The Firm Size-Leverage Relationship and Its Implications for Entry and Business Concentration

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Disclaimer: The views expressed in this paper are those of the authors and do not necessarily represent the views of the Federal Reserve Bank of Philadelphia or the Federal Reserve System.
What We Do

▶ Among U.S. firms, leverage is positively related to size

▶ Develop a model of firm dynamics with default risk that can explain this pattern

▶ Explore the implications of a decline in the risk-free rate for startup rates and business concentration
Overview
Leverage-Size Relationship

- Size is determined by the number of varieties a firm manages.
- Each variety has a probability $\phi$ of going extinct.
  - By property of the binomial distribution, the variance of growth rate is lower for larger firms (for evidence, see Davis, Haltiwanger, Jarmin, Krizan, Javier, Nucci, and Sandusky (2007)).
- Business owners are more impatient than lenders and seek to borrow against the future cash flow of their companies.
Overview

Why Larger Firms Have Higher Leverage?

- Example, if $\phi = 0.2$ then for $K = 1$, default probability will be 20% for any positive debt
  
  - If time ends next period, a $K = 100$ firm can borrow up to 70% of its cash flow with a default risk of (roughly) 1%
  
  - In a model where default is to be avoided (say, because of costs) larger firms will have higher leverage

- The standard model (Cooley and Quadrini (2001)) has the opposite implication
Entry and firm growth are the result of the arrival of ideas for new products (similar to Chatterjee and Rossi-Hansberg (2012)).

Workers get ideas with some probability and randomly match with an existing firm to negotiate with.

The worker/inventor can sell her idea to the firm or start her own firm.
Overview
Sale vs Startup

▶ The success probability might be different in the startup vs the firm

▶ The existing firm can borrow more against the idea’s cash-flow than a startup

▶ These factors will affect where the idea is implemented

▶ In the final section, analyze how risk-free rate affects this decision
Positive relationship b/w firm size and leverage


Negative relationship b/w firm size and employment volatility


Low interest rates and misallocation

## Relationship Between Size and Leverage

### Panel

<table>
<thead>
<tr>
<th>Dep Var</th>
<th>Lev I</th>
<th>Lev II</th>
<th>Lev I</th>
<th>Lev II</th>
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<tr>
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<td></td>
<td>(21.83)</td>
<td>(20.39)</td>
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<td>(22.40)</td>
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<td>$R^2$</td>
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<td>0.18</td>
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Notes: COMPSTAT nonfinancial and nonutility firms reporting in USD with sales and market value greater than 1 M $, 1978-2015
# Relationship Between Size and Leverage

## Cross-Section

<table>
<thead>
<tr>
<th>Dep Var</th>
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<td>(49.22)</td>
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<td>Time FE</td>
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<td>$R^2$</td>
<td>0.26</td>
<td>0.31</td>
<td>0.32</td>
<td>0.30</td>
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</table>

Notes: COMPUSTAT nonfinancial and nonutility firms reporting in USD with sales and market value greater than 1 M $, 1978-2015
MODEL
Three sub-periods

- Ideas arrive, sale/startup decision made
- Variety extinction shocks are realized
- Default and new debt issuance decisions made
Model Elements
Sub-period 1

▶ \((K, B)\): \# of varieties owned and debt,
  
  ▶ \(K \in \{0, 1, 2, \ldots, K_{\text{max}}\}, B \in \mathbb{I}\)

▶ An idea for a new variety is encountered with prob \(\rho K\)
  
  ▶ \(0 < \rho K_{\text{max}} < 1\)

▶ If the idea is implemented in the firm, it succeeds with prob \(s \sim F(s)\)

▶ If implemented in a startup it succeeds with prob \(\sigma\)
Let $W(K, B)$ be the value of the firm with $K$ varieties at the beginning of the second subperiod

Let $s^*$ solve:

$$s^*[W(K + 1, B) - W(K, B)] = \sigma W(1, 0)$$

If $s > s^*(K, B)$, the idea is purchased for $\sigma W(1, 0)$ and implemented in the firm

