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THE "MYSTERY OF THE PRINTING PRESS"

MONETARY POLICY AND SELF-FULFILLING DEBT CRISES

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First Version: December 2012. This version: July 2016

ABSTRACT

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The "Mystery of the Printing Press" Monetary Policy and Self-fulfilling Debt Crises*

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Abstract

We study the conditions under which unconventional (balance-sheet) monetary policy can rule out self-fulfilling sovereign default in a model with optimizing but discretionary fiscal and monetary policymakers. When purchasing sovereign debt, the central bank effectively swaps risky government paper for monetary liabilities only exposed to inflation risk, thus yielding a lower interest rate. As central bank purchases reduce the (ex ante) costs of debt, we characterize a critical threshold beyond which, absent fundamental fiscal stress, the government strictly prefers primary surplus adjustment to default. Because default may still occur for fundamental reasons, however, the central bank faces the risk of losses on sovereign debt holdings, which may generate inefficient inflation. We show that these losses do not necessarily undermine the credibility of a backstop, nor the monetary authorities' ability to pursue its inflation objectives. Backstops are credible if either the central bank enjoys fiscal backing or fiscal authorities are sufficiently averse to inflation.

JEL classification: E58, E63, H63

Key words: Sovereign risk and default, Lender of last resort, Seigniorage, inflationary financing

> "[T]he proposition [is] that countries without a printing press are subject to self-fulfilling crises in a way that nations that still have a currency of their own are not."

Paul Krugman, "The Printing Press Mystery", The conscience of a liberal, August 17, 2011.

> "Public debt is in aggregate not higher in the euro area than in the US or Japan. [T]he central bank in those countries could act and has acted as a backstop for government funding. This is an important reason why markets spared their fiscal authorities the loss of confidence that constrained many euro area governments' market access."

Mario Draghi, Luncheon Address: Unemployment in the Euro Area, Jackson Hole Symposium, August 22, 2014.

1 Introduction

The sovereign debt crisis in the euro area and the launch of the Outright Monetary Transactions (OMTs) program by the European Central Bank (ECB) in September 2012 have revived the debate on the role of monetary policy in shielding a country from belief-driven speculation in the sovereign debt market.[1] In the quote above, the ECB president Mario Draghi argues that providing a backstop for government debt is among the functions normally performed by a central bank. This argument raises two crucial questions, namely: what are the instruments and mechanisms a central bank can rely upon, to perform such a function successfully? Would providing a backstop to government debt necessarily compromise the central bank's ability to pursue its primary objectives of inflation and macroeconomic stability?

The contribution of this paper consists of analyzing the core mechanisms by which monetary authorities can rule out self-fulfilling sovereign

[1] As shown by Calvo (1988), self-fulling default is possible when, by determining the equilibrium interest rate on public debt, agents expectations impact on the ex-post choices by the fiscal and monetary authorities. If agents arbitrarily coordinate their expectations on the anticipation of default on public debt, they will require a high sovereign interest rate. Facing a high debt service, ex post, a discretionary government will choose to default (partially or fully) on its liabilities, over the alternative of adjusting the primary surplus, thus validating agents' expectations. See also Lorenzoni and Werning (2014), and Ayres, Navarro, Nicolini and Teles (2015); De Grauwe (2012) interprets the debt crisis in the euro area as an instance of multiple equilibria.

crises, relying on either conventional or unconventional monetary policies. We specify a model in which welfare-maximizing fiscal and monetary authorities optimally choose their policy under discretion. Ex post, the fiscal authorities set taxes and may choose outright repudiation by imposing losses (haircuts) on bond holders. Monetary authorities set inflation generating seigniorage and reducing the real value of debt. Hence default can occur via haircuts and/or inflationary debt debasement.[2] In addition to pursuing (conventional) inflation policy, however, monetary authorities can engage in (unconventional) balance sheet policy, through outright purchases of government bonds. Whether via conventional or unconventional policy, the central bank can be effective in ruling out self-fulfilling sovereign default only if its policies are credible, i.e., feasible and welfare-improving from the vantage point of monetary policymakers.

We show that conventional monetary policy may enable a central bank to affect the range of debt over which the economy is vulnerable to belief-driven default, but is generally insufficient to eliminate multiplicity—a point also stressed by Aguiar, Amador, Farhi and Gopinath (2015) and Cooper and Camous (2014).[3] However, the scope for a successful backstop is enhanced by the use of unconventional balance sheet policies. At the core of such policies is the ability of a central bank to issue nominal liabilities at a lower interest rate than a government subject to default risk. Central bank liabilities—currency and reserves, possibly interest-bearing—are claims to cash: monetary authorities stand ready to honor them by redeeming them at their nominal value—in our model, we assume that they do so under any circumstances (see Bassetto and Messer 2013, Del Negro and Sims 2014 and Hall and Reis 2015). So, when purchasing government paper while simultaneously issuing currency and reserves, the central bank effectively swaps government debt, possibly subject to outright default, for own liabilities

[2] The empirical evidence suggests that domestic default (usually but not necessarily in conjunction with default on external debt) tends to occur under extreme macroeconomic duress—see, e.g., Reinhart and Rogoff (2011). Relying on a large sample of countries, these authors document that output declines by 4 percent and inflation rises to 170 percent on average, in the year a country defaults on domestic debt.

[3] Under steeply increasing costs of inflation, conventional policy relying on ex-post debt debasement works under stringent conditions. In Aguiar, Amador, Farhi and Gopinath (2015), for instance, these conditions include a lengthening of the maturity of public debt, so that debasement can be accomplished via sustained but moderate inflation over time—essentially, smoothing the costs of inflation debasement across periods. In Cooper and Camous (2014), these conditions include the central bank's ability to pre-commit to high inflation when self-fulfilling expectations of default materialize.

with a guaranteed face value, hence subject only to the risk of inflation.[4] Because of the implied interest differential, central bank interventions in the debt market contain the overall cost of debt service, altering the trade-offs faced by a discretionary fiscal authority in favor of adjusting the primary surplus (rather than choosing outright default). Indeed, we show that, conditional on market investors requiring a high interest rate to finance the government driven by expectations of default not justified by fundamentals, there is a minimum scale of central bank interventions at which outright default becomes a welfare-dominated option and is therefore avoided by fiscal policymakers.

Yet, even if a backstop policy is successful to rule out belief-driven crises, default may still occur ex-post due to adverse realizations of the fundamentals—raising the risk of losses on the central bank balance sheet.[5] To the extent that these losses make it impossible for the central bank to honor its own liabilities without deviating substantially from its optimal inflation plans, monetary authorities become reluctant to intervene ex ante. The risk of losses thus calls into question the credibility of the backstop.

In our model, whether prospective losses undermine the backstop strategy, crucially depends on the interactions between fiscal and monetary authorities. If the government accepts to make contingent transfers to the central bank, i.e., the fiscal authority provides "fiscal backing" to monetary policy (as in the analysis by Del Negro and Sims 2014), prospective losses do not compromise the credibility of the backstop. The central bank can always pursue the optimal inflation plans, whereby inflation is lower under the backstop than under self-fulfilling default.

Conversely, if the two authorities operate under strict budget separation—the central bank is required to be solely responsible for its own balance sheet—debt purchases unavoidably create inflation risk, which in turn raises the output distortions and welfare costs associated with a backstop. Paradoxically, however, the inflationary consequences of balance sheet losses in case of default may strengthen the effectiveness of the backstop. This is so when fiscal and monetary authorities share the same objective function—or

[4] See Gertler and Karadi (2012) for a similar notion of unconventional monetary policy applied to outright purchases of private assets.

[5] A key feature that qualifies our contribution to the literature is that we model fundamental default in addition to belief-driven crises. A proper discussion of the credibility of unconventional backstop policy is not possible in model economies where default exclusively results from self-fulfilling expectations. Ruling out prospective losses from fundamental default by assumption, central bank interventions in the debt market—carried out on a sufficient scale—eliminate default altogether, and with it any risk of having to run the printing press inefficiently.

more in general if the fiscal authorities are sufficiently averse to inflation, so that they internalize the effects of own policy choices on the overall distortions. Intuitively, with strict budget separation, the high inflation stemming from the monetization of large losses adds to the welfare costs of default, altering the trade-offs faced by the fiscal authorities in favor of servicing the outstanding public debt in full.

Our analysis has relevant implications for policy in light of two often-voiced concerns. On the one hand, it is argued that the central bank may not have the ability to expand its balance sheet on a sufficient scale to effectively backstop government debt. On the other hand, it is argued that large-scale purchases of government debt unavoidably force monetary authorities to accept high inflation rates, even when the backstop is successful in ruling out belief-driven crises. Our results suggest that an effective backstop neither has to match the full scale of the government financing, nor has to guarantee the government in all circumstances at the cost of high inflation.

While our framework builds upon Calvo (1988), our model and results are related to a vast and growing literature on self-fulfilling debt crises, most notably Cole and Kehoe (2002) and more recently Lorenzoni and Werning (2014) and Nicolini et al. (2015), as well as to the literature on sovereign default and sovereign risk, see e.g. Arellano (2008) and Uribe (2006) among others. Jeanne (2012) and Roch and Uhlig (2011) analyze the role of an external lender of last resort. Cooper (2012) and Tirole (2015) study debt guarantees and international bailouts in a currency union. With the exception of Uribe (2006), these contributions focus on real (indexed) government debt.

A few recent papers and ours complement each other in the analysis of sovereign default and monetary policy. In a dynamic framework, Aguiar, Amador, Farhi and Gopinath (2015) analyze a similar problem as in our paper with optimizing fiscal and monetary authorities, focusing on (conventional) inflation policies, rather than (unconventional) balance sheet policies. The conventional policies in their paper lower the government's borrowing costs ex post, after debt has been issued at high interest rates. The unconventional policies we study in our paper, instead, lower borrowing costs ex-ante, at the time of debt issuance. Bacchetta, Perazzi and Van Wincoop (2015) introduce the sovereign default model of Lorenzoni and Werning (2014) in a new-Keynesian framework with short- and long-term debt and exogenous monetary and fiscal rules. These authors, like us, study conventional and unconventional (balance sheet) monetary policy—in analyzing the latter, however, they focus exclusively on the case of zero interest rate policies. Similarly to us, but assuming exogenous monetary and fiscal rules, Reis (2013) examines the inflationary consequences of a monetary backstop by modeling the central bank balance sheet, and interest-bearing, default-free monetary liabilities (the "new style central banking").[6]

The text is organized as follows. Section 2 presents our model economy, and Section 3 characterizes equilibrium multiplicity under conventional monetary policy. Section 4 derives and discusses our main results concerning monetary backstops. Section 5 concludes.

2 A model of self-fulfilling sovereign crises and monetary policy

In this section we describe the model, discuss policy instruments and distortions, and characterize the optimal fiscal and monetary strategies. As in Calvo (1988), we study the mechanism by which, given the government financing needs, outright default is precipitated by agents' expectations, and develop our analysis in a two-period economy framework.

2.1 The model setup

Consider a two-period endowment economy, populated by a continuum of identical risk-neutral agents who derive utility from consuming in period 2 only. Initially, in period 1, agents are endowed with a stock of financial wealth W, which they can invest in three assets: public debt, B, central bank liabilities $\mathcal{H}$ (if any are issued), and a safe asset K. In period 2, they receive a random output realization, and the payoffs from their assets; they pay taxes and consume. The economy can be in one of three states, High,

[6] The working of a monetary backstop is most easily understood referring to a situation in which the (risk-free) nominal interest rate is at its lower bound. In this case, studied by Bacchetta, Perazzi and Van Wincoop (2015), the central bank is able to issue fiat money at will and buy government paper, without any impact on current prices. These purchases reduce the cost of servicing the debt and thus eliminate any vulnerability to self-fulfilling crises (as fiat money is subject only to inflation risk). However, to avoid undesirable inflation developments, appropriate fiscal and monetary policies are required in the future to deal with the increased money stock. Our model of unconventional monetary policy can be viewed as an extension to the case in which central bank liabilities are issued at the equilibrium interest rate — namely, at a rate consistent with expectations of future inflation. Also in this case, the central bank can expand its balance sheet by issuing interest-bearing monetary liabilities (that will always be convertible into cash/fiat money at face value), with no immediate inflationary consequences. As discussed below, it will do so only upon fully accounting for the implications of its purchases on the future choice of primary surpluses, in turn affecting its ability to shrink its balance sheet and withdraw monetary liabilities from the market without compromising its inflation objectives.

Average, or Low (H, A, L), occurring with (strictly positive) probability $1-\gamma$, $\gamma\mu$ and $\gamma(1-\mu)$, respectively.

Fiscal and monetary authorities are both benevolent—they maximize the same objective function given by the utility of the representative agent—but act under discretion and independently of each other. In the first period, the fiscal authority (the government) faces an exogenously given financing need equal to B, and issues bonds at the market-determined nominal rate R_B.[7] The monetary authority may decide to purchase a share $\omega \in [0,1]$ of the outstanding debt at some policy rate $\overline{R}_B$ which may differ from the market rate. To finance its debt purchases, the central bank issues interest bearing liabilities $\mathcal{H} = \omega B$, at the risk-free nominal rate R. So, out of total debt, $(1-\omega) B$ is held by private investors, ωB is on the central bank balance sheet. Consumers' wealth in the first period is thus equal to $W = (1-\omega) B + K + \mathcal{H}$, where the safe asset, supplied with infinite elasticity, pays a constant real rate, denoted by ρ.

In the second period, taking interest rates and central bank policy as given, the fiscal authority sets taxes T and may choose to impose a haircut $\theta_i \in [0,1]$ on bond holders, including the central bank. By the same token, taking interest rates and fiscal policy as given, the monetary authority sets inflation π_i and makes good on any liability it may have, paying $R\mathcal{H}$ to private investors.

2.1.1 Policy instruments and distortions

The instruments of fiscal policy Taxation (primary surplus) and default induce distortions that affect net output and may aggravate the budget. Taxation results in a dead-weight loss of output indexed by $z(T_i, Y_i)$, where from now on a subscript i will refer to the output state $i = H, A, L$.[8] The

[7] As long as the initial financial need of the government is given, it is immaterial whether we follow Calvo's specification or we model discount bonds—see Lorenzoni and Werning (2014). The set of equilibria would instead be different in a model after Cole and Kehoe (2000), where multiplicity arises via discretionary default on the initial stock of liabilities.

