The Firm Size-Leverage Relationship and Its Implications for Entry and Business Concentration

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What We Do

- Among U.S. firms, leverage is positively related to size
- Develop a model of firm dynamics with default risk that can explain this pattern
- Explore the implications of a decline in the risk-free rate for startup rates and business concentration

Overview

Leverage-Size Relationship

- Size is determined by the number of varieties a firm manages
- Each variety has a probability ϕ of going extinct
  - By property of the binomial distribution, the variance of growth rate is lower for larger firms (for evidence, see Davis, Haltiwanger, Jarmin, Krizan, Javier, Nucci, and Sandusky (2007))
- Business owners are more impatient than lenders and seek to borrow against the future cash flow of their companies

Overview

Why Larger Firms Have Higher Leverage?

- Example, if $\phi = 0.2$ then for $K = 1$, default probability will be 20 % for any positive debt
  - If time ends next period, a $K = 100$ firm can borrow upto 70 % of its cash flow with a default risk of (roughly) 1 %
  - In a model where default is to be avoided (say, because of costs) larger firms will have higher leverage
- The standard model (Cooley and Quadrini (2001)) has the opposite implication

Overview

Ideas

- Entry and firm growth are the result of the arrival of ideas for new products (similar to Chatterjee and Rossi-Hansberg (2012))
- Workers get ideas with some probability and randomly match with an existing firm to negotiate with
- The worker/inventor can sell her idea to the firm or start her own firm.

Overview

Sale vs Startup

- The success probability might be different in the startup vs the firm
- The existing firm can borrow more against the idea's cash-flow than a startup
- These factors will affect where the idea is implemented
- In the final section, analyze how risk-free rate affects this decision

Literature I

Positive relationship b/w firm size and leverage

- Rajan and Zingales (1995), Dinlersoz, Kalemli-Ozcan, Hyatt, and Penciakova (2019)

Negative relationship b/w firm size and employment volatility

- Davis, Haltiwanger, Jarmin, Krizan, Javier, Nucci, and Sandusky (2007)

Low interest rates and misallocation

- Gopinath, Kalemli-Ozcan, Karabarbounis, and Villegas-Sanchez (2017), Caggese and Perez-Orive (2019), Liu, Mian, and Sufi (2019),

Relationship Between Size and Leverage

	Panel			
Dep Var	Lev I	Lev II	Lev I	Lev II
	Market		Book	
lnSale	0.034	0.037	0.036	0.038
	(21.83)	(20.39)	(20.85)	(20.13)
Cap_Ratio	0.287	0.440	0.634	0.592
	(22.40)	(27.14)	(49.01)	(43.65)
Profit_Ratio	-0.030	-0.137	-0.010	-0.010
	(-3.08)	(-9.43)	(-2.04)	(-1.89)
Firm FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
Num of Obs	182,984	183,496	211,096	203,009
Num of Groups	18,580	18,605	20,937	20,582
R^2	0.15	0.18	0.18	0.16

Notes: COMPUSTAT nonfinancial and nonutility firms reporting in USD with sales and market value greater than 1 M $, 1978-2015

Relationship Between Size and Leverage

	Cross-Section			
Dep Var	Lev I	Lev II	Lev I	Lev II
	Market		Book	
lnSale	0.013	0.025	0.021	0.038
	(49.22)	(79.20)	(62.77)	(109.49)
Cap_Ratio	0.293	0.400	0.486	0.378
	(107.72)	(127.90)	(126.06)	(91.98)
Profit_Ratio	0.004	-0.127	-.010	-0.012
	(0.97)	(-28.17)	(-16.90)	(-18.60)
Subindustry FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
Num of Obs	177,678	178,178	203,834	195,999
R^2	0.26	0.31	0.32	0.30

Notes: COMPUSTAT nonfinancial and nonutility firms reporting in USD with sales and market value greater than 1 M $, 1978-2015

MODEL

Timing

Three sub-periods

- Ideas arrive, sale/startup decision made
- Variety extinction shocks are realized
- Default and new debt issuance decisions made

Model Elements

Sub-period 1

- (K, B): # of varieties owned and debt,
  - $K \in \{0, 1, 2, \ldots, K_{\max}\}$, $B \in \mathbb{I}$
- An idea for a new variety is encountered with prob ρK
  - $0 < \rho K_{\max} < 1$
- If the idea is implemented in the firm, it succeeds with prob $s \sim F(s)$
- If implemented in a startup it succeeds with prob σ

Model Elements

Sub-Period 1: Sale vs Startup (Chatterjee and Rossi-Hansberg (2012))

- Let $W(K, B)$ be the value of the firm with K varieties at the beginning of the second subperiod
- Let s^* solve:

$$s^*[W(K+1, B) - W(K, B)] = \sigma W(1, 0)$$

- If $s > s^*(K, B)$, the idea is purchased for $\sigma W(1, 0)$ and implemented in the firm
- Else, the idea is implemented in a startup

Model Elements

Sub-Period 2: Product extinction shock

- Existing firms arrive with (K, B) or $(K+1, B)$ and a startup with $(1, 0)$
- Any variety can go extinct with prob ϕ
- Varieties remaining after extinction shocks is K'

Model Elements

Sub-Period 3: Choice of B'

$$V(K', B) = \max_{B'} \pi K' - B + q(K', B')B' + \beta\Omega(K', B')$$

s.t.