Else, the idea is implemented in a startup
Model Elements

Sub-Period 2: Product extinction shock

- Existing firms arrive with $(K, B)$ or $(K + 1, B)$ and a startup with $(1, 0)$
- Any variety can go extinct with prob $\phi$
- Varieties remaining after extinction shocks is $K'$
Model Elements
Sub-Period 3: Choice of $B'$

\[
V(K', B) = \max_{B'} \pi K' - B + q(K', B')B' + \beta \Omega(K', B')
\]

s.t.
\[
\pi K' - B + q(K', B')B' \geq 0
\]
\[
d(K', B') \leq \theta
\]

No access to outside equity
Upper bound on default prob
Model Elements
Sub-Period 3: Default Decision

If $G(K')$ is max revenue from bond issuances s.t. default prob not exceeding $\theta$, then $\pi K' + G(K')$ is the most cash the firm can raise

Default happens if $\pi K' + G(K') \equiv \overline{B}(K') < B$

In the event of default, lenders get \((\pi K' + G(K'))/B\) per unit of debt
Let $\alpha$ be the recovery rate on a unit bond

$$
\alpha(K'', B') = \begin{cases} 
1 & \text{if } B' \leq \bar{B}(K'') \\
\frac{1}{B'(K'')} & \text{otherwise}
\end{cases}
$$

Then,

$$
q(K', B') = (1 + r)^{-1} \sum_{K''=0}^{K' + 1} \alpha(K'', B') \Pr(K''|K', B').
$$
The measure of new ideas arriving in a period is a constant $M$

$\rho$ solves:

$$\rho = \frac{M}{\sum_K K H_\rho(K)}$$

where $H_\rho(\cdot)$ is the steady state firm-size distribution
Transition

Determination of $\pi$ along the transition

\[ U = \left[ \int_0^{N'} c(n)^\gamma dn \right]^{1/\gamma}, \quad \gamma \in (0, 1) \]

- Workers are equally distributed across varieties in production
- Real profits per variety will be a function of total number of varieties $N'$
  \[ \pi(N') \propto N'^{\left(\frac{1}{\gamma} - 2\right)} \]
CALIBRATION
Calibration

Calibrate to 1997

Independently set parameters:

\[ r = 1.8\%, \ \beta = 0.975, \ F(s) \sim \text{Truncated Normal}(0.7, \omega = 0.08) \]

Jointly set parameters:

<table>
<thead>
<tr>
<th>Description of Target</th>
<th>Data/Model</th>
<th>Param</th>
<th>Value</th>
</tr>
</thead>
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<tr>
<td>Probability of Default (%)</td>
<td>1.0**</td>
<td>( \theta )</td>
<td>0.08</td>
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<tr>
<td>Survival rate of 1-yr old firms in '97 (%)</td>
<td>86.0*</td>
<td>( \phi )</td>
<td>0.20</td>
</tr>
<tr>
<td>Empl Share of New Entrants in '97 (%)</td>
<td>2.7*</td>
<td>( \sigma )</td>
<td>0.62</td>
</tr>
</tbody>
</table>

* BDS; ** Corbae and D’Erasmo (2017)
## Calibration

### Non-Targeted Moments

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Response of Leverage to $\ln$ Sales</td>
<td>0.013 – 0.038</td>
<td>0.024</td>
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<tr>
<td>Survival rate of 0-yr old firms*</td>
<td>0.76</td>
<td>0.83</td>
</tr>
<tr>
<td>Survival rate of 2-yr old firms*</td>
<td>0.88</td>
<td>0.87</td>
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<tr>
<td>Survival rate of 3-yr old firms*</td>
<td>0.90</td>
<td>0.88</td>
</tr>
<tr>
<td>Survival rate of 4-yr old firms*</td>
<td>0.91</td>
<td>0.89</td>
</tr>
</tbody>
</table>

* BDS, HP Trend
Calibration
Non-Targeted Moments

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empl growth of 0-yr old firms*</td>
<td>1.01</td>
<td>0.97</td>
</tr>
<tr>
<td>Empl growth of 1-yr old firms*</td>
<td>0.97</td>
<td>0.97</td>
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<tr>
<td>Empl growth of 2-yr old firms*</td>
<td>0.97</td>
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<tr>
<td>Empl growth of 3-yr old firms*</td>
<td>0.97</td>
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<tr>
<td>Empl growth of 4-yr old firms*</td>
<td>0.98</td>
<td>0.97</td>
</tr>
</tbody>
</table>