[8] It can be easily shown that the function $z()$ corresponds to the distortions caused by income taxes on the allocation in an economy with an endogenous labor supply. In general, while we encompass trade-offs across different distortions in a reduced-form fashion, in doing so we draw on a vast literature, ranging from the analysis of the macroeconomic costs of inflation in the Kydland-Prescott but especially in the new-Keynesian tradition (see e.g. Woodford 2003), to the analysis of the trade-offs inherent in inflationary financing (e.g. Barro 1983), or the role of debt in shaping discretionary monetary and fiscal policy (e.g. Diaz, Giovannetti, Marimon and Teles 2008 and Martin 2009), and, last but not least, the commitment versus discretion debate in public policy (e.g. Persson and Tabellini 1993).

function $z(.)$ is convex in T_i, satisfying standard regularity conditions. We realistically assume that, to raise a given level of tax revenue, the lower the realization of output, the larger the dead-weight losses, and the faster these losses grow in T_i, that is:

$$\begin{aligned} z(T;Y_L) &> z(T;Y_A) > z(T;Y_H), \\ z'(T;Y_L) &> z'(T;Y_A) > z'(T;Y_H). \end{aligned} \tag{1}$$

Since what matters in our analysis is the size of the primary surplus, rather than the individual components of the budget, for simplicity we posit that government spending is constant at the non-defaultable level G, and use taxation and primary surplus interchangeably. Also for notational simplicity, when unambiguous, we will write the function $z(T_i;Y_i)$ omitting the output argument, $z(T_i)$.

Sovereign default may entail different types of costs, associated with a contraction of economic activity and transaction costs in the repudiation of government liabilities. In the theoretical literature, some contributions (see e.g. Arellano 2008 and Cole and Kehoe 2000) posit that a default causes output to contract by a fixed amount. In other contributions (see e.g. Calvo 1988), the cost of default falls on the budget and is commensurate to the extent of the loss imposed on investors. While the relative weight of different default costs is ultimately an empirical matter (see e.g. Cruces and Trebesch 2013), alternative assumptions are consequential for policy trade-offs and the properties of equilibria—as shown below, multiplicity of well-behaved equilibria can only arise with fixed costs; with variable costs, the equilibrium rate of default responds to central bank interventions. For these reasons, we prefer not to restrict our model to one type of costs only. Rather, we posit that outright default in period 2 entails a loss of Φ units of output regardless of the size of default and the state of the economy,[9] and aggravate the budget in proportion to the size of default. Upon defaulting, the government incurs a financial outlay equal to a fraction $\alpha \in (0,1)$ of the total size of default on private agents $\theta_i (1-\omega) BR_B$. The costs of defaulting on the central bank are discussed below.[10]

[9] The fixed component of the cost squares well with the presumption that the decision to breach government contracts, even with a small haircut, marks a discontinuity in the effects of such policy on economic activity. As we show below, effectively this assumption entails that there is a minimum threshold for the haircut θ applied by the government under default.

[10] Calvo (1988) motivates variable costs of default stressing legal and transaction fees associated to debt repudiation. In a broader sense, one could include disruption of financial intermediaries (banks and pension funds) that may require government support. Note that

The instruments of monetary policy Inflation has also distortionary effects on economic activity, that we assume to be isomorphic to those of taxation: it causes a loss of output according to a convex function $C(\pi_i)$. Without loss of generality, we normalize this function such that $C(0) = C'(0) = 0$—a standard instance being $C(\pi) = \frac{\lambda}{2}\pi^2$. For simplicity, as in Calvo (1988), we assume that inflation in period 2 generates seigniorage revenue according to the function

$$\text{Seigniorage} = \frac{\pi_i}{1+\pi_i}\kappa. \tag{2}$$

where a constant κ implies that there is no Laffer curve.[11]

In addition to setting inflation in period 2, monetary authorities have the option to purchase government debt by issuing interest-bearing "reserves" in period 1. This option captures a key institutional development in modern central banking, concerning the distinction between conventional (inflation) policy and unconventional (balance-sheet) policy. To pursue the latter, the central bank crucially relies on the ability to expand its liabilities in line with its preferred inflation path (see Reis 2013). In our model, we posit that, in period 1, the central bank issues monetary liabilities without affecting the initial price level, bearing an equilibrium interest rate fully consistent with its discretionary choice of inflation (the conventional monetary instrument) in period 2.[12]

2.1.2 Budget constraints of the fiscal and the monetary authority

In order to write the budget constraint of the government and the central bank, there are at least three interrelated issues that need to be addressed. These concern whether (i) a government that opts to default is able/willing

our results would go through if the variable costs of default were in output units, rather than affecting the budget. The main difference would be that the perceived marginal benefit of default for the fiscal authority would be 1 instead of $1-\alpha$; the marginal cost would remain equal to α.

[11] We refer to the specification in Calvo (1988), who restricts the demand for (non-interest bearing) fiat money to the case of a constant velocity, and abstracts from specifying a terminal condition. Note that our setup can be easily generalized to encompass an inflation Laffer curve, making κ a decreasing function of inflation.

[12] See footnote 6. In dynamic monetary models, buying government debt by increasing the money stock does not necessarily result in higher current inflation, as the latter mainly reflects future money growth (see, e.g., Diaz, Giovannetti, Marimon and Teles 2008 and Martin 2009, placing this consideration at the heart of their analysis of time inconsistency in monetary policy).

to discriminate between private investors and monetary authorities, applying different haircuts; (ii) defaulting on the central bank generates budget costs and (iii) the central bank can receive fiscal transfer, i.e., in the definition by Del Negro and Sims (2014), the bank can rely on a "fiscal backing".

The first two issues have no substantial implications for our analysis. For clarity of exposition and analytical convenience, we assume that, first, the government applies the same haircut θ_i rate to all debt holders, corresponding to a *pari passu* clause in government paper; second, the budget cost of defaulting on the central bank is isomorphic, but not necessarily identical, to the costs of defaulting on the private investors—we stipulate $1 > \alpha > \alpha_{CB} \geq 0$. Under a *pari passu* rule, and allowing for $0 \leq \alpha_{CB} \leq \alpha$, the budget constraint of the fiscal authority reads:

$$T_i - G = [1 - \theta_i (1 - \alpha)] \frac{R_B}{1 + \pi_i} (1 - \omega) B + [1 - \theta_i (1 - \alpha_{CB})] \frac{\overline{R}_B}{1 + \pi_i} \omega B - \mathcal{T}_i \tag{3}$$

where R_B is the market interest rate at which agents buy the share of government debt $(1 - \omega) B$ not purchased by the central bank, $\overline{R}_B$ is the intervention rate at which the central bank purchases bonds, and $\mathcal{T}_i$ denotes transfers from the central bank to the fiscal authority in state i. The budget constraint of the central bank in the second period is:

$$\begin{aligned} \mathcal{T}_i &= \frac{\pi_i}{1 + \pi_i} \kappa + \frac{(1 - \theta_i) \overline{R}_B}{1 + \pi_i} \omega B - \frac{R}{1 + \pi_i} \mathcal{H} = \\ &= \frac{\pi_i}{1 + \pi_i} \kappa + \left(\frac{(1 - \theta_i) \overline{R}_B}{1 + \pi_i} - \frac{R}{1 + \pi_i} \right) \omega B. \end{aligned} \tag{4}$$

The third issue—"fiscal backing"—is instead quite consequential. If the constraint on fiscal transfers to the central bank (if any) is either relaxed or not binding, the two budget constraints of the authorities can be consolidated as follows:

$$\begin{gathered} T_i - G + \frac{\pi_i}{1 + \pi_i} \kappa = \\ \frac{[1 - \theta_i (1 - \alpha)] R_B}{1 + \pi_i} (1 - \omega) B + \left[\frac{R}{1 + \pi_i} + \frac{\alpha_{CB} \theta_i \overline{R}_B}{1 + \pi_i} \right] \omega B. \end{gathered} \tag{5}$$

The consolidated budget above clarifies that, no matter how large the increase in the central bank balance sheet (ωB) in period 1 is, a large enough primary surplus (net of the ex post interest bill of the government) allows the central bank to redeem its nominal liabilities from the market in period

2, without impinging on the desired level of inflation. Conversely, if transfers to the central bank are ruled out by (an unbreakable) rule (i.e., $\mathcal{T}_i \geq 0$ always), by (4) it is apparent that monetary authorities will have to ensure repayment of their liabilities by raising seigniorage:

$$\pi_i \kappa + \left((1-\theta_i)\,\overline{R}_B - R\right)\omega B = 0.$$

So, if the central bank intervenes in the sovereign debt market and exposes its balance sheet to default risk, the need to make up for ex-post losses will weigh on monetary policy decisions. Honoring the outstanding stock of nominal liabilities $\mathcal{H}$ at face value will require monetary authorities to run a rate of inflation large enough to satisfy the above constraint in all circumstances.

In advanced countries, budget interactions between the fiscal and monetary authorities are regulated by institutional rules that typically hold central banks responsible for backing their own liabilities—constraining the modalities and the size of fiscal transfers to the central bank. These institutional constraints on transfers, however, are not always binding. Indeed, they tend to be relaxed precisely in exceptional crisis circumstances—when the fiscal authorities may be especially concerned with the possibility that balance sheet losses could condition monetary policy. In light of these considerations, we find it important to analyze monetary backstops under either scenario—of fiscal backing and budget separation.

2.2 Optimality conditions

In this subsection, we characterize the optimal debt pricing by the risk neutral consumers in the economy in period 1, and the policy plan set by the fiscal and monetary authorities under discretion in period 2.

2.2.1 Debt pricing by risk neutral agents

Under risk neutrality, the utility of the representative agent coincides with consumption in period 2:

$$\begin{aligned} U_i \;&=\; Y_i - z\left(T_i; Y_i\right) - T_i - \frac{\pi_i}{1+\pi_i}\kappa + \\ &\quad KR + \frac{(1-\theta_i)\,R_B}{1+\pi_i}\left(1-\omega\right)B + \frac{R}{1+\pi_i}\mathcal{H} - \mathcal{C}\left(\pi_i\right), \end{aligned} \tag{6}$$

where, ex post, the net real asset payoffs are determined by the realization of default and inflation. Ex ante, the expected real returns on government bonds are equalized to the constant, safe return on the real asset:

$$R_B \left\{ (1-\gamma) \frac{1-\theta_H}{1+\pi_H} + \gamma \left[\mu \frac{1-\theta_A}{1+\pi_A} + (1-\mu) \frac{1-\theta_L}{1+\pi_L} \right] \right\} = \rho. \tag{7}$$

The interest parity condition pins down the price of government debt as a function of both expected default and expected inflation rates.

When both B and $\mathcal{H}$ are traded, there is a second equilibrium interest parity condition. To generate a demand for reserves, which we assume to be free from the risk of outright default, the interest rate R offered by the central bank must be equal to the real rate ρ adjusted by expected inflation:

$$R \left[\frac{1-\gamma}{1+\pi_H} + \gamma \left(\frac{\mu}{1+\pi_A} + \frac{(1-\mu)}{1+\pi_L} \right) \right] = \rho. \tag{8}$$

In equilibrium, the interest rate on government debt must exceed the interest paid on central bank's liabilities by the expected rate of default.

2.2.2 Optimal discretionary plans for inflation, taxation and default

The two authorities independently maximize the same objective function, given by (6), subject to the consolidated budget constraint (5) (with multiplier λ), and the constraint $\mathcal{T}_i \geq 0$ on the central bank budget (with multiplier $\lambda^{CB} \geq 0$). In doing so, they take as given (i) the rates of return on all assets set in period 1, (ii) the scale of interventions by the central bank in period 1, and (iii) each other instruments. As, from the perspective of each policy authority, the solution to the policy problem identifies "best responses" to the policy set by the other authority and to private sector expectations, below we will denote the optimal discretionary plan for inflation, taxation and default with the superscript σ, that is: $\pi_i^\sigma = \pi_i \left(R_B, B, R, \omega, \overline{R}_B \right)$, $T_i^\sigma = T_i \left(R_B, B, R, \omega, \overline{R}_B \right)$ and $\theta_i^\sigma = \theta_i \left(R_B, B, R, \omega, \overline{R}_B \right)$.

The plans in general The fiscal authority will choose to default when the welfare(=consumption) gains from reducing distortionary taxation after the implementation of an optimal haircut exceeds the fixed and variable costs of default, net of output losses due to inflation:

$$U_i \left(\theta_i^\sigma > 0 \right) \geq U_i \left(\theta_i^\sigma = 0 \right). \tag{9}$$

Above, utility is assessed conditional on the optimal plans for taxation and inflation given the government decision to either default (on the left hand side), or service its debt in full (on the right hand side).

As regards the optimal fiscal plan, under optimal default the constraint on the admissible haircut rate $\theta_i \in (0,1]$ may/may not be binding. If the constraint is not binding, the optimal taxation plan obeys the following first order condition of the fiscal authority:

$$z'\left(T_i^{\sigma};Y_i\right)=\frac{\alpha\left(1-\omega\right)R_B+\left(\alpha_{CB}+\lambda_i^{CB}\right)\cdot\omega\overline{R}_B}{\left(1-\alpha\right)\left(1-\omega\right)R_B-\alpha_{CB}\omega\overline{R}_B}, \tag{10}$$

In an interior solution, the fiscal authorities set taxes trading off the output costs of distortionary taxation, with the benefits of reducing the haircut rates so to contain the budget (variable) costs of default. Note that, when the central bank budget constraint is binding (the multiplier λ_i^{CB} is strictly positive), a government that cares about inflation will tend to set higher taxes and reduce the optimal haircut rate, so to contain the inflationary consequences of losses on the central bank balance sheet. Given the optimal T_i^{σ}, the optimal haircut rate θ_i^{σ} is then obtained from the budget constraint of the government (3). If the constraint $\theta_i \in (0,1]$ is binding, under optimal default the government imposes a 100 percent haircut on bond holders, and sets taxation T_i^{σ} to satisfy the budget constraint (3) evaluated at $\theta_i^{\sigma}=1$. Similarly, under no default, the government adjusts taxation T_i^{σ} to service its liabilities in full. In this case, the relevant budget constraint is (5), evaluated at $\theta_i^{\sigma}=0$.

As regards the optimal inflation plan, the first order condition of the monetary authority problem is:

$$\begin{aligned}\left(1+\pi_i^{\sigma}\right)^2\mathcal{C}'\left(\pi_i^{\sigma}\right)&=z'\left(T_i;Y_i\right)\left[\kappa+\left(1-\omega\right)BR_B+R\omega B\right]\\&+\theta_i^{\sigma}\left\{\begin{array}{c}z'\left(T_i;Y_i\right)\left[-\left(1-\alpha\right)\left(1-\omega\right)BR_B+\alpha_{CB}\overline{R}_B\omega B\right]\\+\alpha\left(1-\omega\right)BR_B+\left[\alpha_{CB}\overline{R}_B\right]\omega B\end{array}\right\}\\&+\lambda_i^{CB}\cdot\left[\kappa+\left(\left(1-\theta_i\right)\overline{R}_B-R\right)\omega B\right],\end{aligned} \tag{11}$$

The central bank sets inflation by trading off its costs with the output benefits from reducing distortionary income taxation net of the costs of default (if any). Under discretion, the monetary authorities will always choose a non-negative rate of inflation—even if printing money generates no seigniorage revenue ($\kappa=0$). This is because a discretionary monetary authority will not resist the temptation to inflate nominal debt, if only moderately so (according to the condition above). Positive and rapidly rising costs of inflation nonetheless prevent policymakers from attempting to wipe away the debt with a bout of very high inflation. Note that, in state of the world in which there is no default (for $\theta_i^{\sigma}=0$), the constraint on the central bank budget never binds ($\lambda^{CB}=0$).