$$\pi K' - B + q(K', B')B' \geq 0$$

$$d(K', B') \leq \theta$$

No access to outside equity

Upper bound on default prob

Model Elements

Sub-Period 3: Default Decision

If $G(K')$ is max revenue from bond issuances s.t. default prob not exceeding θ, then $\pi K' + G(K')$ is the most cash the firm can raise

Default happens if $\pi K' + G(K') \equiv \overline{B}(K') < B$

In the event of default, lenders get $(\pi K' + G(K'))/B$ per unit of debt

Model Elements

Sub-period 3: Determination of $q(K', B')$

Let α be the recovery rate on a unit bond

$$\alpha(K'', B') = \begin{cases} 1 & \text{if } B' \leq \bar{B}(K'') \\ \frac{\bar{B}(K'')}{B'} & \text{otherwise} \end{cases}$$

Then,

$$q(K', B') = (1+r)^{-1} \sum_{K''=0}^{K'+1} \alpha(K'', B') \Pr(K''|K', B').$$

Model Elements

Closing the Model

- The measure of new ideas arriving in a period is a constant M
- ρ solves:

$$\rho = \frac{M}{\sum_K K\, H_\rho(K)}$$

where $H_\rho(\cdot)$ is the steady state firm-size distribution

Transition

Determination of π along the transition

- $U = \left[\int_0^{N'} c(n)^{\gamma} dn\right]^{1/\gamma}, \ \ \gamma \in (0,1)$
- Workers are equally distributed across varieties in production
- Real profits per variety will be a function of total number of varieties N'
  - $\pi(N') \propto N'^{(\frac{1}{\gamma}-2)}$

CALIBRATION

Calibration

Calibrate to 1997

Independently set parameters:

$$r = 1.8\%,\ \beta = 0.975,\ F(s) \sim \text{Truncated Normal}(0.7, \omega = 0.08)$$

Jointly set parameters:

Description of Target	Data/Model	Param	Value
Probability of Default (%)	1.0**	θ	0.08
Survival rate of 1-yr old firms in '97 (%)	86.0*	ϕ	0.20
Empl Share of New Entrants in '97 (%)	2.7*	σ	0.62

* BDS; ** Corbae and D'Erasmo (2017)

Calibration

Non-Targeted Moments

Moments	Data	Model
Response of Leverage to ln Sales	$0.013 - 0.038$	0.024
Survival rate of 0-yr old firms*	0.76	0.83
Survival rate of 2-yr old firms*	0.88	0.87
Survival rate of 3-yr old firms*	0.90	0.88
Survival rate of 4-yr old firms*	0.91	0.89

* BDS, HP Trend

Calibration

Non-Targeted Moments

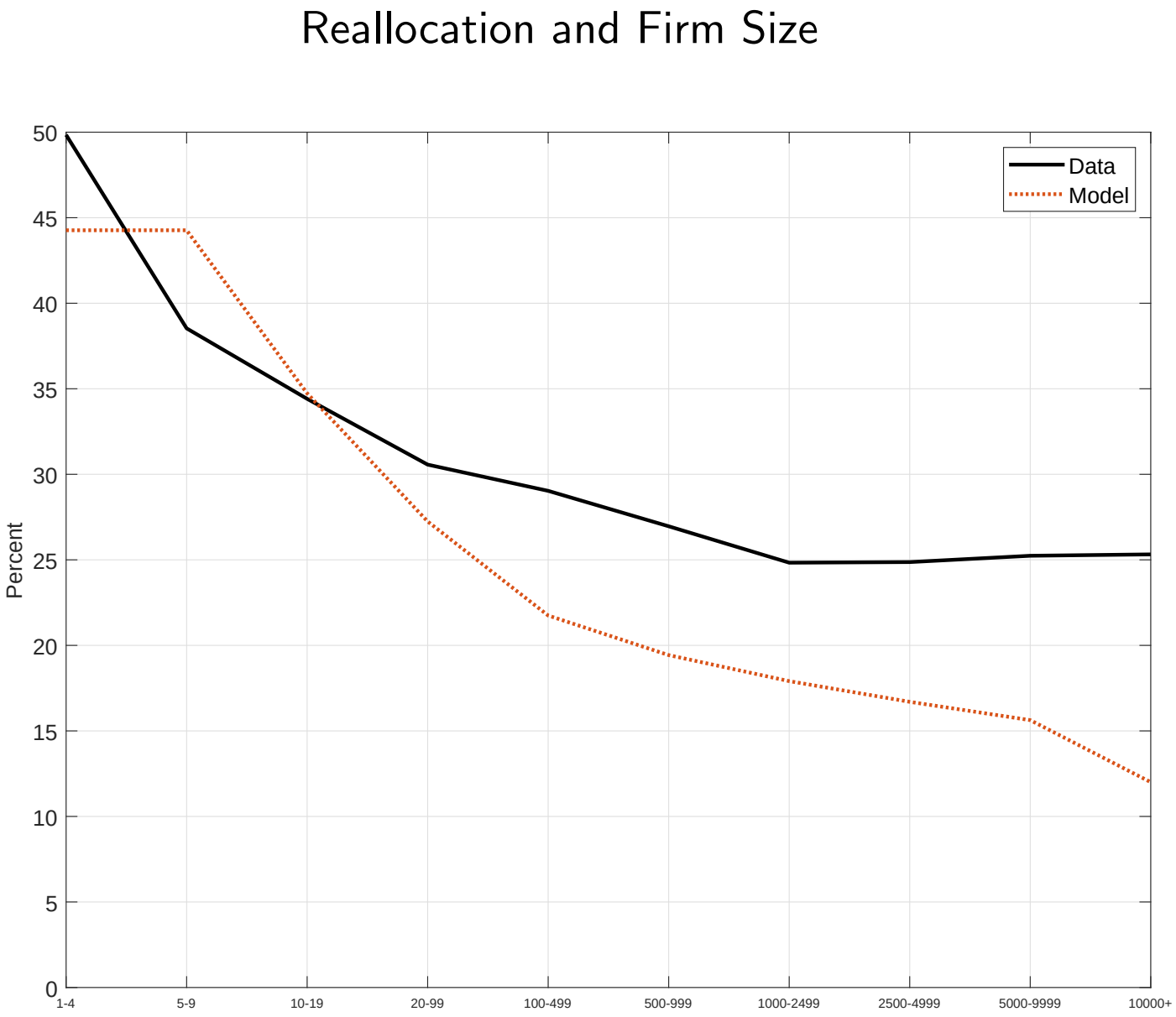
Moments	Data	Model
Empl growth of 0-yr old firms*	1.01	0.97
Empl growth of 1-yr old firms*	0.97	0.97
Empl growth of 2-yr old firms*	0.97	0.97
Empl growth of 3-yr old firms*	0.97	0.97
Empl growth of 4-yr old firms*	0.98	0.97

