Interbanking Networks, Efficiency and Taxation

Pier-André Bouchard St-Amant

Institute for New Economic Thinking

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Interbanking Networks, Efficiency and Taxation

1. How tax instruments can mitigate interbanking externalities?
2. Can we use them to restrain cascades of bankruptcy?
3. What would the tax structure look like?
4. This is work in progress.

Paper Structure

1. Model & Endogenous Framework
2. Tax instruments in a static setup.
3. Tax instruments in a dynamic setup (e.g. with cascades).

Results

1. Interbanking Networks Amplifies Risk & Returns of Bank Portfolios.
2. The network externality predominantly increases risk. A decentralized equilibrium leads to underinvestment in other banks (Not enough failures!).
3. Provided "enough" tax instruments, a decentralized first best can be reached.
4. Enough: depends on the bank asymmetries. In the worst case: link specific instruments.
5. (Conjecture:) Optimal tax design in a dynamic setup leads to controlled failures.

Literature

1. Analysis of cascades: Elliot, Golub & Jackson (forth), Blume, Easly, Kleinberg^2 & Tadros.
2. Traffic Networks and Tolls: Kelly (2008, 2009, 2011), Karaskotas & al (2006).
3. Banks & optimal taxation: Bierbauer (forth).
4. Engineering: failure control literature & approximations.

1. There are fundamental assets $f \in F$, each has a given variance σ_f^2.
2. There are banks $b \in B$, whose mean/variance depends on investment strategies.
3. Banks can invest in fundamental assets or other banks (x_{ba} for $a \in B \cup F$).

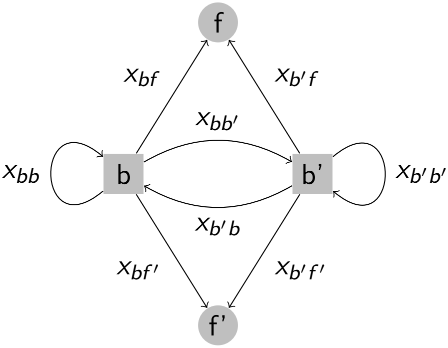

Figure : **A Simple Banking Network**

1. Each bank has its own return profile $\tilde{r}_{bf}$ on each fundamental assets.
2. Bank own their own (unit) share and a given fundamental asset $f(b)$.

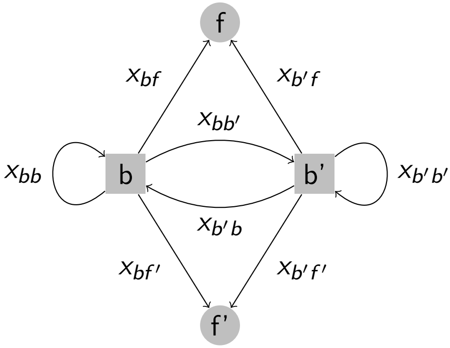

Figure : **A Simple Banking Network**

1. Denote X_B and X_F the interbanking adjacency matrix and the "bank to asset" adjacency matrix. Given shocks ϵ_f on fundamental assets, banks face a shock (and thus variance):

$$\epsilon_b(X_B, X_F) \equiv (I - X_B)^{-1} X_F \epsilon_f \tag{1}$$

$$\Rightarrow \boldsymbol{\sigma}_b^2(X_B, X_F) = E\left[\epsilon_b(X_B, X_F)\epsilon_b(X_B, X_F)'\right] \tag{2}$$

2. This is a convex function in X_B (linear/quadratic in X_F).
3. Explicit consequence: at least one bank does not fully sell its shares.

In a similar fashion: returns on banks are given by:

$$\tilde{\mathbf{r}}_b(X_B, X_F) \equiv (I - X_B)^{-1} X_{Fr} \tilde{\mathbf{r}}_F \tag{3}$$

with:

$$X_{Fr} \equiv \begin{bmatrix} x_{b_1 f_1} & \dots & x_{b_1 f_F} & 0 & \dots & 0 & \dots & 0 & \dots & 0 \\ 0 & \dots & 0 & x_{b_2 f_1} & \dots & x_{b_2 f_F} & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \dots & 0 & 0 & \dots & 0 & \dots & x_{b_B f_1} & \dots & x_{b_B f_F} \end{bmatrix} \tag{4}$$

Banks have a simple mean/variance payoff function:

$$u(x_b, x_f) = \sum_a \tilde{r}_{ba}(X_B, X_F)x_{ba} - h\left(\sum_a x_{ba}^2 \frac{\sigma_a^2(X_B, X_F)}{2}\right) \quad (5)$$

and solve:

$$\max_{x_b} u(x_b, x_f) \text{ s.t. } p_b + p_{f(b)} \geq \sum_a p_a x_{ba} \quad (6)$$

The equilibrium concept is a feasible allocation ($\sum_{b'} x_{ba}^* \leq 1$) and a class of prices that yields market clearing (at least) on fundamental assets.