Overall, since policymakers are benevolent and share the same objective function, the optimal plan described above minimizes the joint distortions induced by taxation and default on the one hand, and inflation on the other hand. In general, policymakers will want to use all available instruments—ruling out an uneven resort to extreme inflation as a substitute for outright default.

For notational clarity, from now on we will distinguish variables across the cases of interior default as opposed to complete default, using a 'hat' $\hat{}$, and a 'tilde' $\tilde{}$, respectively. To keep complexity at a minimum, we also abstract from budget costs of default on central bank holdings of sovereign bonds, setting $\alpha_{CB} = 0$.

The case of a consolidated budget constraint in detail To gain insight on the above plans, we find it useful to provide a close-up analysis of the case of a consolidated budget constraint, i.e. $\lambda_i^{CB} = 0$. This will be an endogenous outcome in Section 3 below, when there are no central bank interventions, and a maintained hypothesis implied by fiscal backing in Section 4.1.

When $\lambda_i^{CB} = 0$, in the case of an interior default, (10) simplifies to:

$$z'\left(\widehat{T}_i^{\sigma}; Y_i\right) = z'\left(\widehat{T}_i; Y_i\right) = \frac{\alpha}{1-\alpha}. \tag{12}$$

implying that taxation does not depend on inflation and agents' expectations—so that we can drop the best-response superscript σ. Likewise, the condition for the optimal inflation plan (11):

$$\left(1+\widehat{\pi}^{\sigma}\right)^2 \mathcal{C}'\left(\widehat{\pi}^{\sigma}\right) = \frac{\alpha}{1-\alpha}\left[\kappa + \left(1-\omega\right) BR_B + \omega BR\right] \tag{13}$$

shows that $\widehat{\pi}^{\sigma}$ is constant across states of the world. Given $\widehat{T}_i$, the optimal default rate is derived from the consolidated budget constraint:

$$\widehat{\theta}_i^{\sigma} = \frac{\left(1-\omega\right) BR_B + \omega RB - \left(1+\widehat{\pi}^{\sigma}\right)\left(\widehat{T}_i - G\right) - \widehat{\pi}^{\sigma}\kappa}{\left(1-\alpha\right)\left(1-\omega\right) BR_B}.$$

In the case of complete default, taxation $\widetilde{T}_i^{\sigma}$ is set to satisfy the consolidated budget constraint:

$$\widetilde{T}_i^{\sigma} = G - \frac{\widetilde{\pi}_i^{\sigma}}{1+\widetilde{\pi}_i^{\sigma}}\kappa + \frac{\alpha R_B}{1+\widetilde{\pi}_i^{\sigma}}\left(1-\omega\right) B + \frac{R}{1+\widetilde{\pi}_i^{\sigma}}\omega B,$$

and is always state contingent. So is the inflation plan, determined by:

$$\left(1+\widetilde{\pi}_i^{\sigma}\right)^2 \mathcal{C}'\left(\widetilde{\pi}_i^{\sigma}\right) = z'\left(\widetilde{T}_i^{\sigma}\right)\left[\kappa+\alpha\left(1-\omega\right)BR_B+\omega BR\right]+\alpha\left(1-\omega\right)BR_B,$$

For the case of an interior solution for the haircut rate, the optimal outright default condition (9) becomes

$$\Phi + z\left(\widehat{T}_i\right)+\mathcal{C}\left(\widehat{\pi}^{\sigma}\right)+\alpha\widehat{\theta}_i^{\sigma}\frac{R_B}{1+\widehat{\pi}^{\sigma}}\left(1-\omega\right)B \leq z\left(T_i^{\sigma}\right)+\mathcal{C}\left(\pi_i^{\sigma}\right). \qquad (14)$$

Written with an equality sign, the above condition can be solved for the minimum default rate which the government finds optimal in each state of the world, denoted by $\underline{\theta}_i$. Because of the fixed output costs Φ, optimal default only occurs at strictly positive rates, hence $\underline{\theta}_i > 0$. If an interior solution does not exist, $\widehat{\theta}_i^{\sigma}$, $\hat{T}_i$, and $\widehat{\pi}^{\sigma}$ in (14) are replaced with 1, $\widetilde{T}_i^{\sigma}, \widetilde{\pi}_i^{\sigma}$, and the default condition reads:

$$\text{If } \widehat{\theta}_i^{\sigma} > 1: \qquad \Phi+z\left(\widetilde{T}_i^{\sigma}\right)+\mathcal{C}\left(\widetilde{\pi}_i^{\sigma}\right)+\alpha\frac{R_B}{1+\widetilde{\pi}_i^{\sigma}}\left(1-\omega\right)B \leq z\left(T_i^{\sigma}\right)+\mathcal{C}\left(\pi_i^{\sigma}\right). \qquad (15)$$

Note that, since

$$z'(\widetilde{T}_i^{\sigma}) \geq \alpha/\left(1-\alpha\right). \qquad (16)$$

taxes (and tax distortions) under full default cannot be lower than taxes under partial default, namely $\widetilde{T}_i^{\sigma} \geq \hat{T}_i$.

2.3 Regularity conditions

In the rest of the paper, we will define and analyze the equilibrium across different possible policy scenarios, allowing for either conventional or unconventional monetary policy or both (Sections 3 and 4), with and without "fiscal backing" (Subsections 4.1 and 4.2). We will present our results both analytically and resorting to numerical examples.

In spite of its stylized nature, the model in its general form admits many equilibria, not all of interest for an analysis of monetary backstops. Before proceeding, we set conditions under which fundamental default may or may not occur in the 'bad' state L, but there is no fundamental reason for defaulting in the relatively 'good' states A, H. Moreover, we would like to study equilibria that are well-behaved, i.e., stable by the Walrasian criterion discussed, e.g., by Lorenzoni and Werning (2014). By this criterion, a small increase in the supply of government bonds should not decrease the interest rate charged by the markets.[13]

Below we spell out two assumptions such that our model admits a well-defined (stable) equilibrium with

1. no default for a sufficiently low initial B;
2. full default in state L and no default in the other states for intermediate levels of B;
3. full default in L and partial default in A, for a sufficiently high B.

We refer to the above as the ND (no-default) equilibrium, the D (default in L) equilibrium, and the DD (default in L and A) equilibrium. In each equilibrium, we will denote variables with the corresponding superscript ND, D and DD.

The first assumption establishes a reasonable ordering between the primary surplus under interior default in the high (H) state and the average (A) state, and stipulates that both must be larger than required to service the maximum level of debt sustainable in a state of fiscal stress (L) at the real risk free rate ρ, denoted by $\overline{B}_L$. The second assumption restricts the probability of the state (H) in which bond holders are paid in full to be large enough, and the probability of the intermediate state (A) not to be too large.

Assumption 1. The primary surplus under interior default across states of the world satisfies the following restrictions:

$$\overline{B}_L < (1-\gamma)\left(\widehat{T}_A - G\right)/\rho < \left(\widehat{T}_A - G\right)/\rho < \left(\widehat{T}_H - G\right)/\rho \qquad (17)$$

Assumption 2. The probabilities of state H, $1-\gamma$, and of state A, $\gamma\mu$, are such that:

$$\begin{aligned} 1-\gamma &> \alpha, \qquad (18) \\ \frac{1+\hat{\pi}^{DD}}{1+\pi_H^{DD}} &> \mu > 0. \end{aligned}$$

[13] In (Walrasian-)unstable equilibria, such as the one discussed by Calvo (1988), the economy is vulnerable to self-fulfilling crisis for small levels (but not for high levels) of debt, and the sovereign rates are decreasing in the stock of debt. In an analysis of backstops, interventions by the central bank should be negative, i.e., the central bank should actually sell government debt in response to the threat of a run on debt. By the same argument set forth by Lorenzoni and Werning (2014), in what follows we will abstract from these equilibria, on the ground that they have pathological and unpalatable implications for policy.

where the superscript DD refers to the third equilibrium above. Note that the last condition above is always satisfied in two cases: first, if $\pi_i \to 0$ for all i—which is the case when either policymakers are extremely averse to inflation, or when debt is real (indexed) and seigniorage is zero; second, if policy distortions are such that inflation in state H remains sufficiently low even for levels of sovereign debt that lead the government to default in both L and A, so that $\pi_H^{DD} \leq \hat{\pi}^{DD}$.[14]

A third assumption—that the primary surplus in state L is at most zero when seigniorage revenue is at its maximum κ—is imposed for the sake of analytical tractability

Assumption 3:

$$\left(\widehat{T}_L - G\right) + \kappa \to 0. \tag{19}$$

As shown below, this implies that that, if the government optimally chooses to default in the low state, the haircut will always be 100 percent. This last assumption will be relaxed in our numerical examples.

3 Inflation and macroeconomic resilience to self-fulfilling crises

We start our study of monetary policy in economies vulnerable to self-fulfilling debt crises restricting the central bank to rely exclusively on conventional policies, i.e. the central bank only sets inflation and $\omega = 0$. As stressed by Calvo (1988), some degree of "stealth repudiation" is a natural outcome in an economy in which public liabilities are denominated in nominal terms, because unexpected changes in inflation rates affect their ex-post real returns. In our model, indeed, repudiation in period 2 can take the form of either outright default on the nominal value of debt, or inflation surprises reducing the real value of debt, or both.[15]

In this section, we will first characterize the properties of the optimal inflation plans and define the equilibrium. Second, we will discuss equilibrium multiplicity. We will show that in general conventional inflation policy does not rule out belief-driven outright default via haircuts on bond holders. Yet

[14] It is important to note that (18) does not rule out the existence of other, not well-behaved, equilibria, i.e., our model may admit equilibria that are "unstable" by the Walrasian criterion, coexisting with the stable ones on which we focus our analysis.

[15] This is different from Calvo (1988), where default is implemented alternatively through outright repudiation (in the real version of the model), or inflation only (in the monetary version).

convex costs of inflation will prevent self-fulfilling expectations of "stealth repudiation" via inflationary debt debasement à la Calvo.

3.1 Properties of the optimal inflation plans

We have seen that, under discretion, government spending will be financed at least in part through seigniorage and some debasement of outstanding (ex-default) public liabilities, so that inflation rates will always be positive in equilibrium. Then (with $\omega = 0$) under our assumptions the central bank budget constraint never binds and $\lambda^{CB} = 0$.

For convenience, we summarize the optimal policy plans in Table 1 below. In the table, the equation (28) that defines the minimum (interior) haircut rate at which the government optimally defaults, $\underline{\theta}_i$, is obtained from the default condition (14) by setting $\widehat{\theta}_i^\sigma = \underline{\theta}_i$, and using the fact that, for $\omega = 0$, the cost of debt issuance can be written as[16]

$$BR_B = \frac{(1+\hat{\pi})\left(\hat{T}_i - G\right) + \hat{\pi}\kappa}{1 - \widehat{\theta}_i^\sigma (1-\alpha)} > 0. \tag{20}$$

According to the table: under the optimal default plan with an interior haircut rate, $\underline{\theta}_i \leq \widehat{\theta}_i^\sigma \leq 1$, inflation is identical across states of the world, i.e., $\widehat{\pi}_A^\sigma = \widehat{\pi}_L^\sigma = \widehat{\pi}^\sigma$—see the conditions (22) and (23).[17] If the constraint $\widehat{\theta}_i^\sigma \leq 1$ is binding and default is complete, inflation plans are instead state dependent, as taxes and seigniorage will have to adjust to cover current non-interest expenditure according to (24) and (25). The same applies under no outright default ($\theta_i^\sigma = 0$), whereas the revenue from taxation and seigniorage needs to adjust according to (26) and (27) to finance the government real expenditure and interest bill in full. Note that, in either corner solution for default, π_i^σ and $\widetilde{\pi}_i^\sigma$ always comove positively with T_i^σ and $\widetilde{T}_i^\sigma$, hence inflation inherits the same properties as taxation. For this reason, the condition (16) on taxation implies that output distortions due to inflation under full default cannot be lower than under partial default.

[16] When (28) is satisfied for a $\underline{\theta}_i$ exceeding 100 percent (as is the case if the primary surplus under interior default $\left(\hat{T}_i - G\right) + \hat{\pi}\kappa / (1+\hat{\pi})$ is non-positive) the government opts for complete default.

[17] This property of the optimal inflation rate depends on the simplifying assumption that the cost of inflation does not vary with the state of the world. It would be easy to relax this assumption, at the cost of cluttering the notation without much gain in terms of economic intuition.

The expressions in the table highlight several properties of inflation of particular interest for our analysis. As stated in the following lemma, inflation rates are increasing in the ex-ante interest rate R_B and the stock of debt B. At either corner solution for the default rate, $\theta_i^\sigma = \{0, 1\}$, the better the state of the economy, the lower the inflation rates (and taxation).