* BDS, HP Trend

Empl growth in t-yr old firms: Total empl of $(t+1)$-yr old firms in 1997 $\div$ total empl in t-yr old firms in 1996

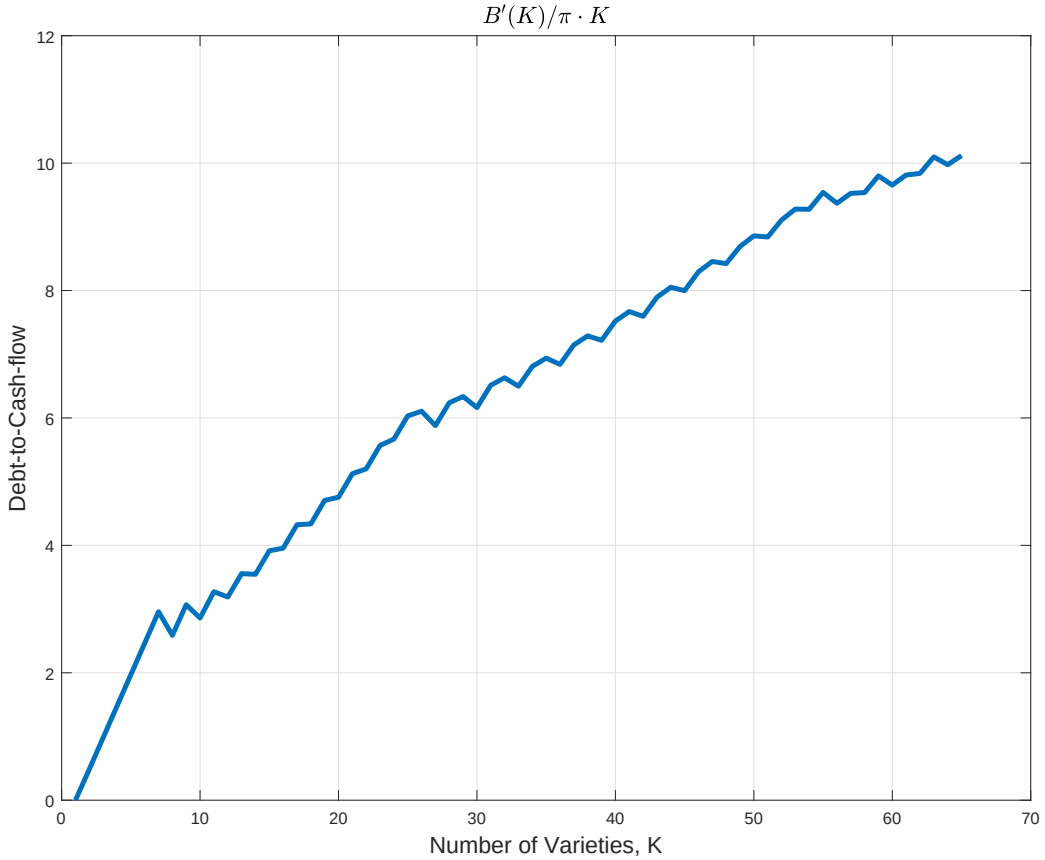
Calibration

Non-Targeted Moments



Model Properties

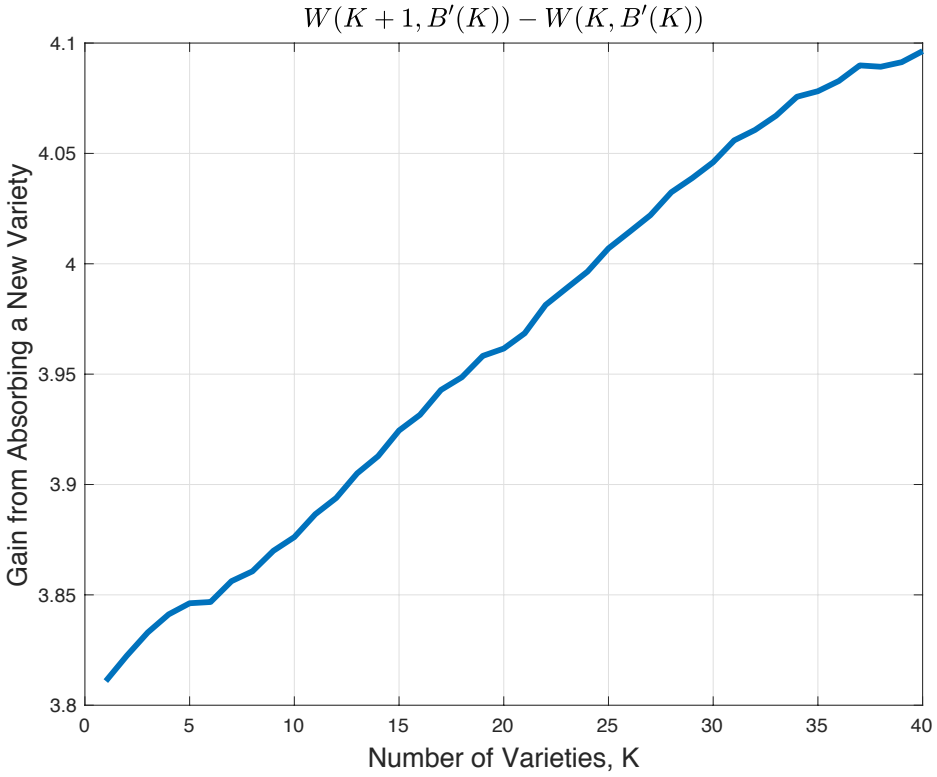
Debt-to-Cash-flow and Firm Size



