2,s}, x'_{3,s}, n_2{=}1) > v_2(x^*_2, x_3{=}0, n_2{=}0)$, *and* $v_1(x'_{1,s}, x'_{2,s}, x'_{3,s}, n_1{=}1, n_2{=}1) > v_1(x^*_1, x^*_2, x_3{=}0, n_1{=}1, n_2{=}0)$.

Proof: Generation 1 is better off in the model with commitment as the condition

$$v_1(x'_{1,s}, x'_{2,s}, x'_{3,s}, n_1 = 1, n_2 = 1) > v_1(x^*_1, x^*_2, x_3 = 0, n_1 = 1, n_2 = 0) \tag{26}$$

defines the region characterized in Proposition 2. Note that condition (26) can be rewritten as:

$$\frac{1+\beta\delta}{\delta}\ln\left(\frac{(1-\delta)(1+\beta\delta)}{1-(1-\beta)\delta}\right) + \frac{\beta\delta}{1-\delta}\ln\left(\frac{\alpha\beta\delta^2 K}{1-(1-\beta)\delta}\right) > 0. \tag{27}$$

Generation 2 is better off in the model with commitment in the region characterized in Proposition 2 if and only if

$$v_2(x'_{2,s}, x'_{3,s}, n_2 = 1) > v_2(x^*_2, x_3 = 0, n_2 = 0)$$

which holds if and only if

$$\ln\left(\frac{(1-\delta)(1+\beta\delta)}{1-(1-\beta)\delta}\right) + \beta\delta\ln\left(\frac{\alpha\beta\delta^2 K}{1-(1-\beta)\delta}\right) > 0. \tag{28}$$

Inequality (27) implies that inequality (28) is satisfied. ■