Table 1

$$\textbf{If } 1 \geq \widehat{\theta}_i^\sigma = \frac{1}{1-\alpha}\left[1 - \frac{(1+\widehat{\pi}^\sigma)\left(\hat{T}_i - G\right) + \widehat{\pi}^\sigma \kappa}{BR_B}\right] \geq \underline{\theta}_i > 0 : \tag{21}$$

$$\theta_i^\sigma = \widehat{\theta}_i^\sigma \qquad T_i^\sigma = \widehat{T}_i = z'^{-1}\left(\tfrac{\alpha}{1-\alpha}; Y_i\right) \tag{22}$$

$$\text{and} \qquad (1+\widehat{\pi}^\sigma)^2 \mathcal{C}'(\widehat{\pi}^\sigma) = \tfrac{\alpha}{1-\alpha}(\kappa + BR_B) \tag{23}$$

$$\textbf{If } \widehat{\theta}_i^\sigma > 1 : \qquad \theta_i^\sigma = 1 \qquad \widetilde{T}_i^\sigma = G + \alpha \tfrac{R_B}{1+\widetilde{\pi}_i^\sigma} B - \tfrac{\widetilde{\pi}_i^\sigma}{1+\widetilde{\pi}_i^\sigma}\kappa \tag{24}$$

$$\text{and} \qquad (1+\widetilde{\pi}_i^\sigma)^2 \mathcal{C}'(\widetilde{\pi}_i^\sigma) = z'(\widetilde{T}_i^\sigma)(\kappa + \alpha BR_B) + \alpha BR_B \tag{25}$$

$$\textbf{If } \widehat{\theta}_i^\sigma < \underline{\theta}_i : \qquad \theta_i^\sigma = 0 \qquad T_i^\sigma = \tfrac{R_B}{1+\pi_i^\sigma} B + G - \frac{\pi_i^\sigma}{1+\pi_i^\sigma}\kappa \tag{26}$$

$$\text{and} \qquad (1+\pi_i^\sigma)^2 \mathcal{C}'(\pi_i^\sigma) = z'(T_i^\sigma)(\kappa + BR_B) \tag{27}$$

$$\underline{\theta}_i \text{ solves } \Phi = z\left(G - \frac{\pi_i^\sigma}{1+\pi_i^\sigma}\kappa + \frac{1+\hat{\pi}}{1+\pi_i^\sigma}\frac{\left(\hat{T}_i - G\right) + \frac{\hat{\pi}}{1+\hat{\pi}}\kappa}{1-\underline{\theta}_i(1-\alpha)}\right) - z\left(\hat{T}_i\right) \tag{28}$$

$$+\mathcal{C}(\pi_i^\sigma) - \mathcal{C}(\hat{\pi}) - \alpha\underline{\theta}_i \frac{\left(\hat{T}_i - G\right) + \frac{\hat{\pi}}{1+\hat{\pi}}\kappa}{1-\underline{\theta}_i(1-\alpha)}.$$

Lemma 1 *Inflation best responses* $(\pi_i^\sigma, \hat{\pi}, \widetilde{\pi}_i^\sigma)$ *are i) increasing in the expected sovereign rate* R_B *and the stock of debt* B*, where* $\hat{\pi} \leq \widetilde{\pi}_i^\sigma$*; ii) such that* $(\pi_i^\sigma, \widetilde{\pi}_i^\sigma) > \left(\pi_j^\sigma, \widetilde{\pi}_j^\sigma\right)$ *if* $z(\cdot; Y_i) > z(\cdot; Y_j)$*, while* $\hat{\pi}$ *is constant across states.*

Omitting a formal proof, we note that these properties are intuitive in light of our assumption of convex costs of inflation, $\mathcal{C}(\pi_i^\sigma)$, which translates into decreasing marginal benefits from its use. Property *i)* follows from inspection of the inflation reaction function, rewritten here in general form:

$$(1+\pi_i^\sigma)^2\, \mathcal{C}'(\pi_i^\sigma) = z'(T_i^\sigma)(\kappa + BR_B) + \theta_i^\sigma BR_B\left[\alpha - z'(T_i^\sigma)(1-\alpha)\right] \quad (29)$$

In this expression, the right-hand-side is increasing in $R_B B$, since taxes are weakly increasing in the interest rate bill. Moreover, by (16), taxes under full default are at least as high as taxes under partial default ($\widetilde{T}_i^\sigma \geq \hat{T}_i$). Property ii) descends directly from our ordering of tax distortions $z(\cdot; Y_i)$ across states, stipulating that distortions are worse then the fundamentals are weaker.

3.2 Equilibrium

A rational-expectation equilibrium is defined by the pricing condition (7), together with the consolidated budget constraint (5) with $\omega = 0$, the optimal tax rates—either (22), or (24), or (26)—, given the default threshold (28), and the optimal inflation rate—either (23) or (25) or (27).

By assumption 3, (28) implies that in equilibrium there will be either no default or complete default in state L. Partial default will instead be a possibility in the A state—whereas the interior optimal rate $\widehat{\theta}_A^{DD}$ in the DD equilibrium is given by:

$$\widehat{\theta}_A^{DD} = \frac{B\rho - \left[(1-\gamma)\frac{1+\widehat{\pi}^{DD}}{1+\pi_H^{DD}} + \gamma\mu\right]\left[\left(\hat{T}_A - G\right) + \frac{\widehat{\pi}^{DD}}{1+\widehat{\pi}^{DD}}\kappa\right]}{(1-\alpha)B\rho - \gamma\mu\left[\left(\hat{T}_A - G\right) + \frac{\widehat{\pi}^{DD}}{1+\widehat{\pi}^{DD}}\kappa\right]} \geq \underline{\theta}_A^{DD} > 0. \quad (30)$$

Under the assumptions above, the sovereign interest rates R_B^j and the risk-free nominal rates R^j in the equilibrium $j = ND, D, DD$ are as follows:

$$R_B^{ND} = R^{ND} = \rho\left\{\frac{(1-\gamma)}{1+\pi_H^{ND}} + \gamma\left[\frac{\mu}{1+\pi_A^{ND}} + \frac{(1-\mu)}{1+\pi_L^{ND}}\right]\right\}^{-1} \quad (31)$$

$$R_B^D = \rho \left\{ \frac{(1-\gamma)}{1+\pi_H^D} + \frac{\gamma\mu}{1+\pi_A^D} \right\}^{-1} > R^D = \rho \left\{ \frac{(1-\gamma)}{1+\pi_H^D} + \gamma \left[\frac{\mu}{1+\pi_A^D} + \frac{(1-\mu)}{1+\widetilde{\pi}_L^D} \right] \right\}^{-1} \tag{32}$$

$$R_B^{DD} = \rho \left\{ \frac{(1-\gamma)}{1+\pi_H^{DD}} + \gamma\mu \frac{1-\widehat{\theta}_A^{DD}}{1+\widehat{\pi}^{DD}} \right\}^{-1} > R^{DD} = \rho \left\{ \frac{(1-\gamma)}{1+\pi_H^{DD}} + \gamma \left[\frac{\mu}{1+\widehat{\pi}^{DD}} + \frac{(1-\mu)}{1+\widetilde{\pi}_L^{DD}} \right] \right\}^{-1} \tag{33}$$

where inflation rates are determined according to (25) in states with full default (i.e. state L in the D and DD equilibria); according to (27) in states with partial default (namely state A in the DD equilibrium), and according to (23) in (all other) states with no default. Note that the risk-free nominal rate is not the same across equilibria, because inflation rates differ.

To characterize the range of B in which our model admits the equilibria ND, D and DD, we now define two pairs of debt thresholds, $\left\{\underline{B}_L, \overline{B}_L\right\}$ and $\left\{\underline{B}_A, \overline{B}_A\right\}$. Focusing on the first pair: $\underline{B}_L$ is defined as the *minimum* level of B at which, if markets coordinate their expectations on anticipating a 100 per cent haircut in the low output state (and thus charge a destabilizing high market rate R_B^D), the government is indifferent between not defaulting and validating ex post markets' expectations (thus defaulting in the low output state). This threshold is obtained from the counterpart of (15), written as an equality and evaluated at the sovereign rate R_B^D:

$$\Phi + \alpha \frac{R_B^D \underline{B}_L}{1+\widetilde{\pi}_L^D} + z \left(G + \alpha \frac{R_B^D \underline{B}_L}{1+\widetilde{\pi}_L^D} - \frac{\widetilde{\pi}_L^D}{1+\widetilde{\pi}_L^D}\kappa \right) + \mathcal{C}\left(\widetilde{\pi}_L^D\right) \tag{34}$$
$$= z \left(G + \frac{R_B^D \underline{B}_L}{1+\pi_L^D} - \frac{\pi_L^D}{1+\pi_L^D}\kappa \right) + \mathcal{C}\left(\pi_L^D\right).$$

The second threshold in the pair, $\overline{B}_L$, is defined as the *maximum* level of B at which, if markets expect no default and thus charge the risk free rate, the government will be indifferent between default and no default in any state of the world. The threshold $\overline{B}_L$ is also obtained from (15), again written as an equality but now evaluated at the sovereign rate R_B^{ND}:

$$\Phi + \alpha \frac{R_B^{ND} \overline{B}_L}{1+\widetilde{\pi}_L^{ND}} + z \left(G + \alpha \frac{R_B^{ND} \overline{B}_L}{1+\widetilde{\pi}_L^{ND}} - \frac{\widetilde{\pi}_L^{ND}}{1+\widetilde{\pi}_L^{ND}}\kappa \right) + \mathcal{C}\left(\widetilde{\pi}_L^{ND}\right) \tag{35}$$
$$= z \left(G + \frac{R_B^{ND} \overline{B}_L}{1+\pi_L^{ND}} - \frac{\pi_L^{ND}}{1+\pi_L^{ND}}\kappa \right) + \mathcal{C}\left(\pi_L^{ND}\right).$$

Note that, when debt is above the threshold $\overline{B}_L$, the government would default in the weak fundamental state even if markets charge the risk free rate. For $B \geq \overline{B}_L$, then, ND cannot be an equilibrium. The thresholds, $\underline{B}_A$ and $\overline{B}_A$ are analogously defined—except that default in state A occurs at the minimum haircut rate defined in (28), rather than at 100 per cent—we omit the definitions to save space.

So, holding assumptions (17) through (18), our model admits a (rational-expectations) ND equilibrium in which the government borrows at R_B^{ND} and there is no default for $0 < B < \overline{B}_L$; a D equilibrium in which the government borrows at the rate R_B^D and default only occurs in the L state for $\overline{B}_L \leq B < \overline{B}_A$; a DD equilibrium in which the government borrows at R_B^{DD}, and there is a 100 percent haircut in the low output state and at least partial default in state A, for $B \geq \underline{B}_A$. When debt is above the threshold $\overline{B}_A$, default in the intermediate state becomes inevitable, and the D equilibrium no longer exists.

3.3 Uniqueness of inflation versus self-fulfilling "stealth default"

An important result by Calvo (1988) is the possibility of self-fulfilling hikes in interest rates driven by anticipations of debt debasement, i.e. "stealth default", via inflation. In the context of our model, this possibility revolves around the question of whether the inflation rate is uniquely determined for given equilibrium haircut rates, i.e., for a given equilibrium level of the interest rate. A unique inflation rate would indeed rule out the policy scenario envisioned by Calvo. We now show that this is the case with convex costs of inflation.

To gain insight, consider the best-response of inflation in the case of no-default (27), rewritten here for convenience:

$$\mathcal{C}'(\pi j) = z'\left(\frac{R_B^j}{1+\pi_i}B + G - \frac{\pi_i}{1+\pi_i}\kappa\right)\frac{BR_B^j + \kappa}{(1+\pi_i)^2}, \qquad j = ND, D, DD$$

where the expressions for R_B^j are given by (31) through (33). Inflation uniqueness is ensured if the RHS of the above expression—evaluated at equilibrium accounting for the fact that R_B^j will be generally increasing in expected inflation—is either decreasing in π_i, or increasing in π_i at a rate slower than $C'(\pi_i)$ on the LHS. This condition fails in Calvo's seminal contribution, essentially because $C'(\pi_i)$ is not restricted to be positive.[18]

[18] Indeed, in the Calvo model, inflation multiplicity would disappear with convex costs, as we posit in our analysis. Multiplicity essentially requires the cost of inflation to be bounded and/or a Laffer Curve for seigniorage revenues.

The following lemma states that, with convexity of inflation costs, inflation rates are uniquely determined in our equilibria. For the sake of analytical tractability, the lemma is stated in reference to equilibria where the default rate is at a corner, either zero or 100 percent.

Lemma 2 *For optimal haircut rates equal to either 0 or 100 percent, with convex costs of inflation, the inflation rates are unique in the equilibria* ND, D, DD.

Proof. See the appendix. ■

We stress that our analysis by no means dismisses the main lesson of Calvo (1988)—rather, it casts new light on it. Namely, belief-driven hikes in interest rates on public debt rooted in the discretionary behavior of monetary authorities cannot occur if the costs of inflation rise sufficiently fast in the policymakers' preferences. They may occur (independently of self-fulfilling expectations of outright default) if inflation costs are sufficiently flat.[19]

3.4 Multiple equilibria for intermediate levels of sovereign debt

While our assumptions guarantee that the first pair $\{\underline{B}_L, \overline{B}_L\}$ is strictly below $\{\underline{B}_A, \overline{B}_A\}$, they do not restrict the ranking of the thresholds within each pair. The equilibrium would be unique if $\overline{B}_L < \underline{B}_L$ and $\overline{B}_A < \underline{B}_A$. To have multiplicity, it must be that either $\overline{B}_L > \underline{B}_L$ or $\overline{B}_A > \underline{B}_A$, or both.

Specifically, for B comprised in the range between $\underline{B}_L$ and $\overline{B}_L$, an equilibrium with no default (ND) coexists with another equilibrium with sovereign risk (D). In this range, the D equilibrium is "non-fundamental," in the sense that default in L is determined by market expectations, and would not occur if investors have bought government debt at the risk-free rate R_B^{ND}. The D equilibrium instead becomes "fundamental" for higher B's, since then the financing need of the government are high enough that the government would default under macroeconomic stress even if investors bought debt at R_B^{ND}. For B comprised between $\overline{B}_L$ and $\underline{B}_A$, moreover, the (fundamental) D equilibrium is also unique: there is no other equilibrium in which the government would find it optimal to repudiate debt in better states of the economy than the weak one. Default in more states of the world becomes

[19] A key implication for policy is worth stressing. Contrary to popular arguments, low social costs of inflation provide no firm foundations for the central bank to rule out non-fundamental sovereign debt crises, since high tolerance for inflation may feed self-fulfilling beliefs of stealth (inflationary) default.

instead a possibility for higher values of B, in the range comprised between $\underline{B}_A$ and $\overline{B}_A$. The D equilibrium with fundamental default in the weak output state coexists with a DD equilibrium, where self-fulfilling expectations precipitate (non-fundamental) default *also* under average macroeconomic conditions.

Below, we set the stage of the analysis by proving that $\overline{B}_L > \underline{B}_L$ and $\overline{B}_A > \underline{B}_A$ in the limiting case of $\pi_i \to 0$. Then, we reconsider multiplicity with positive inflation. We will show that inflation and seigniorage do affect the debt range over which multiplicity obtains—but the option to inflate debt away does not necessarily rule out the possibility of self-fulfilling non-fundamental sovereign crises.

3.4.1 Multiplicity in the limiting case $\pi_i \to 0$

In our model, multiplicity always obtains when $\pi_i \to 0$ in all states of the world for all values of debt.[20] This case is analytically equivalent to assuming that debt is (indexed) real and seigniorage is zero, as in the first section of Calvo (1988) and in Lorenzoni and Werning (2014)—we can naturally map our contribution in this literature.