- Proportion of surviving varieties next period is roughly $(1-\phi)(1+\rho)$ on average and its variance is $[\phi(1-\phi)/(1+\rho)K]$
- Variance declines with K

Model Properties

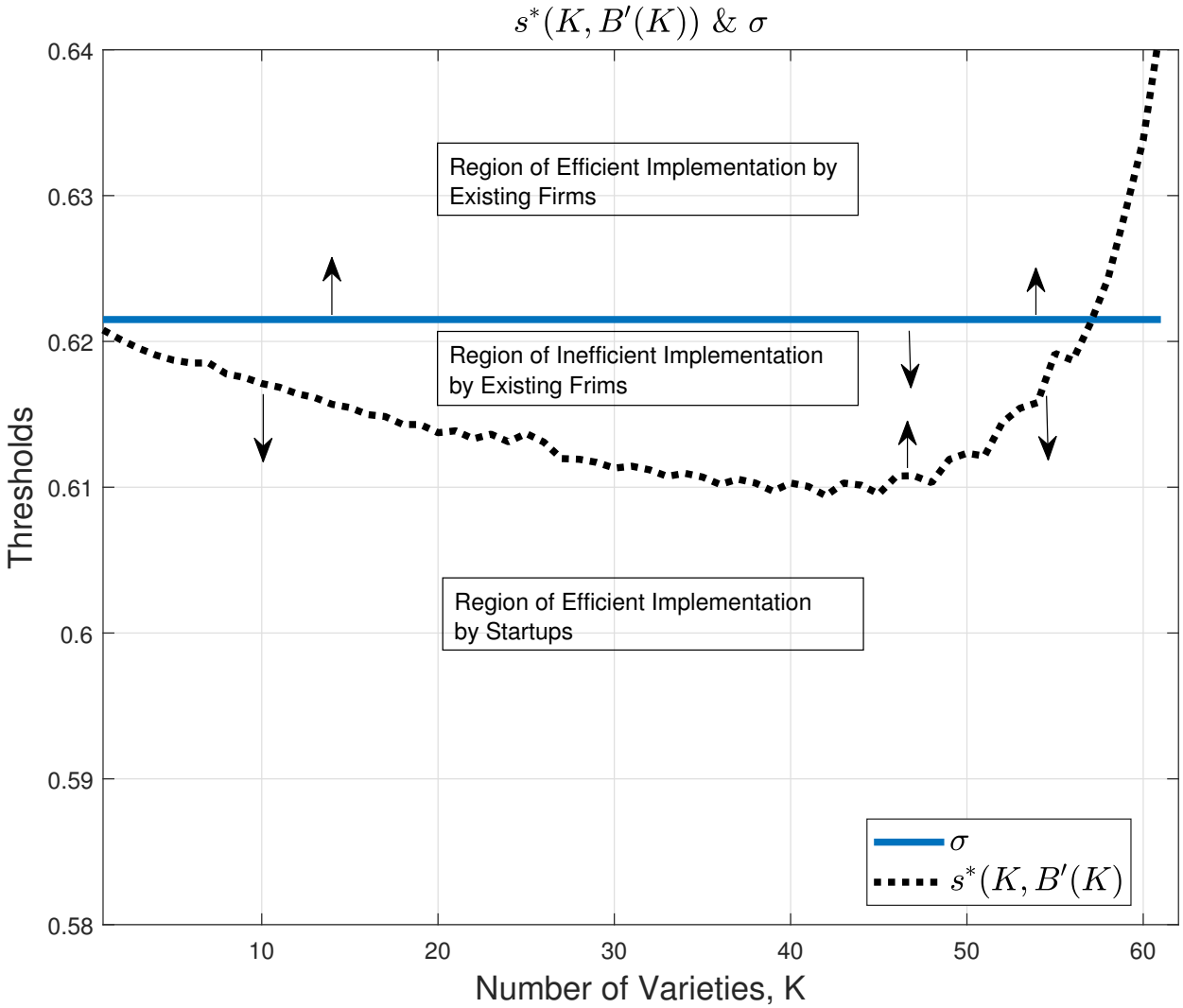
Positive Relationship between Size & Gain from Absorbing a New Variety



- The reason is that larger firms are able to borrow a greater fraction of the pdv of a variety's cash flow