FOC (for some asset a):

$$Lp_a = \underbrace{r_{ba} - h'(\cdot)\sigma_a^2 x_{ba}}_{\text{Classic Valuation}} + \underbrace{\sum_{b' \in B} \frac{\partial r_{b'}}{\partial x_{ba}} x_{bb'}}_{\text{Return Amplification}} \underbrace{- h'(\cdot) \sum_{b' \in B} \frac{\partial \sigma_{b'}^2}{\partial x_{ba}} \frac{x_{bb'}^2}{2}}_{\text{Variance Amplification}} \tag{7}$$

1. The solution X_B^*, X_F^* exists and is unique.
2. x_{bf}^* is increasing in $\tilde{r}_{bf}$, which means...
3. $x_{b'b}^*$ is non-decreasing in r_{bf}.
4. A decrease in the net average return on assets $(r_{ba} \equiv (1 - \tau_{ba})\tilde{r}_{ba})$ decrease investments in a similar fashion.

1. Decompose the interbanking matrix in its eigenvector/eigenvalue form:

$$X_B^* = V \lambda V^{-1}. \tag{8}$$

2. It is useful to think of V of the **distributive effect** of the graph and of λ of the **amplification effect** of the graph:

$$(I - X_B^*)^{-1} = V \underbrace{\sum_{i=0}^{\infty} \lambda^i}_{\equiv \Lambda} V^{-1} \tag{9}$$

3. Because the largest eigenvalue on X_B^* is smaller than one, Λ has its largest eigenvalue larger than one.

1. Take a single fundamental return and make it arbitrarily large ($r_{bf} \to \infty$). Then, the largest eigenvalue ($\Lambda_{max} \to \infty$) becomes arbitrarily large.
2. An increase in "attractiveness" attracts investments, but they also concentrate risk.

Some Comparative Statics

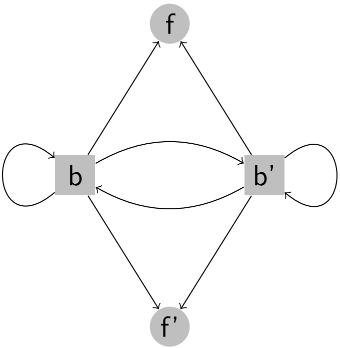

Figure : **As returns of b' increase...**

$\xrightarrow[r_{b'} \to \infty]{}$

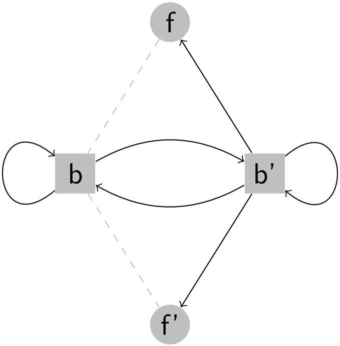

Figure : **...the network converges to a star**

Theorem (Conjecture)

Let $F(\boldsymbol{\sigma}^2)$ be a finite set of fundamental assets and

$$\mathcal{B} \equiv \{b_1(\mathbf{r}_1, f(b_1)), \ldots, b_n(\mathbf{r}_n, f(b_n))\}$$

be the set of bank profiles defining returns and ownership on those fundamental assets. Define $\lambda_{max}(\mathcal{B})$ as the largest equilibrium eigenvalue given by X_B^*. Then, $\lambda_{max}(\mathcal{B})$ is minimal only if profiles are identical ($b_i = b_j \ \forall i, j$).

1. Similar problem: increasing the number of attractive banks reduces the amplification effect.
2. Intuition: the interbanking network converges to a balanced complete graph (Theorem 4 in Acemoglu, Carvahlo, Ozdaglar & Tahbaz-Salehi).

1. Optimum: Assume first the government can control banks' decisions.
2. Let ω_b be a set of pareto weights. The centralized problem is defined as:

$$W_C^{opt} \equiv \max_{X_B, X_F} \sum_b \omega_b u(x_b, x_f) \text{ s.t } \sum_b x_{ba} \leq 1 \ \forall a \tag{10}$$

1. Denote L_a the lagrange multiplier with asset a. The FOCs are:

$$\underbrace{\omega_b \left[\tilde{r}_{ba} - h'_b(\cdot)_b \sigma_a^2 x_{ba}\right]}_{\text{Standard Valuation}} = L_a - \underbrace{\sum_{b' \in B} \omega_{b'} \left[\sum_{c \in B} \frac{\partial \tilde{r}_{b'c}}{\partial x_{ba}} x_{b'c} - h'_{b'}(\cdot) \frac{\partial \sigma_c^2}{\partial x_{ba}} \frac{x_{b'c}^2}{2}\right]}_{\text{Identical for each bank}} \quad (11)$$

2. A central values investment as in standard portfolio theory.
3. Only the shadow price of asset is different.

1. The government seeks to maximize:

$$W_D^{opt} \equiv \max_{\tau_{bf}} \sum_b \omega_b u(x_b^*, x_f^*) \text{ s.t } \sum_{b,a} \tau_{ba} r_{ba} x_{ba}^* = 0 \tag{12}$$