Proposition 3 *With $\pi_i \to 0$, $\overline{B}_L > \underline{B}_L$ and $\overline{B}_A > \underline{B}_A$, and there is multiplicity between the ND and the D equilibrium, and between the D and the DD equilibrium, all equilibria being well-behaved.*

Proof. To show that $\overline{B}_L > \underline{B}_L$, we note that, as inflation is zero in equilibrium, there is no seigniorage revenue and, by (19) $\widehat{T}_L - G = 0$. Moreover, $R_B^D = \frac{\rho}{(1-\gamma)+\gamma\mu} > \rho$. Combining the equations determining $\underline{B}_L$ and $\overline{B}_L$ we obtain:

$$\begin{aligned} \Phi &= z\left(G+\rho\overline{B}_L\right) - z\left(G+\alpha\rho\overline{B}_L\right) - \alpha\rho\overline{B}_L \qquad (36) \\ &= z\left(G+\frac{\rho}{(1-\gamma)+\gamma\mu}\underline{B}_L\right) - z\left(G+\frac{\alpha\rho}{(1-\gamma)+\gamma\mu}\underline{B}_L\right) - \frac{\alpha\rho}{(1-\gamma)+\gamma\mu}\underline{B}_L, \end{aligned}$$

The expressions on the first and the second lines are both increasing in B: this follows from the fact that when $\widehat{T}_L - G = 0$, taxes and distortions under no default and full default are always larger than under partial default—see (16). Since the sovereign (real) rate is higher when agents anticipate default, if evaluated at the same level of B, the expression on the second line would

[20] The proof can be easily generalized to the case in which π_i converges to a constant value.

be bigger than the expression on the first line: it must be that $\overline{B}_L > \underline{B}_L$.
To establish that the D and the ND equilibria (both admissible for B within the two thresholds) are well-behaved, we note that, as (31) and (32) are constant when $\pi_i \to 0$, the ex-ante interest rate payments $R_B^j B$ are always increasing in B for $j = ND, D$.
To show that $\overline{B}_A > \underline{B}_A$, we combine the equations determining these thresholds for $\pi_i \to 0$, to write:

$$[(1-\gamma)+\gamma\mu]\left(\hat{T}_A - G\right) > \left[(1-\gamma)+\left(1-\underline{\theta}_A^{DD}\right)\gamma\mu\right]\left(\hat{T}_A - G\right) \qquad (37)$$

where $\underline{\theta}_A^{DD}$ solves (28) in state A. The above inequality is always satisfied because $\underline{\theta}_A^{DD}$ is strictly positive, i.e., $\underline{\theta}_A^{DD} > 0$. The second term in the inequality is also larger than $\rho\overline{B}_L$ by Assumption 1, implying $\underline{B}_A > \overline{B}_L$.
This equilibrium is well behaved as Assumption 2 ensures that $R_B^{DD}B = \frac{(1-a)B\rho-\gamma\mu\left(\hat{T}_A-G\right)}{1-a-\gamma+\gamma\alpha(1-\mu)}$ is increasing in B. ■

An important property of multiplicity that, to our knowledge, has not been noted so far in the literature is summarized by the following remark.

Remark 4 *For $\alpha \to 0$ the lower range of multiplicity $[\underline{B}_L, \overline{B}_L)$ has size $\gamma(1-\mu)\overline{B}_L$, where $\gamma(1-\mu)$ is (approximately) the spread between the non-fundamental and the fundamental values of the sovereign rate R_B^D and R_B^{ND}. For $\alpha > 0$, the upper range of multiplicity $[\underline{B}_A, \overline{B}_A)$ has size $\gamma\mu\underline{\theta}_A^{DD}\left(\hat{T}_A - G\right)/\rho$, proportional to the (minimum) spread between the non-fundamental and the fundamental value of the sovereign rate, R_B^{DD} and R_B^D. This spread is approximately equal to the term $\gamma\mu\underline{\theta}_A^{DD}$, which, with an endogenous haircut rate, is the post-default primary surplus in state A, $\left(\hat{T}_A - G\right)$.*

In other words: there is a strict relation between the size of the multiplicity range and the size of the spread in interest rates across the fundamental and non-fundamental equilibria. By way of instance, with only fixed default costs and no fundamental default, as in Lorenzoni and Werning (2014), the range $[\underline{B}_L, \overline{B}_L)$ will generally be larger, the larger the spread between R_B^D and R_B^{ND}, and the higher $\overline{B}_L$.

3.4.2 Multiplicity with state-contingent inflation

Away from the limiting case $\pi_i \to 0$, monetary policy affects the fiscal policy trade-offs. Since inflation produces seigniorage revenues and unexpected inflation reduces the ex-post real burden of debt, conventional monetary policy contains taxes and tax-related output distortions and thus lowers the costs of primary surplus adjustment relative to default.

Recall that the benevolent monetary authorities internalize the benefit from lowering tax distortions. In particular, this means that the higher the elasticity of seigniorage revenue to inflation, the more the central bank is willing to raise inflation, making outright default less attractive for fiscal policymakers. In principle, if raising inflation could generate a very large amount of fiscal resources, seigniorage could eliminate multiplicity altogether (see Calvo 1988 footnote 15 on page 656). For this to be the case, however, the elasticity of seigniorage should be implausibly high—implying that there should be no Laffer curve for seigniorage.[21]

Multiplicity between the ND and D equilibrium Our considerations above suggests that, since in our model seigniorage is increasing in inflation (as in Calvo 1988), multiplicity exists provided that κ is not exceedingly high. Moreover, recall that, to ensure that our equilibria are "well-behaved", Assumption 2 restricts the size of α to be not too large. For the sake of tractability we state and prove the proposition on multiplicity below, further restricting both parameters, i.e., under the sufficient condition $\alpha, \kappa \to 0$. Under this condition, when the government chooses to default, the optimal haircut rates will tend to 100 percent. Even if seigniorage revenues approach zero, discretionary monetary authorities will still set positive inflation rates ex post, with the goal of reducing (if only at the margin) the burden of government debt in real terms.

The following proposition states our main result, establishing a lower multiplicity region for B.

Proposition 5 *For $\kappa, \alpha \to 0$, $\underline{B}_L < \overline{B}_L$ and the two equilibria ND and D coexist and are well behaved for B in the range $\underline{B}_L \leq B < \overline{B}_L$.*

Proof. See the appendix. ■

Multiplicity between the D and the DD equilibrium Omitting a formal proposition and focus directly on the condition for multiplicity, we

[21] As argued by Bacchetta, Perazzi and Van Wincoop (2015), a similar result (that the benefits of inflation are not enough to rule out multiplicity) carries over in models where monetary policy is not neutral, and a monetary expansion can boost tax revenues and the primary surplus also by improving economic activity, on top and above seigniorage.

write the counterpart of (37) with non-zero inflation rates:

$$\left[(1-\gamma)\frac{1+\widehat{\pi}^D}{1+\pi_H^D}+\gamma\mu\frac{1+\widehat{\pi}^D}{1+\pi_A^D}\right]\left[\left(\hat{T}_A-G\right)+\frac{\widehat{\pi}^D}{1+\widehat{\pi}^D}\kappa\right]> \tag{38}$$
$$\left[(1-\gamma)\frac{1+\widehat{\pi}^{DD}}{1+\pi_H^{DD}}+\left(1-\underline{\theta}_A^{DD}\right)\gamma\mu\right]\left[\left(\hat{T}_A-G\right)+\frac{\widehat{\pi}^{DD}}{1+\widehat{\pi}^{DD}}\kappa\right]$$

By the same logic of Proposition 5, it is easy to verify the inequality above for $\alpha,\kappa\to 0$, whereas the minimum default threshold is $\underline{\theta}_A^{DD}\to 1$. Under Assumption 2, the DD equilibrium is well behaved—i.e., the ex-ante interest rate bill $R_B^{DD}B$ in nominal terms will be rising in the initial level of nominal liabilities B. Namely, in the non-fundamental DD equilibrium:

$$BR_B^{DD}=\frac{(1-\alpha)\,B\rho-\gamma\mu\left[\left(\hat{T}_A-G\right)+\frac{\hat{\pi}^{DD}}{1+\hat{\pi}^{DD}}\kappa\right]}{\frac{(1-\gamma)(1-\alpha)}{1+\pi_H^{DD}}-\frac{\alpha\gamma\mu}{1+\hat{\pi}^{DD}}}.$$

Provided κ is not exceedingly large, $1-\gamma>\alpha$ and $\frac{1+\hat{\pi}^{DD}}{1+\pi_H^{DD}}>\mu>0$ will ensure that $R_B^{DD}B$ is increasing in B.

In either case, multiple equilibria exist when the (ex-post) real spread between the non-fundamental and the fundamental sovereign rate is positive: the ND and the D equilibria co-exist when $\frac{R_B^D}{1+\pi_L^D}>\frac{R_B^{ND}}{1+\pi_L^{ND}}$; the D and the DD equilibria co-exist when $\frac{R_B^{DD}}{1+\widehat{\pi}^{DD}}>\frac{R_B^D}{1+\widehat{\pi}^D}$. Observe that in both instances, higher inflation under default ex post does not compensate for the increase in the nominal rate due to self-fulfilling anticipations of default in an additional state—L in the first, A in the second range of multiplicity.

3.5 Numerical illustration

Figure 1 summarizes the main properties of our model economy in the absence of a successful backstop. The figure plots the interest costs of issuing public debt, R_BB, measured on the y-axis, against the initial financing need of the government, denoted by B, on the x-axis. As explained above, the market interest rate R_B in the figure is set by risk-neutral rational investors, forming expectations of taxation, default and inflation one-period ahead, knowing that policymaking is discretionary and (exogenous) macroeconomic conditions vary randomly.

In the figure, the interest costs faced by the government are overall increasing in B but not continuously so. Because default has fixed costs, the interest rate R_B jumps up, marking a sharp increase in $R_B B$, at the debt thresholds $\underline{B}_L$ and $\underline{B}_A$. There are three $R_B B$ lines. From the left to the right, the lower line corresponds to the ND equilibrium; the middle to the D equilibrium; the upper line to the right the DD equilibrium. These lines have different slopes, steeper for higher interest costs line to the right of the figure, and overlap over two ranges of B over which there is multiplicity (marked by a shaded area). As the initial financing need of the government grows larger, the higher interest costs imply that the fiscal authority may find it optimal to default more, and in more states of the world, rather than facing the economic distortions associated with adjusting the primary surplus to service debt in all circumstances.

To draw Figure 1, we impose Assumptions 1 and 2, but abstract from Assumption 3, which was motivated only on the grounds of analytical convenience. The primary surpluses in the L and A states are set equal to, respectively, $\hat{T}_L - G = 0.30$, $\hat{T}_A - G =$0.52 (where $G = 0$ without loss of generality), and the maximum value of seigniorage equal to $\kappa = 0.50$. Note that, given the two-period structure of our model, these figures can be interpreted in terms of present discounted values. That is, normalizing expected output to 1, they can be read as, respectively, 30, 52 and 50 percent of expected GDP. By the same token, $B = 1$ in the figure corresponds to 100 percent of expected GDP. As long as endogenous inflation is always close to zero in the H state, outcomes in this state are practically irrelevant for the equilibria we study. We then set $\hat{T}_H - G =$2.22 (in line with the last inequality on the right in Assumption 1) and the probabilities such that $\mu = 1/2$ and $\gamma = 0.2$ (in line with Assumption 2). The sovereign rate spread over the risk free rate is then around 10 percent in the D equilibrium, and 20 percent in the DD equilibrium. The fixed and variable costs of default are, respectively, $\Phi = 0.1$ (equal to 10 percent of expected GDP) and $\alpha = 0.1$ (equal to 10 percent of GDP if total debt service is as high as GDP). The cost of inflation is assumed to be quadratic and equal to $C(\pi) = 1.125 \cdot \pi^2$. This implies that an inflation rate of 10 percent ($\pi = 0.1$) causes output costs (in present discounted) equal to 1.1 percent of expected GDP. Similarly, tax distortions are set to $z(\cdot; Y_i) = \psi_i \cdot T_i^2$, where $2 \cdot \psi_i \cdot \hat{T}_i = \alpha / (1 - \alpha)$. By way of example, this implies that in state A $\psi_A = 0.11$ and, in present discounted value, the output cost of a primary surplus of 50 percent of expected GDP is around 2.5 percent of expected GDP. From this parameterization, we obtain minimum haircut rates around 75 percent in the L and the A state (precisely, $\underline{\theta}_L =$ 0.7904 and $\underline{\theta}_A =$0.7230). As shown in the figure, equilibrium multiplicity is possible for B in the following ranges [1.07,1.15), and [1.39,1.49).

In the economic environment illustrated by Figure 1, our question is now whether and how central bank interventions in the debt market can prevent a rise in interest rates driven by arbitrary anticipations of outright default, *de facto* eliminating the overlap between segments.

4 Ruling out bad equilibria with a credible monetary backstop

When multiple equilibria are possible, social welfare is lower if markets coordinate on the equilibrium with default in more states of the world. A high interest bill driven by self-fulfilling expectations of default causes unwarranted output and budget costs—associated to non-fundamental default when this occurs, but also with the need to raise distortionary primary surpluses in states of the world where the government opts for servicing the debt in full. The fact that equilibria with non-fundamental default are detrimental to social welfare motivates the search for effective backstop policies.

In this section we analyze the workings of monetary backstops, whereby the central bank announces that it will stand ready to purchase an amount of public debt ωB if agents coordinate their expectations **on non-fundamental default.** As is customary in the literature, we posit that market coordination across equilibria is regulated by a mechanism akin to a "traffic-signal" device that switches between red and green: when red appears, agents coordinate their expectations on the non-fundamental equilibrium, provided this equilibrium exists (see e.g. Evans, Honkapohja and Romer (1998)). If the central bank backstop policy is successful, however, the very announcement of debt purchases rules out the non-fundamental interest rate as an equilibrium outcome, and markets expectations cannot but coordinate on the unique fundamental equilibrium.

Debt purchases do not need to be carried out in equilibrium, but, to have the desired effect, the policy announcement has to be credible. Namely, the backstop has to satisfy the requirement that, if debt purchases were to be effectively carried out, the ensuing outcome would be feasible, unique, and welfare-improving from the vantage point of the policymakers. So, even if balance sheet losses and/or the possibility of elevated inflation as a consequence of central bank purchases of debt are merely off-equilibrium outcomes, their assessment is crucial to the design of a backstop policy. Should central bank interventions result in expected welfare losses relative to a non-fundamental equilibrium, the announcement of a backstop would clearly not be credible, hence would not have any influence on market's expectations.

As discussed in Section 3, backstops can be implemented under different regimes of fiscal and monetary interactions. We focus on the two polar assumptions of either a consolidated or a separate budget constraint, whereby the transfers by the central bank to the fiscal authorities are restricted to be non-negative.