Model Properties

Efficient and Inefficient Organizational Choice



APPLICATION

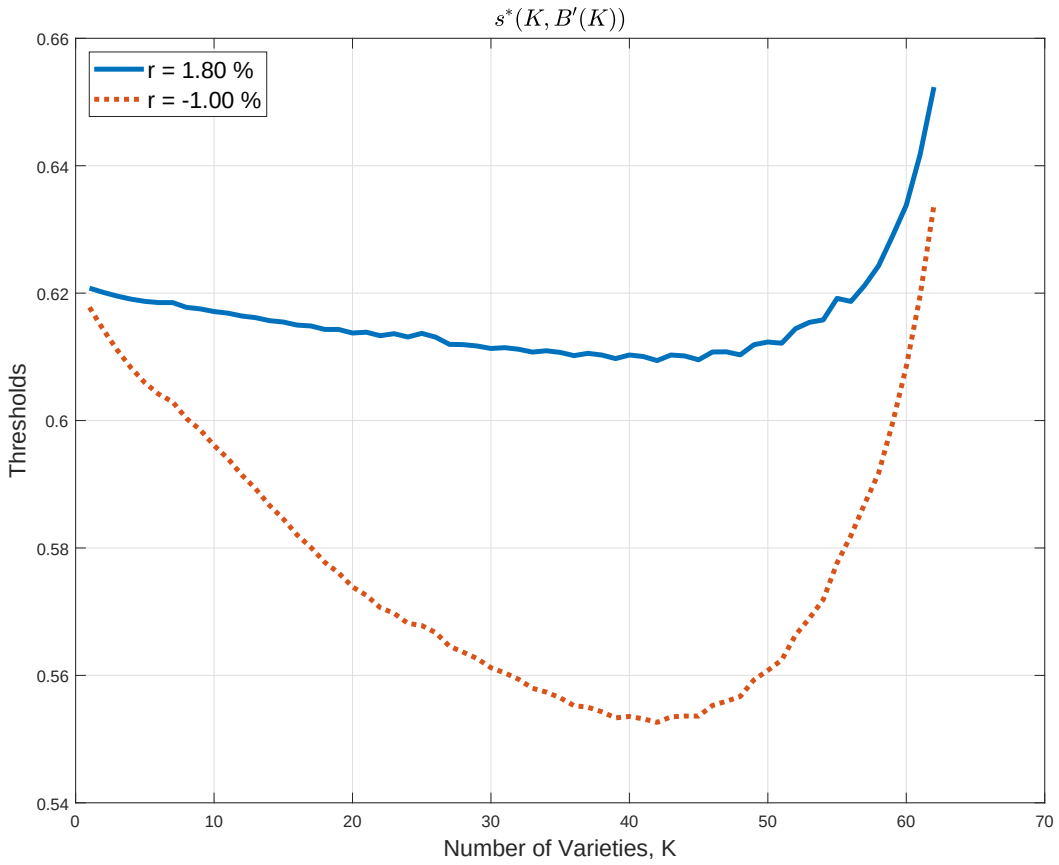
Application

Implications of a Decline in r

- Lower interest rates benefit larger firms more because they have access to borrowing
- More of the arriving ideas get sold instead of resulting in start-ups
- $\Rightarrow$ lower entry rate and higher concentration

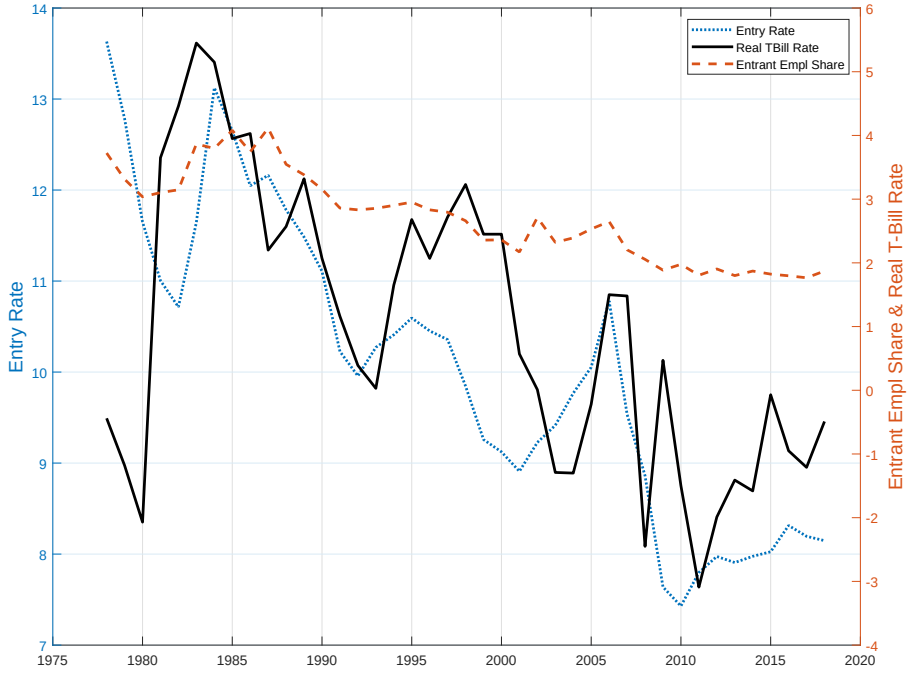
Model

Implications of a Decline in r



Application

Why Is This Relevant?