* BDS, HP Trend
Empl growth in $t$-yr old firms: \[
\text{Total empl of } (t + 1)\text{-yr old firms in 1997} \div \text{total empl in } t\text{-yr old firms in 1996}
\]
Calibration
Non-Targeted Moments

Reallocation and Firm Size

Data
Model
Proportion of surviving varieties next period is roughly \((1 - \phi)(1 + \rho)\) on average and its variance is \([\phi(1 - \phi)/(1 + \rho)K]\).

Variance declines with \(K\).
The reason is that larger firms are able to borrow a greater fraction of the pdv of a variety’s cash flow.
Model Properties
Efficient and Inefficient Organizational Choice

$s^*(K, B'(K))$ & $\sigma$

Region of Efficient Implementation by Existing Firms
Region of Inefficient Implementation by Existing Firms
Region of Efficient Implementation by Startups

Thresholds

Number of Varieties, $K$

$s^*(K, B'(K))$ & $\sigma$

$s^*(K, B'(K))$

$\sigma$

Number of Varieties, $K$

10 20 30 40 50 60
APPLICATION
Application

Implications of a Decline in \( r \)

- Lower interest rates benefit larger firms more because they have access to borrowing
- More of the arriving ideas get sold instead of resulting in startups
- \( \Rightarrow \) lower entry rate and higher concentration
Model

Implications of a Decline in $r$

\begin{align*}
&\text{Number of Varieties, } K \\
&\text{Thresholds} \\
&r = 1.80 \% \\
&r = -1.00 \%
\end{align*}
Declines in $r$, startup rates, and entrant employment share

Decline in $r$ accelerated in the late 1990s: Caballero, Farhi, and Gourinchas (2008), Mendoza, Quadrini, and Ríos-Rull (2009), Del Negro, Giannone, Giannoni, and Tambalotti (2017),
Application
Why Is This Relevant?

- Rising concentration of innovation

Application

Why Is This Relevant?

▶ Rising concentration of ownership of new ideas

Share of Patents of Top 1% Buyer Firms

A puzzle for most theories of entry: high profits $\Rightarrow$ more entry

In our model also, high profits could potentially increase the flow of new ideas ($M$) and, therefore, entry

But our model also features another channel: where new varieties are implemented
Application
Assessing the strength of the interest rate channel

- We feed in the trend in the risk-free rate between 1997 and 2018 as fully anticipated and compute transition path.

- We picked the s.d. of $s$ to generate the trend decline in employment share of entrants between 1997 and 2018.
  - Fell from 2.7 percent to 1.8 percent.

- We keep $M$ – number of new ideas arriving per period – fixed.
Application

Assessing the strength of the interest rate channel
Application
Survival Rates: Model and Data
Application

Employment Growth: Model and Data
## Application

**Implications for Business Concentration**

<table>
<thead>
<tr>
<th>Measure of Firms</th>
<th>Share of Output Base</th>
<th>Low r</th>
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<tbody>
<tr>
<td>Top 5% by Size in Base</td>
<td>5.8</td>
<td>24.1</td>
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<tr>
<td>Top 10% by Size in Base</td>
<td>11.7</td>
<td>37.7</td>
</tr>
<tr>
<td>Top 15% by Size in Base</td>
<td>17.5</td>
<td>47.7</td>
</tr>
<tr>
<td>Measure of Varieties</td>
<td>387.13</td>
<td>387.06</td>
</tr>
</tbody>
</table>

Autor, Dorn, Katz, Patterson, and van Reenen (2017) document that share of sales of top 4 (top 20) firms has risen in most industries since the 90s.
Summary

Presented a model where . . .

- larger firms are less risky and have higher leverage and

- a decline in the interest rate makes purchasing a variety more attractive for firms, resulting in fewer startups and greater concentration of sales