4.1 The central bank budget constraint is not binding (fiscal backing)

Under either fiscal backing or a non-binding budget separation ($\lambda^{CB} = 0$,), any haircut on the bonds held by the central bank automatically generates an equivalent increase in tax liabilities: haircuts reduce the central bank transfer to the fiscal authority and may turn them negative. *De facto,* central bank holdings of debt reduce the "tax base" (the outstanding stock of debt held by private investors) on which the fiscal authority can impose haircuts and produce net budget saving. *Conditional on default,* even accounting for possible budget savings on variable default costs with $\alpha_{CB} \leq \alpha$, a higher ω tends to translate into higher taxation and (given that when $\lambda^{CB} = 0$ taxes and inflation comove positively under the optimal policy) higher output distortions overall. But debt purchases reduce the overall cost of issuing debt ex ante. *Conditional on no default,* a higher ω means that the government has to generate lower primary surpluses, at lower social costs.

So, when $\lambda^{CB} = 0$, central bank interventions unambiguously make the resort to default less attractive for the fiscal policymaker: they raise tax distortions under default, while still facilitating the full service of debt at lower costs. To appreciate this point, consider debt levels in the multiplicity range, $\underline{B}_L \leq B < \overline{B}_L$, where the two thresholds satisfy the condition for full default in state L conditional on $\omega = 0$. Posit that markets coordinate expectations on an equilibrium with 100 percent haircut charging R_B^D. Central bank purchases of debt by ωB enter the condition for optimal default in state L as follows:

$$\Phi \leq z\left(T_L^D\right) - z\left(\widetilde{T}_L^D\right) + \mathcal{C}\left(\pi_L^D\right) - \mathcal{C}\left(\widetilde{\pi}_L^D\right) - \alpha \frac{R_B^D}{1+\widetilde{\pi}_L^D}\left(1-\omega\right)B \tag{39}$$

where taxes are given by

$$\begin{aligned} \widetilde{T}_L^D &= G - \frac{\widetilde{\pi}_L^D}{1+\widetilde{\pi}_L^D}\kappa + \frac{\alpha R_B^D + \omega\left(R^D - \alpha R_B^D\right)}{1+\widetilde{\pi}_L^D}B \\ T_L^D &= G - \frac{\pi_L^D}{1+\pi_L^D}\kappa + \frac{R_B^D - \omega\left(R_B^D - R^D\right)}{1+\pi_L^D}B, \end{aligned}$$

and inflation rates under central bank interventions are determined by the counterparts of (11) in Section 3. For $R_B^D > R^D > \alpha R_B^D$, $\widetilde{T}_L^D$ and $\widetilde{\pi}_L^D$ are increasing in ω, while T_L^D and π_L^D are decreasing in ω: larger debt purchases unambiguously reduce the expression on the right-hand side of the inequality (39).

Given the fixed cost $\Phi > 0$, a sufficiently large amount of interventions will overturn the fiscal policy decisions to default. The following proposition synthesizes our main result for the lower multiplicity region.

Proposition 6 *Assume that there is a range of debt $\underline{B}_L \leq B < \overline{B}_L$, where $\underline{B}_L$ and $\overline{B}_L$ are defined in (34) and (35), respectively, for which both equilibria ND and D exist. Then there exists a minimum level of announced purchases, $1 > \underline{\omega}_L(B) > 0$, for which the only equilibrium is ND. Sufficient conditions are that $\alpha \to 0$ or $\pi_i \to 0$.*

Proof. See the Appendix. ■

In the second multiplicity region $\underline{B}_A \leq B < \overline{B}_A$, matters are somewhat complicated by the fact that the haircut rate in state A is endogenous when default is interior. Since interventions reduce the stock of liabilities held by private investors, when choosing to default the government may optimally impose a higher haircut rate. This is apparent in the limiting case $\pi_i \to 0$. The equation determining $\underline{\theta}_A$ conditional on $\omega > 0$ is:

$$\Phi = z\left(G + \frac{\left(\hat{T}_A - G\right) - (1-\alpha)\,\underline{\theta}_A \rho\omega B}{1-(1-\alpha)\,\underline{\theta}_A}\right) - z\left(\hat{T}_A\right) - \alpha\underline{\theta}_A \frac{\left(\hat{T}_A - G\right) - \rho\omega B}{1-(1-\alpha)\,\underline{\theta}_A}.$$

Since $z'(\cdot) > \alpha/(1-\alpha)$, the expression on the right hand side of the equation falls with an increase in ω. Therefore, the endogenous haircut threshold $\underline{\theta}_A$ will have to rise with ω in order for the expression to hold with equality.

As long as default is interior ($\underline{\theta}_A < 1$), the fact that the haircut rate unambiguously rises with ω means that interventions exert opposing effects on the overall cost of debt. On the one hand, central bank purchases lower the interest rate on ωB below market rates. On the other hand, market rates on $(1-\omega)B$ rise since agents anticipate a higher haircut.[22]

But once interventions are large enough, $\underline{\theta}_A$ reaches its upper bound equal to 1: if the government decides to default in state A, the optimal haircut rate will invariably be 100 percent. At that point, the same logic of

[22] In the limiting case of $\pi_i \to 0$, we can show that, as long as default is interior, increasing the scale of interventions ω initially raises, then lowers the debt threshold $\underline{B}_A(\omega)$. To wit: with $\omega > 0$, the lower threshold $\underline{B}_A$ is determined by the following

Proposition 6 also applies to the higher multiplicity region. For interventions on a sufficient scale that $\underline{\theta}_A = 1$, there will be a minimum size of debt purchases by the central bank that will result in an unique equilibrium without default in state A.

Relative to the non-fundamental DD-equilibrium, the economy is better off not only in the D-equilibrium, but also off equilibrium—if the central bank actually carries out debt purchases in the first period, in response to markets anticipating default in both the L and the A states. This follows from the fact that, off the equilibrium path interventions prevent debt service from rising with the high rates charged by the market: the economy does not incur the suboptimal costs of default in state A, and inflation and taxes are lower in all states of the world.[23]

Figures 2a,b illustrate these results based on our numerical example—in reference to the two regions of multiplicity shown Figure 1. The upper panel of the figure plots the minimum level of interventions required to eliminate multiplicity. The lower panel reports (ex ante) welfare conditional on no intervention ($\omega = 0$) and conditional on the minimum-level interventions shown in the panel above. Two features of the numerical example stand out. First, multiplicity over the relevant range disappears for values of ω between 1/4 and 1/2—corresponding to debt purchases targeting between 25 and 50 percent of the initial financing need of the government. Second, welfare conditional on actual (minimum) interventions is always higher than welfare in a non-fundamental equilibrium—confirming that the minimum-intervention backstop is feasible and welfare-improving as off-equilibrium outcome.

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expression

$$\underline{B}_A = \frac{[(1-\gamma) + (1-\underline{\theta}_A)\gamma\mu]\left(\hat{T}_A - G\right)/\rho}{[1-(1-\alpha)\underline{\theta}_A] + \omega\left[(1-\alpha)\underline{\theta}_A - \gamma\left(1-(1-\underline{\theta}_A)\mu\right)\right]}.$$

The coefficient multiplying ω in the denominator is negative for $\underline{\theta}_A \to 0$ and positive for $\underline{\theta}_A \to 1$, hence the debt threshold $\underline{B}_A$ will be initially increasing, then will become decreasing, as the central bank picks higher $\omega's$, until $\underline{\theta}_A = 1$.

[23] Observe that very large debt purchases by the central bank may increase the threshold for fundamental default in state L—up to the point in which, conditional on central bank interventions, the equilibrium may feature no default in any state. But to the extent that the two multiplicity regions are further from each other (as they are in our specification), there will be some level of interventions that will rule out non-fundamental default in state A without ruling out fundamental default in state L.

4.2 The central bank budget constraint is binding (budget separation)

The transmission mechanism by which a backstop operates is radically different under budget separation, with a binding constraint ($\lambda^{CB} \neq 0$). Namely, central bank purchases of debt no longer reduce the 'tax base' for a default. By contrast, they raise the possibility of high inefficient inflation. This is because, if only off equilibrium, debt purchases may result in balance sheet losses—implying that, in case of default, the monetary authorities will have to deviate from the optimal policy and run the printing press at inefficiently high rates. The question is whether such deviation impinges on the credibility of backstops.

Relative to the case of budget consolidation, high prospective inflation in case of default reduces welfare conditional on interventions: since monetary authorities cannot pursue their efficient policy plans, distortions are no longer optimally smoothed across policy instruments, and inflation is inefficiently high. But exactly for this reason, *as long as preferences over inflation are sufficiently similar across policy makers,* the large output distortions from high prospective inflation also weigh on the decision to default by the fiscal authority. Indeed, the main result of this section is that, as long as fiscal policymakers are sufficiently averse to inflation, the consequences of budget separation for the conduct of monetary policy act as a deterrent against outright debt repudiation.

Focus first on the low multiplicity region, for $\underline{B}_L \leq B < \overline{B}_L$. As long as no default takes place, the central bank budget constraint does not bind: debt purchases by the central bank result in *lower taxes* and *lower inflation* (and thus a lower costs of debt servicing), as in the case with budget consolidation. This is apparent from inspecting the government budget constraint under no default when markets charge the destabilizing interest rate R_B^D, and the optimality conditions for inflation, rewritten below for convenience:

$$T_i^D = G - \frac{\pi_i^D}{1+\pi_i^D}\kappa + \frac{R_B^D}{1+\pi_i^D}(1-\omega)B + \left[\frac{R_B^D}{1+\pi_i^D}\right]\omega B,$$

$$\left(1+\pi_i^D\right)^2 \mathcal{C}'\left(\pi_i^D\right) = z'\left(T_i^D\right)\left[\kappa + (1-\omega)BR_B^D + \omega BR^D\right].$$

When default occurs and the central bank constraint binds, instead, debt purchases have opposing effects on taxes and inflation. By the budget constraints of the two policy authorities evaluated under full default $\theta_L^D = 1$, a larger ω results in *lower taxation* (due to savings on the variable costs of default), but *higher inflation* (as the monetary authorities need to run the printing press to honor their nominal liabilities at face value):

$$\widetilde{T}_L^D = G + \alpha \frac{R_B^D}{1+\widetilde{\pi}_L^D} (1-\omega) B,$$

$$\widetilde{\pi}_L^D \geq \frac{\omega}{\kappa} R^D B.$$

To appreciate the consequences on the default decision by fiscal authorities, reconsider condition (39) imposing that the central bank constraint binds (i.e. $\widetilde{\pi}_L^D = \frac{\omega}{\kappa} R^D B$). By this condition, the fiscal authorities only choose debt repudiation in state L if the cost of defaulting is lower than the cost of servicing debt in full:

$$\Phi \leq z\left(T_L^D\right) + \mathcal{C}\left(\pi_L^D\right) +$$
$$-z\left(G + \alpha \frac{R_B^D}{1+\frac{\omega}{\kappa} R^D B} (1-\omega) B\right) - \alpha \frac{R_B^D}{1+\frac{\omega}{\kappa} R^D B} (1-\omega) B - \mathcal{C}\left(\frac{\omega}{\kappa} R^D B\right).$$

Observe that a larger ω implies lower T_L^D and π_L^D and a higher $\widetilde{\pi}_L^D$; therefore, for $\alpha \to 0$, higher purchases reduce the cost of servicing debt in full over the cost of defaulting. Furthermore, with $\omega \to 1$, the right-hand side of the above inequality has to be lower than Φ by construction: T_L^D and π_L^D are set to service the debt at the risk-free rate R^D, while $\widetilde{\pi}_L^D$ must be higher than its optimal counterpart under default (obviously barring the possibility that seigniorage is so high that the central bank constraint does not bind). So, unless the costs of inflation are either bounded or downplayed by the fiscal authorities (and/or seigniorage revenues are unrealistically high and elastic to inflation), there will be a threshold above which debt purchases by the central bank rule out multiplicity.

The same argument applies for level debt in the higher multiplicity region, $\underline{B}_A \leq B < \overline{B}_A$. In this range, when the central bank is constrained, taxes and inflation under non-fundamental default in state A are given by

$$\widehat{T}_A - G = \left[1 - \underline{\theta}_A (1-\alpha)\right] \frac{R_B^{DD}}{1+\pi_A} (1-\omega) B + \left[1 - \underline{\theta}_A\right] \frac{\overline{R}_B}{1+\pi_A} \omega B,$$

$$\pi_A \kappa = \left(R^{DD} - \left(1 - \underline{\theta}_A\right) \overline{R}_B\right) \omega B.$$

For a given $\underline{\theta}_A$, debt purchases by the central bank reduce overall taxation and increase inflation under default. But now debt purchases (and their inflationary consequences in case of default) also impinge on the minimum threshold $\underline{\theta}_A$. Moreover, even when successfully ruling out non-fundamental default in state A, central bank purchases still carry the risk of balance sheet losses due to fundamental default in state L. While these complications do not substantially alter our conclusions, they make the analytical characterization of the minimum intervention levels required to rule out the DD equilibrium quite cumbersome. Therefore, we illustrate our main result via a numerical example.

Figure 3a,b illustrate the effects of interventions under budget separation based on the same parameterization and layout of the previous figure. The upper panel shows the minimum level of interventions required to eliminate multiplicity. The lower panel reports ex ante welfare conditional on no intervention ($\omega = 0$) and conditional on the successful (minimum-level) interventions shown in the panel above.

As in Figure 2, welfare conditional on actual (minimum) interventions is always higher than welfare in a non-fundamental equilibrium—confirming that the minimum-intervention backstop is feasible and welfare-improving also under budget separation. There is however a notable difference relative to the previous figure: multiplicity over the relevant range disappears for values of ω between 1/20 and 1/10, much lower than in the case of a consolidated budget constraint. The reason is clearly spelt out in our analysis in this section: provided fiscal authorities are adverse to inflation and budget separation is an inflexible (hence credible) rule, budget separation does not undermine at all monetary backstops. Committing the central bank to be responsible for its own budget constraint can actually strengthen its ability to backstop government debt.

4.3 Discussion

There are a number of factors and considerations that are to be taken into account in the design of a successful backstop. We have already discussed a crucial one in Subsection 3.3, concerning the possibility of multiplicity in equilibrium inflation rates—in the conclusion below we will briefly consider the possibility of non-market interventions by the central bank, e.g. measures affecting banks' reserves. Here we briefly discuss the issue of whether a backstop could feed opportunistic behavior by the fiscal authority *in equilibrium*, exacerbating fiscal fragility and therefore the likelihood of (fundamental) default.[24] "Moral hazard" is a widely-debated issue that would require

[24] We have seen above that, off-equilibrium, the actual implementation of debt purchases may affect the optimal default rate in the case of an interior solution. The elasticity of θ to ω is however irrelevant in the equilibrium allocation resulting from a successful backstop.

extensive analysis; we limit our discussion to a basic consideration.