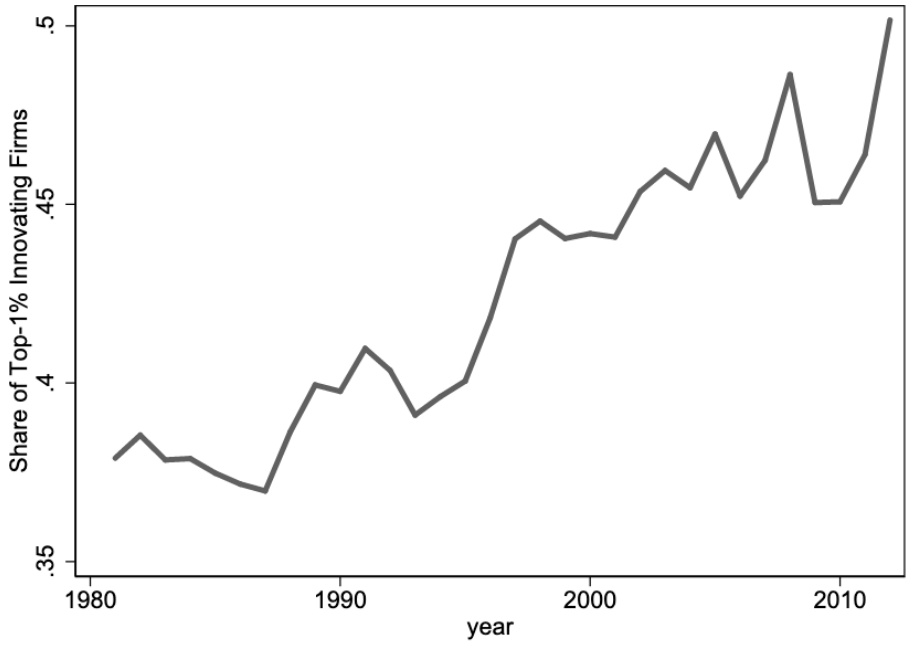
- Declines in r, startup rates, and entrant employment share
- Decline in r accelerated in the late 1990s: Caballero, Farhi, and Gourinchas (2008), Mendoza, Quadrini, and Ríos-Rull (2009), Del Negro, Giannone, Giannoni, and Tambalotti (2017),

Application

Why Is This Relevant?

- Rising concentration of innovation

Share of Patents of Top 1% Patenting Firms



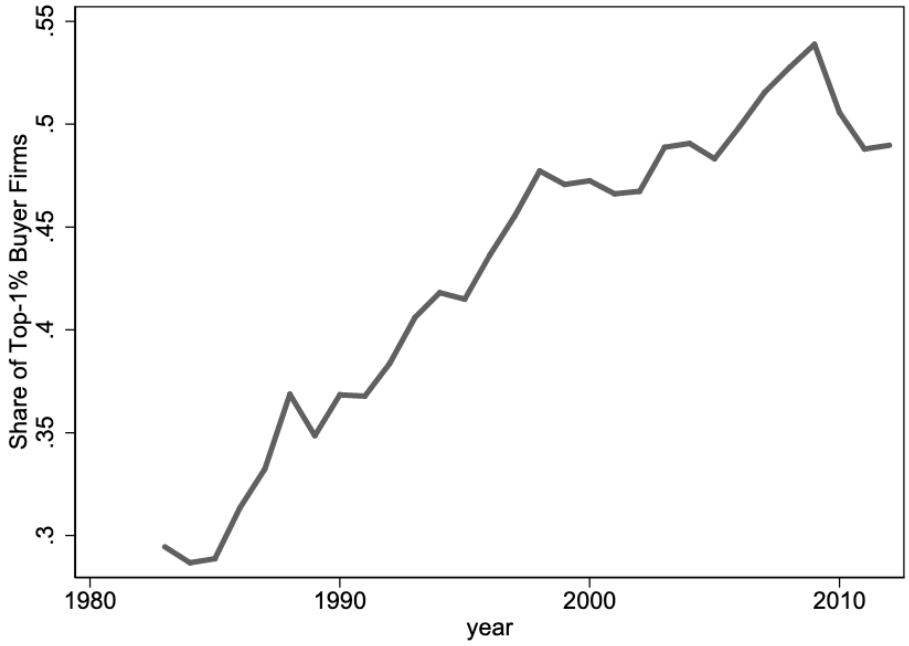
- *Source: "What Happened to Business Dynamism?" by U. Akcigit & S. Ates, NBER WP No. 25756, May 2019*

Application

Why Is This Relevant?

- Rising concentration of ownership of new ideas

Share of Patents of Top 1% Buyer Firms



- *Source: "What Happened to Business Dynamism?" by U. Akcigit & S. Ates, NBER WP No. 25756, May 2019*

Application

Why Is This Relevant?

Share of After-tax Corporate Profits in GDP

- A puzzle for most theories of entry: high profits $\Rightarrow$ more entry
- In our model also, high profits could potentially increase the flow of new ideas (M) and, therefore, entry
- But our model also features another channel: where new varieties are implemented