2. E.g.: how to redistribute returns to achieve an efficient allocation?

Marginal change in payoff for bank b given a change in τ_{dc}:

$$\frac{\partial u_b^*}{\partial \tau_{dc}} = \underbrace{-\mathbb{1}_{d=b}\tilde{r}_{bc}^* x_{bc}^*}_{\text{Direct Effect}} - \underbrace{L_b \sum_a p_a \frac{dx_{ba}^*}{d\tau_{dc}}}_{\text{Value Effect}} \dots$$

$$\dots + \sum_a x_{ba}^* \underbrace{\left[\frac{dr_{ba}^*}{d\tau_{dc}} - \sum_{x_{ba'} \in \mathbf{x}_b} \frac{\partial r_{ba}}{\partial x_{ba'}^*} \frac{dx_{ba'}^*}{d\tau_{dc}} \right]}_{\text{Return Externality}} \dots$$

$$\dots - h_b' \sum_a \frac{x_{ba}^{*2}}{2} \underbrace{\left[\frac{d\sigma_a^{2*}}{d\tau_{dc}} - \sum_{x_{ba'} \in \mathbf{x}_b} \frac{\partial \sigma_a^{*2}}{\partial x_{ba'}^*} \frac{dx_{ba'}^*}{d\tau_{dc}} \right]}_{\text{Risk Externality}}, \qquad (13)$$

Total derivative

FOC of the government:

$$\sum_b \omega_b \frac{\partial u_b^*}{\partial \tau_{dc}} + L\left(r_{dc} x_{dc}^* + \underbrace{\sum_{ab} \tau_{ba} \frac{dr_{ba}}{dx_{ba}} x_{ba}^* + \tau_{ba} \frac{dx_{ba}^*}{d\tau_{dc}}}_{\text{Efficiency: A positive tax reduces the fiscal basis}} \right) = 0 \tag{14}$$

Four additional features are required:

1. A discontinuity in payoffs (if $\epsilon_a < \underline{d}$, there is bankruptcy ($r_a = 0$)).
2. A measure on ϵ that preserves convexity (*à la* Blume).
3. A behaviour for bankrupt banks (do they stay bankrupt indefinitely and so on)?
4. A price behaviour when banks go bankrupt (one shot game? Banks adjust their price?)
5. (Applied): A small state-space, or a good approximation.

The problem can then be modelled as in standard asset pricing problem.

1. Let s_0 be the initial state (all banks are healthy) and S the whole state space. The problem of banks can then be presented as:

$$V_b(x_b, x_f | s_0) = \sum_{s \in S} P(s|s_0) V_b(x_b, x_f | s) \tag{15}$$

2. As long as $P(s|s_0)V_b(x_b, x_f, s)$ remains convex, the problem is similar: V is implicitely defined by the state space and the utility functions.
3. The multiplicity of equilibria shown in Elliot, Golub & Jacskon is resolved: nature (shocks) picks only one in the lattice.
4. Conjecture: Discontinuities make some states too expensive to insure: controlled cascades.

No failure

One failure

Two failures

Complete failure

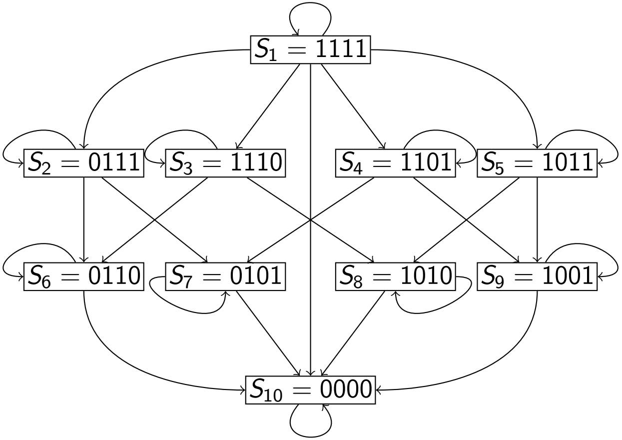

Figure : The State-Space With Two Banks/Assets

1. Develop a better understanding of binding constraints for each banks. How many banks do not fully sell their assets?
2. Shadow banking story: over-investment?
3. Solve the dynamic case.

Contact

Pier-André Bouchard St-Amant
pabsta@gmail.com

Total Derivatives

$$\frac{dx^*_{b'a'}}{d\tau_{dc}} \equiv \frac{\partial x^*_{b'a'}}{\partial \tau_{dc}} + \sum_{a''} \frac{\partial x_{b'a'}}{\partial p_{a''}} \frac{\partial p_{a''}}{\partial \tau_{dc}}, \tag{16}$$

$$\frac{dr^*_{ba}}{d\tau_{dc}} \equiv \sum_{x^*_{b'a'} \in X^*} \frac{\partial r_{ba}}{\partial x^*_{b'a'}} \frac{dx^*_{b'a'}}{d\tau_{dc}}, \tag{17}$$

$$\frac{d\sigma^{2*}_a}{d\tau_{dc}} \equiv \sum_{x^*_{b'a'} \in X^*} \frac{\partial \sigma^{2*}_a}{\partial x^*_{b'a'}} \frac{dx^*_{b'a'}}{d\tau_{dc}}, \tag{18}$$

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