Conceptually, backstops are distinct from bailouts, that take the form contingent transfers occurring with positive probability. The literature has long clarified that, acting as market coordination devices, backstops may actually strengthen the incentives for a government to undertake costly actions that improve economic resilience to fiscal stress—the opposite of the "moral hazard" consequences of a bailout (see Morris and Shin 2006, Corsetti et al. 2005, Corsetti and Dedola 2011 and Nicolini et al. 2014 among others). This is because, without a backstop, the possibility of belief-driven crises tends to reduce the expected future benefits from these actions.

Nonetheless, a backstop does not necessarily eliminate fundamental default. With weak fundamentals creating fiscal stress, a central bank may face the risk of being drawn into quite a different policy regime, one of ex-post debt monetization à la Sargent and Wallace (1981), which may threaten its independence and ability to deliver on its objectives. Minimizing this risk is a first-order issue in the design of institutional arrangements supporting backstop policies. Yet this risk is not an argument for barring monetary backstops independently of a proper assessment of the trade-offs between the social and economic costs of inflation versus belief-driven debt crises.

A further issue is why central banks do not engage more systematically in the sovereign debt market, to ensure that government debt is non-defaultable in nominal terms under any circumstances. The public finance literature has shown that even when the government is benevolent and can commit, under some circumstances it is optimal for the fiscal authorities to manipulate ex-post the value of (non-contingent) government bonds, see, e.g., Adam and Grill (2011). An intriguing direction of research may build on the observation that eliminating default under any circumstances through monetary interventions may not be efficient.

5 Conclusions

This paper has reconsidered the question of whether and how a central bank can rule out self-fulfilling sovereign debt crises. Our model highlights crucial conditions. Firstly, a monetary backstop rests on the ability of the central bank to issue liabilities at a lower interest rate than a government subject to default risk. In our analysis, successful intervention strategies translate into a swap of (default-) risky government debt with nominal liabilities which can always be redeemed against currency. Secondly, monetary policymakers should be sufficiently averse to inflation, so that monetary policy is not itself a source of multiple equilibria in inflation and interest rates. Namely, conditional on a realized haircut, inflation rates should be uniquely determined, ruling out the possibility of high interest rates and taxation in the presence of sound fiscal fundamentals and no default. Lastly, either the fiscal authorities provide "backing" to the monetary authorities—to prevent prospective balance sheet losses in case of fundamental default to force the central bank to run inefficient inflation—or, barring any form of "fiscal backing" of the central bank, the fiscal authorities are themselves sufficiently averse to inflation, and internalize the inflationary costs of default in their decision making.

Our results are at odds with views often voiced in the public debate, claiming that the central bank can freely play the role of lender of last resort to the government because, alternatively, a central bank can always consolidate its liabilities and force private banks to hold them indefinitely, or debase them by a bout of unexpected inflation. In light of our analysis, both views have fundamental weaknesses. The latter view stressing the need for the central bank to impose financial repression over private banks by forcing them to hold reserves, *de facto* introduces the possibility of default on monetary liabilities, without however working out its consequences. If the central bank is expected to tamper with its liabilities, it is easy to see that the arbitrage condition relating the rate on monetary liabilities and the risk free rate would have to include terms in the anticipated central bank's haircut θ_i^{CB}: the optimal monetary policy would have to account for the optimal haircut on the holders of reserves. The logic of self-fulfilling beliefs would then apply to a discretionary central bank as well as to the government.

The alternative, inflationary-debasement view downplays the social costs of running high inflation, historically conducive to financial and macro instability. If anything, in line with Calvo (1988), our analysis suggests that downplaying the costs of inflation may actually raise the prospects of self-fulfilling sovereign debt crises driven by expectations of debt debasement, rather than outright default. Our analysis calls attention on the non-trivial fact that, exactly because high inflation is costly, a monetary backstop is credible even under budget separation. Most importantly, inflation rates are higher in an equilibrium with belief-driven outright defaults: an effective monetary backstop prevents high (let alone runaway) inflation, rather than creating price instability.

An important conclusion from our analysis is that a shared objective function among fiscal and monetary authorities (or enough aversion to inflation costs by the fiscal authority) greatly facilitates the implementation of a monetary backstop. As each authority internalizes the effects of own policy choices on overall distortions, a monetary backstop is effective under reasonably mild conditions, even when the central bank is held responsible for its own balance sheet losses, barring contingent fiscal transfers under any circumstance. It follows that the conditions for a monetary backstop to be credible may be stricter when political economy or distributional considerations cause the two authorities to trade-off self-interested objectives with socially efficient policies.

Although we have developed our model from the perspective of a national economy with an independent currency, our analysis bears lessons for a currency union. In a monetary union among essentially independent states, national governments may pursue conflicting, inward-looking objectives and/or be adverse to extending large-scale fiscal backing to the common central bank. In case of budget separation, the inflationary consequences from budget losses due to default by one country may be quite contained and, most importantly, are diffuse through the entire currency union. This means that a national fiscal authority choosing to default may not face the full inflationary costs of its decision. Even under these circumstances, however, a common central bank can still engineer a successful backstop to member states, to the extent that, as is the case for the OMTs in the euro area, governments have access to the benefit of a backstop only provided that they agree to strict conditionality, ensuring stability of public finances and possibly eliciting stricter cross-border cooperation.

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6 Appendix

6.1 Uniqueness of inflation

We prove uniqueness of inflation for the ND, D and DD equilibrium in turn, assuming that, when default occurs, the haircut rate is 100 percent. Consider first the ND equilibrium (with no default) whereas

$$R_B^{ND} = \frac{\rho}{\frac{(1-\gamma)}{1+\pi_H^{ND}} + \frac{\gamma\mu}{1+\pi_A^{ND}} + \frac{\gamma(1-\mu)}{1+\pi_L^{ND}}}$$

and, for a given interest rate R_B, the inflation reaction functions are given by:

$$\mathcal{C}'(\pi_i)(1+\pi_i) = z'(\frac{R_B}{1+\pi_i}B+G-\frac{\pi_i}{1+\pi_i}\kappa;Y_i)\left(\frac{R_B}{1+\pi_i}B+\frac{\kappa}{1+\pi_i}\right), i = H, A, L.$$

The equations above uniquely define $\pi_i(R_B)$ with $\frac{\partial \pi_i(R_B)}{\partial R_B} > 0$ for $C(\cdot), z(\cdot)$ convex, as the right hand side falls while the left-hand side increases with π_i. Observe that $\pi_L(R_B) > \pi_A(R_B) > \pi_H(R_B)$, given our assumptions on $z'(\cdot;Y_i)$, and that $0 \leq \pi_i(R_B = 0) < \infty$.

The equilibrium rate R_B^{ND} is defined by the following fixed point:

$$V(R_B) \equiv \frac{\rho}{\frac{(1-\gamma)}{1+\pi_H(R_B)} + \frac{\gamma\mu}{1+\pi_A(R_B)} + \frac{\gamma(1-\mu)}{1+\pi_L(R_B)}} = R_B,$$

where $V(R_B) > 0$ when $R_B = 0$. To show that the fixed point exists and is unique, observe that the following inequality holds:

$$V(R_B) \equiv \frac{\rho}{\frac{(1-\gamma)}{1+\pi_H(R_B)} + \frac{\gamma\mu}{1+\pi_A(R_B)} + \frac{\gamma(1-\mu)}{1+\pi_L(R_B)}} \leq \rho\left[1+\pi_L(R_B)\right],$$

where the left-hand side and right-hand side are both increasing in R_B. Therefore, a sufficient condition for $V(R_B)$ to have a unique fixed point at $0 < R_B^{ND} \leq R_B^*$ is that there exists a unique R_B^* such that

$$R_B^* = \rho\left[1+\pi_L(R_B^*)\right].$$

Setting $R_B = \rho(1+\pi_L)$ in the inflation reaction function in state L yields a unique value π_L^* due to convexity of the functions $C(\cdot)$ and $z(\cdot)$:

$$\mathcal{C}'(\pi_L^*)(1+\pi_L^*) = z'(\rho B + G - \kappa\frac{\pi_L^*}{1+\pi_L^*};Y_L)\left(\rho B + \frac{\kappa}{1+\pi_L^*}\right);$$

therefore we have that $R_B^* = \rho\left(1+\pi_L^*\right) = \rho\left[1+\pi_L\left(R_B^*\right)\right].$ This establishes that $V\left(R_B\right)$ also has a unique fixed point.

Consider now the D equilibrium, with full default in state L, in which the sovereign interest rate is given by $R_B^D = \dfrac{\rho}{\dfrac{(1-\gamma)}{1+\pi_H^D}+\dfrac{\gamma\mu}{1+\pi_A^D}}$, and the inflation reaction functions in states A and H under no default, (27), are given by:

$$\begin{aligned}
\mathcal{C}'\left(\pi_A^D\right)\left(1+\pi_A^D\right) &= z'\left(\frac{\rho}{(1-\gamma)\Pi^D+\gamma\mu}B+G-\frac{\pi_A^D}{1+\pi_A^D}\kappa;Y_A\right)\left(\frac{\rho}{(1-\gamma)\Pi^D+\gamma\mu}B+\frac{\kappa}{1+\pi_A^D}\right)\\
\mathcal{C}'\left(\pi_H^D\right)\left(1+\pi_H^D\right) &= z'\left(\frac{\Pi^D\rho}{(1-\gamma)\Pi^D+\gamma\mu}B+G-\frac{\pi_H^D}{1+\pi_H^D}\kappa;Y_H\right)\left(\frac{\Pi^D\rho}{(1-\gamma)\Pi^D+\gamma\mu}B+\frac{\kappa}{1+\pi_H^D}\right),
\end{aligned}$$

where Π^D is defined as the ratio of inflation in the A and the H state, $\Pi^D \equiv \frac{1+\pi_A^D}{1+\pi_H^D}$. According to these equations π_i^D $i=A,H$ are uniquely determined as a function of Π^D, since under convexity of the inflation costs $C(\cdot)$ the left-hand side is increasing in π_i^D, while the right-hand side is decreasing in π_i^D. Namely, the equations implicitly define the two functions $\pi_i^D = f_i\left(\Pi^D\right)$, where f_H (f_A) is increasing (decreasing) in Π^D as $\dfrac{\Pi^D\rho}{(1-\gamma)\Pi^D+\gamma\mu}$ is also increasing (and $\left(\dfrac{\rho}{(1-\gamma)\Pi^D+\gamma\mu}\right)$ is decreasing) in Π^D. In turn, these functions define the following mapping of Π^D into itself:

$$\Pi^D = \frac{\left(1+\pi_A^D\right)}{\left(1+\pi_H^D\right)} = \frac{1+f_A\left(\Pi^D\right)}{1+f_H\left(\Pi^D\right)} \equiv g\left(\Pi^D\right), \Pi^D \in (0,\infty).$$

Therefore, if we can show that $g\left(\Pi^D\right)$ has a unique fixed point, this would establish uniqueness of π_i^D $i=A,H$. Observe first that for $\Pi^D \to 0$, $\lim_{\Pi^D\to 0} f_A\left(\Pi^D\right) > \lim_{\Pi^D\to 0} f_H\left(\Pi^D\right)$, since the right-hand side of the inflation reaction function evaluated at the same inflation rate for $\Pi^D \to 0$ is always higher in state A than in state H:

$$z'\left(\frac{\rho}{\gamma\mu}B+G-\frac{\pi}{1+\pi}\kappa;Y_A\right)\left(\frac{\rho}{\gamma\mu}B+\frac{\kappa}{1+\pi}\right) > z'\left(G-\frac{\pi}{1+\pi}\kappa;Y_H\right)\left(\frac{\kappa}{1+\pi}\right),$$

while the left-hand side is the same. This implies that $\lim_{\Pi^D\to 0} g\left(\Pi^D\right) > 1 > 0$. By the same token, $\Pi^D \to \infty, \lim_{\Pi^D\to\infty} g\left(\Pi^D\right) < 1$ as $\lim_{\Pi^D\to\infty} f_A\left(\Pi^D\right) < \lim_{\Pi^D\to\infty} f_H\left(\Pi^D\right)$. Hence, the mapping $g\left(\Pi^D\right)$ has at list one fixed point.

Then a sufficient condition for the uniqueness of this fixed point is that $g\left(\Pi^D\right)$ is decreasing in Π^D. This is the case if $f_H\left(\Pi^D\right)$ is decreasing and $f_A\left(\Pi^D\right)$ is increasing, a fact that we have already established above. Therefore π_i^D $i = A, H$ are uniquely determined, and so is R_B^D. Finally, as R_B^D does not depend on it, the following uniquely solves for inflation in state L:

$$\mathcal{C}'\left(\widetilde{\pi}_L\right) = \frac{z'(\alpha\frac{R_B^D}{1+\widetilde{\pi}_L}B + G - \frac{\widetilde{\pi}_L}{1+\widetilde{\pi}_L}\kappa; Y_L)\left(\alpha B R_B^D + \kappa\right) + \alpha B R_B^D}{\left(1+\widetilde{\pi}_L\right)^2}.$$

This establishes that inflation rates in the D equilibrium, π_i^D $i = H, A, L$, are unique.

Finally, consider the DD equilibrium, where $R_B^{DD} = \dfrac{\rho}{\dfrac{(1-\gamma)}{1+\pi_H^{DD}} + \dfrac{\gamma\mu\left(1-\widehat{\theta}_A^{DD}\right)}{1+\widehat{\pi}^{DD}}}$.

When $\widehat{\theta}_A^{DD} = \underline{\theta}_A^{DD} = 1$, the same argument just used for the D equilibrium immediately applies, since the following equation has a unique solution for π_H^{DD} :

$$\mathcal{C}'\left(\pi_H^{DD}\right)\left(1+\pi_H^{DD}\right) = z'(\frac{\rho}{1-\gamma}B + G - \frac{\pi_H^{DD}}{1+\pi_H^{DD}}\kappa; Y_H)\left(\frac{\rho}{1-\gamma}B + \frac{\kappa}{1+\pi_H^{DD}}\right);$$

therefore inflation rates are uniquely determined.