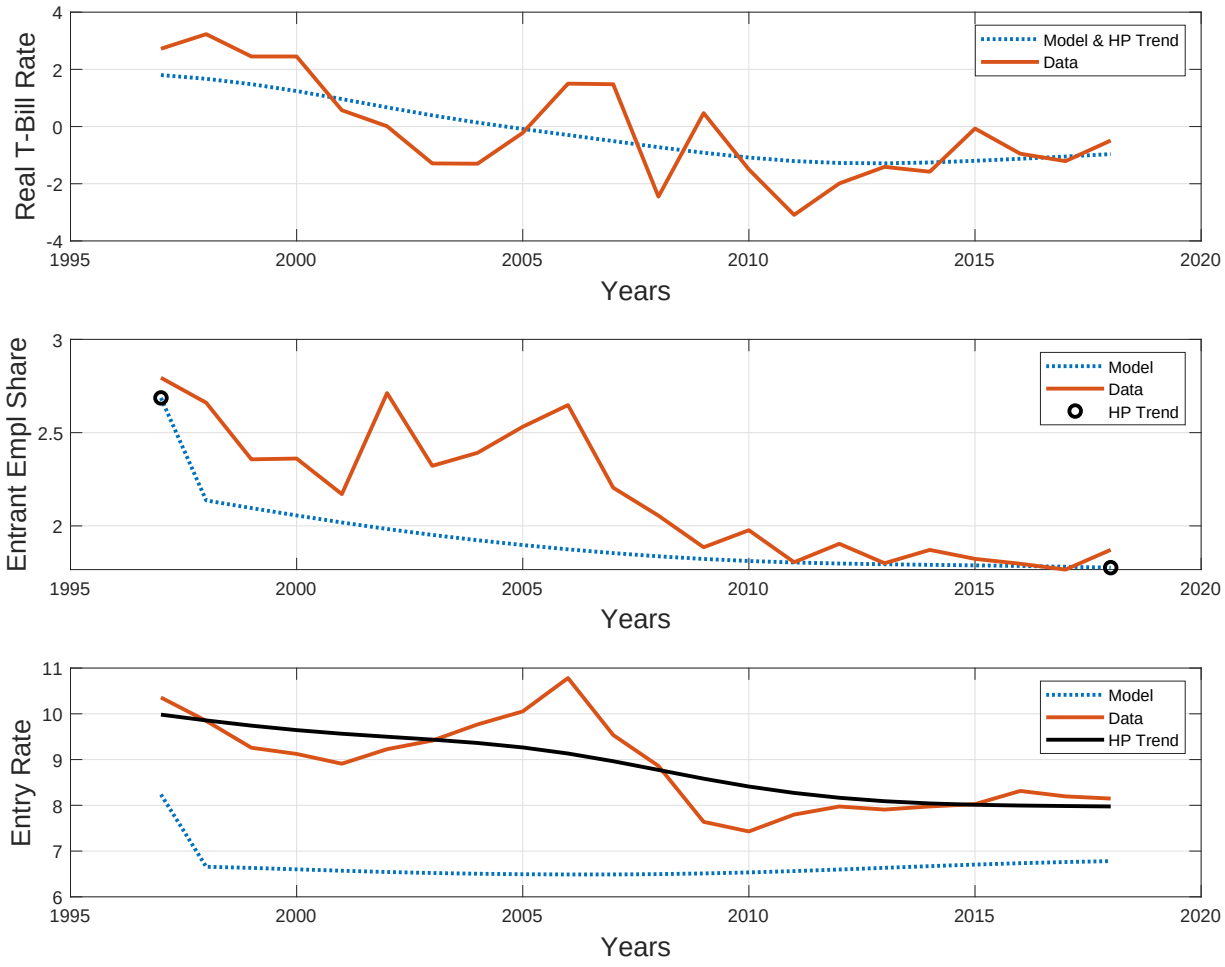
Application

Assessing the strength of the interest rate channel

- We feed in the trend in the risk-free rate between 1997 and 2018 as fully anticipated and compute transition path
- We picked the s.d. of s to generate the trend decline in employment share of entrants between 1997 and 2018
  - Fell from 2.7 percent to 1.8 percent
- We keep M – number of new ideas arriving per period – fixed

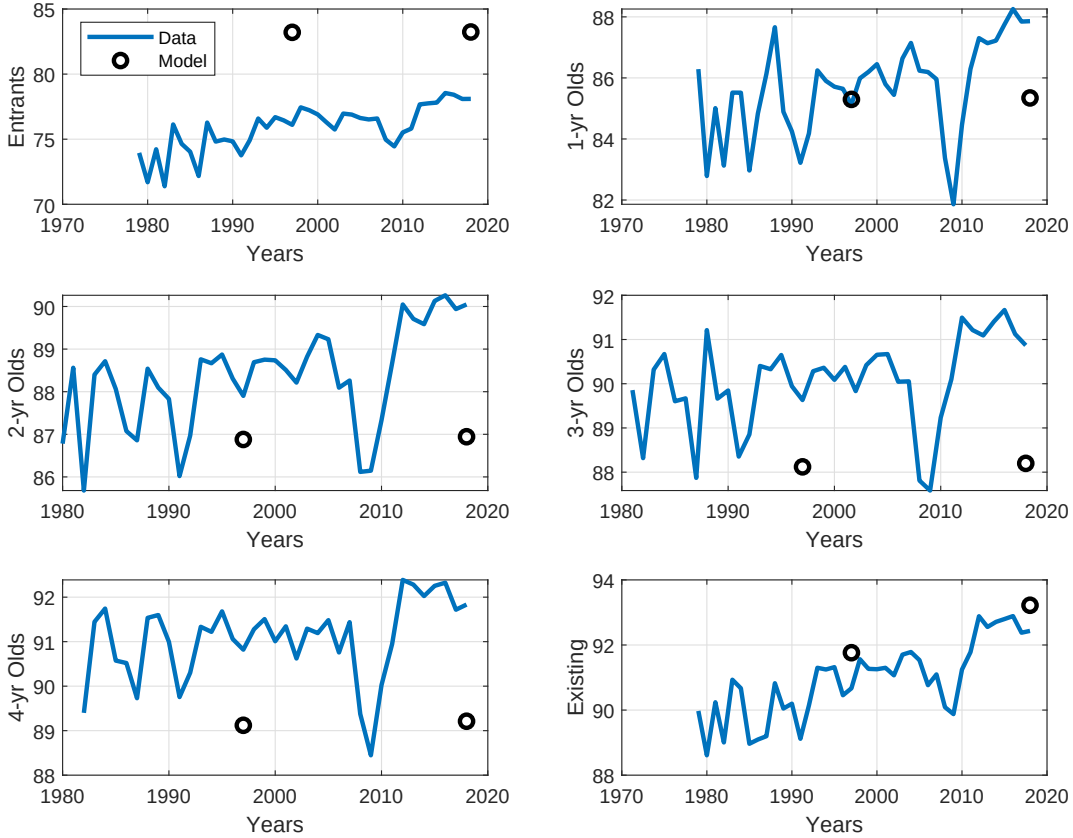
Application

Assessing the strength of the interest rate channel



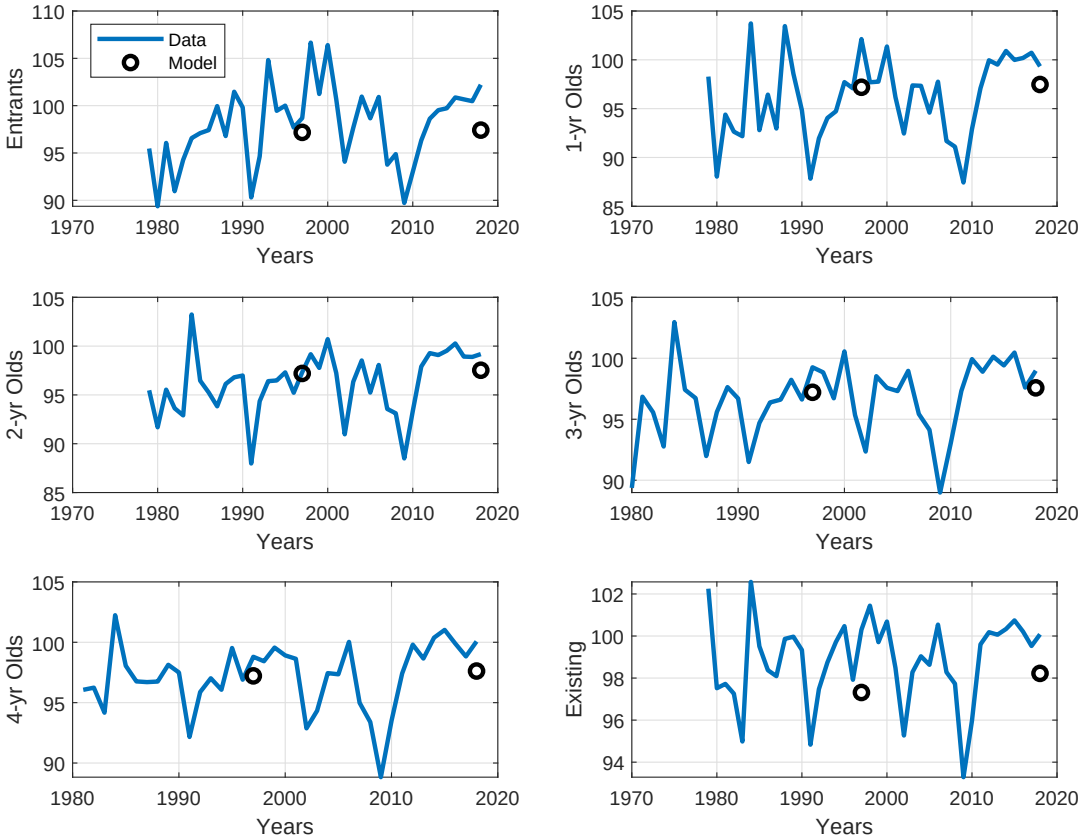
Application

Survival Rates: Model and Data



Application

Employment Growth: Model and Data



Application

Implications for Business Concentration

	Measure of Firms	Share of Output Base	Share of Output Low r
Top 5% by Size in Base	5.8	24.1	29.9
Top 10% by Size in Base	11.7	37.7	45.5
Top 15% by Size in Base	17.5	47.7	56.5
Measure of Varieties		387.13	387.06

Autor, Dorn, Katz, Patterson, and van Reenen (2017) document that share of sales of top 4 (top 20) firms has risen in most industries since the 90s

Summary

Presented a model where . . .

- larger firms are less risky and have higher leverage and
- a decline in the interest rate makes purchasing a variety more attractive for firms, resulting in fewer startups and greater concentration of sales

[]Autor, D., D. Dorn, L. Katz, C. Patterson, and J. van Reenen (2017): "Concentrating on the Fall of the Labor Share," National Bureau of Economic Research, Working Paper No. 23108.

[] Caballero, R. J., E. Farhi, and P.-O. Gourinchas (2008): "An Equilibrium Model of 'Global Imbalances' and Low Interest Rates," *American Economic Review*, 98(1), 358–393.

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[]Chatterjee, S., and E. Rossi-Hansberg (2012): "Spinoffs and the Market for Ideas," *International Economic Review*, 53(1), 53–93.

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[]Davis, S. J., J. Haltiwanger, R. S. Jarmin, C. Krizan, M. Javier, A. Nucci, and K. Sandusky (2007): "Measuring

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