6.2 Multiplicity between the ND and the D equilibrium with state-contingent inflation

The condition for multiplicity of ND and D equilibria with $\alpha, \kappa \to 0$ is:

$$\begin{aligned}\Phi &= z\left(G + \frac{R_B^{ND}}{1+\pi_L^{ND}}\overline{B}_L\right) - z(G) + \mathcal{C}\left(\pi_L^{ND}\right) - \mathcal{C}\left(\widetilde{\pi}_L^{ND}\right)\\ &< z\left(G + \frac{R_B^{D}}{1+\pi_L^{D}}\overline{B}_L\right) - z(G) + \mathcal{C}\left(\pi_L^{D}\right) - \mathcal{C}\left(\widetilde{\pi}_L^{D}\right),\end{aligned}$$

where sovereign bond rates are defined as:

$$\begin{aligned}R_B^{ND} &= \frac{\rho}{\frac{(1-\gamma)}{1+\pi_H^{ND}} + \gamma\left(\frac{\mu}{1+\pi_A^{ND}} + \frac{1-\mu}{1+\pi_L^{ND}}\right)},\\ R_B^{D} &= \frac{\rho}{\frac{(1-\gamma)}{1+\pi_H^{D}} + \frac{\gamma\mu}{1+\pi_A^{D}}};\end{aligned}$$

and, because of full default and no seigniorage revenue, inflation is set at the value that minimizes its cost $C'\left(\widetilde{\pi}_L^{ND}\right) = C'\left(\widetilde{\pi}_L^{D}\right) = 0$, implying $C\left(\widetilde{\pi}_L^{ND}\right) = C\left(\widetilde{\pi}_L^{D}\right)$. Therefore we can rewrite the condition for multiplicity as follows:

$$z\left(G + \frac{R_B^{ND}}{1+\pi_L^{ND}}\overline{B}_L\right) + \mathcal{C}\left(\pi_L^{ND}\right) < z\left(G + \frac{R_B^{D}}{1+\pi_L^{D}}\overline{B}_L\right) + \mathcal{C}\left(\pi_L^{D}\right),$$

where inflation rates are (uniquely) determined according to the following reaction functions:

$$\begin{aligned}\left(1+\pi_L^{ND}\right)\mathcal{C}'\left(\pi_L^{ND}\right) &= z'\left(G + \frac{R_B^{ND}}{1+\pi_L^{ND}}\overline{B}_L\right)\frac{R_B^{ND}}{1+\pi_L^{ND}}\overline{B}_L \\ \left(1+\pi_L^{D}\right)\mathcal{C}'\left(\pi_L^{D}\right) &= z'\left(G + \frac{R_B^{D}}{1+\pi_L^{D}}\overline{B}_L\right)\frac{R_B^{D}}{1+\pi_L^{D}}\overline{B}_L.\end{aligned}$$

Here, π_L^{ND} is an equilibrium inflation rate, while π_L^{D} represents a best deviation, so R_B^D does not depend on it. Observe that $\frac{R_B^D}{1+\pi_L^D} > \frac{R_B^{ND}}{1+\pi_L^{ND}} <=> \pi_L^D > \pi_L^{ND}$, as the function on the left-hand sides, $l(\pi) \equiv (1+\pi)\mathcal{C}'(\pi)$, is increasing in π, and the function on the right-hand side, $r\left(\frac{R_B}{1+\pi}B\right) \equiv z'(\cdot; Y_L)\frac{R_B}{1+\pi}B$, is increasing in the ex-post real rate $\frac{R_B}{1+\pi}$. Therefore, if $\frac{R_B^D}{1+\pi_L^D} > \frac{R_B^{ND}}{1+\pi_L^{ND}}$ (and because $\pi_L^D > \pi_L^{ND}$, equivalently if $R_B^D > R_B^{ND}$) then there will be multiplicity. Thus we only need to show that $R_B^D > R_B^{ND}$**;** namely, since $\gamma(1-\mu)\frac{1+\pi_i^{ND}}{1+\pi_L^{ND}} > 0$, $i = A, H$ we have to show that:

$$\frac{(1-\gamma)}{1+\pi_H^D} + \frac{\gamma\mu}{1+\pi_A^D} \leq \frac{(1-\gamma)}{1+\pi_H^{ND}} + \frac{\gamma\mu}{1+\pi_A^{ND}}.$$

which is verified if $\pi_i^D \geq \pi_i^{ND}$ $i = A, H$. Consider first inflation determination in the ND equilibria, in the states A and H:

$$\begin{aligned}&\left(1+\pi_A^{ND}\right)\mathcal{C}'\left(\pi_A^{ND}\right) \\ = \; & z'\left(G + \frac{\rho}{(1-\gamma)\frac{1+\pi_A^{ND}}{1+\pi_H^{ND}} + \gamma\mu + \gamma(1-\mu)\frac{1+\pi_A^{ND}}{1+\pi_L^{ND}}}\overline{B}_L\right)\frac{\rho}{(1-\gamma)\frac{1+\pi_A^{ND}}{1+\pi_H^{ND}} + \gamma\mu + \gamma(1-\mu)\frac{1+\pi_A^{ND}}{1+\pi_L^{ND}}}\overline{B}_L \\ &\left(1+\pi_H^{ND}\right)\mathcal{C}'\left(\pi_H^{ND}\right) \\ = \; & z'\left(G + \frac{\rho}{(1-\gamma) + \gamma\mu\frac{1+\pi_H^{ND}}{1+\pi_A^{ND}} + \gamma(1-\mu)\frac{1+\pi_H^{ND}}{1+\pi_L^{ND}}}\overline{B}_L\right)\frac{\rho}{(1-\gamma) + \gamma\mu\frac{1+\pi_H^{ND}}{1+\pi_A^{ND}} + \gamma(1-\mu)\frac{1+\pi_H^{ND}}{1+\pi_L^{ND}}}\overline{B}_L.\end{aligned}$$

These equations determine π_i^{ND} as a function of π_L^{ND}, where π_i^{ND} is larger, the larger π_L^{ND}. This follows from the fact that the right-hand side of both equations is increasing in π_L^{ND} and π_j^{ND}, $j \neq i$. Therefore, π_i^{ND} would reach their maximum (but finite) value for $\pi_L^{ND} \to \infty$, for which the term $\gamma(1-\mu)\frac{1+\pi_i^{ND}}{1+\pi_L^{ND}} \to 0$. But inflation determination π_i^D in the D equilibrium can be thought of as exactly this limiting case, because under complete default in state L the term $\gamma(1-\mu)\frac{1+\pi_i^D}{1+\widetilde{\pi}_L^D}$ is multiplied by zero. Indeed it does not appear in the inflation reaction functions, which are otherwise the same as those above:

$$
\begin{aligned}
\left(1+\pi_A^D\right)\mathcal{C}'\left(\pi_A^D\right) &= z'\left(G+\frac{\rho}{(1-\gamma)\frac{1+\pi_A^D}{1+\pi_H^D}+\gamma\mu}\overline{B}_L\right)\frac{\rho}{(1-\gamma)\frac{1+\pi_A^D}{1+\pi_H^D}+\gamma\mu}\overline{B}_L \\
\left(1+\pi_H^D\right)\mathcal{C}'\left(\pi_H^D\right) &= z'\left(G+\frac{\rho}{(1-\gamma)+\gamma\mu\frac{1+\pi_H^D}{1+\pi_A^D}}\overline{B}_L\right)\frac{\rho}{(1-\gamma)+\gamma\mu\frac{1+\pi_H^D}{1+\pi_A^D}}\overline{B}_L.
\end{aligned}
$$

6.3 Existence of threshold for interventions in non-fundamental D equilibrium

In the D equilibrium where the sovereign and the risk-free interest rates are:

$$
\begin{aligned}
R_B^D &= \frac{\rho}{\dfrac{(1-\gamma)}{1+\pi_H^D}+\dfrac{\gamma\mu}{1+\pi_A^D}} \\
R^D &= \frac{\rho}{\dfrac{(1-\gamma)}{1+\pi_H^D}+\dfrac{\gamma\mu}{1+\pi_A^D}+\dfrac{\gamma(1-\mu)}{1+\widetilde{\pi}_L^D}},
\end{aligned}
$$

optimal default in state L implies that the following inequality holds:

$$
\Phi < z\left(T_L^D\right) - z\left(\widetilde{T}_L^D\right) + \mathcal{C}\left(\pi_L^D\right) - \mathcal{C}\left(\widetilde{\pi}_L^D\right) - \alpha\frac{R_B^D}{1+\widetilde{\pi}_L^D}(1-\omega)B; \qquad (40)
$$

whereas we assume that this inequality holds for $\omega = 0$ in the multiplicity range $\underline{B}_L \leq B < \overline{B}_L$. Taxes are given by the budget constraints under full default $\left(\widetilde{T}_L^D\right)$ and under full repayment $\left(T_L^D\right)$:

$$
\begin{aligned}
\widetilde{T}_L^D &= G - \frac{\widetilde{\pi}_L^D}{1+\widetilde{\pi}_L^D}\kappa + \frac{\alpha(1-\omega)R_B^D+\omega R^D}{1+\widetilde{\pi}_L^D}B, \\
T_L^D &= G - \frac{\pi_L^D}{1+\pi_L^D}\kappa + \frac{(1-\omega)R_B^D+\omega R^D}{1+\pi_L^D}B,
\end{aligned}
$$

while inflation rates obey the reaction functions:

$$\left(1+\widetilde{\pi}_L^D\right)^2 \mathcal{C}'\left(\widetilde{\pi}_L^D\right) = z'\left(\widetilde{T}_L^D\right)\left(\kappa + \left[\alpha R_B^D(1-\omega)+\omega R^D\right]B\right) + \alpha R_B^D(1-\omega)B,$$

$$\left(1+\pi_L^D\right)^2 \mathcal{C}'\left(\pi_L^D\right) = z'\left(T_L^D\right)\left(\kappa + \left[R_B^D(1-\omega)+\omega R^D\right]B\right).$$

Observe that the last two equations determine inflation rates, which in turn determine taxes.

We need to show that there is a threshold $0 < \underline{\omega}(B) < 1$ at which (40) holds as an equality, implying that default is no longer optimal. As state above, by assumption this inequality holds for $\underline{B}_L \leq B < \overline{B}_L$ and $\omega = 0$. For $\omega \to 1$, the right-hand side of the inequality goes to zero, becoming lower than Φ. Therefore, we only need to show that the right-hand side is a decreasing function of ω. A sufficient condition is that T_L^D, π_L^D be decreasing in ω, while $\widetilde{T}_L^D, \widetilde{\pi}_L^D$ be increasing in ω. Since $R_B^D > R^D$, under full repayment, debt service $\left[R_B^D(1-\omega)+\omega R^D\right]B$ decreases in ω. It follows that both T_L^D, π_L^D are also decreasing in ω, since a lower debt service will translate into lower distortions at the margin. Debt costs under default, $\left[\alpha R_B^D(1-\omega)+\omega R^D\right]B$, fall with ω if and only $R^D > \alpha R_B^D$, namely:

$$(1-\alpha)\left[\frac{(1-\gamma)}{1+\pi_H^D} + \frac{\gamma\mu}{1+\pi_A^D}\right] > \alpha\gamma(1-\mu)\frac{1}{1+\widetilde{\pi}_L^D}.$$

When $\widetilde{\pi}_L^D \geq \pi_i^D$ this condition is always verified under Assumption 2, as $1-\alpha-\gamma > 0$; this is the case when $\pi_i \to 0$. Obviously, $\alpha \to 0$ is another sufficient condition holding for any inflation rate.

Finally, it is apparent that welfare is higher off the equilibrium path with interventions at the threshold $\underline{\omega}(B)$ than in the D equilibrium with $\omega = 0$, since off-equilibrium the central bank interventions lower the debt costs, implying lower taxes and inflation, and the economy is spared default costs.

Figure 1
Interest costs of issuing public debt
as a function of the initial financing need of the government

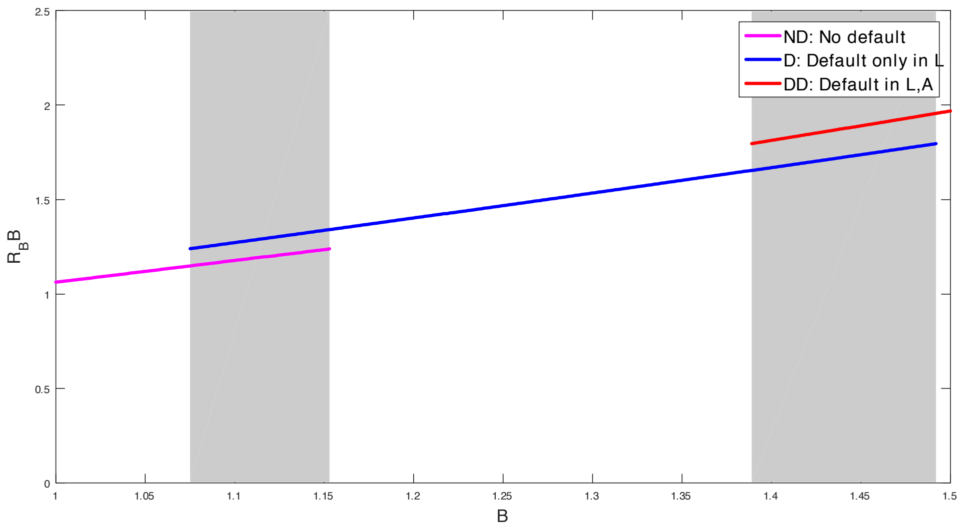

Figure 2a
Minimum Interventions required to eliminate the non fundamental equilibrium
Case of budget consolidation

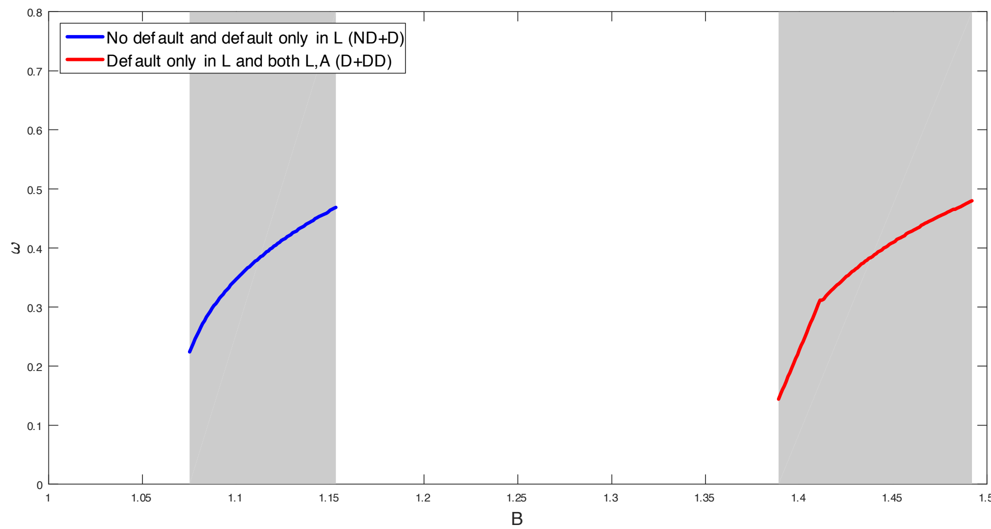

Figure 2b
Welfare with no backstop and with a minimum-intervention backstop
Case of budget consolidation

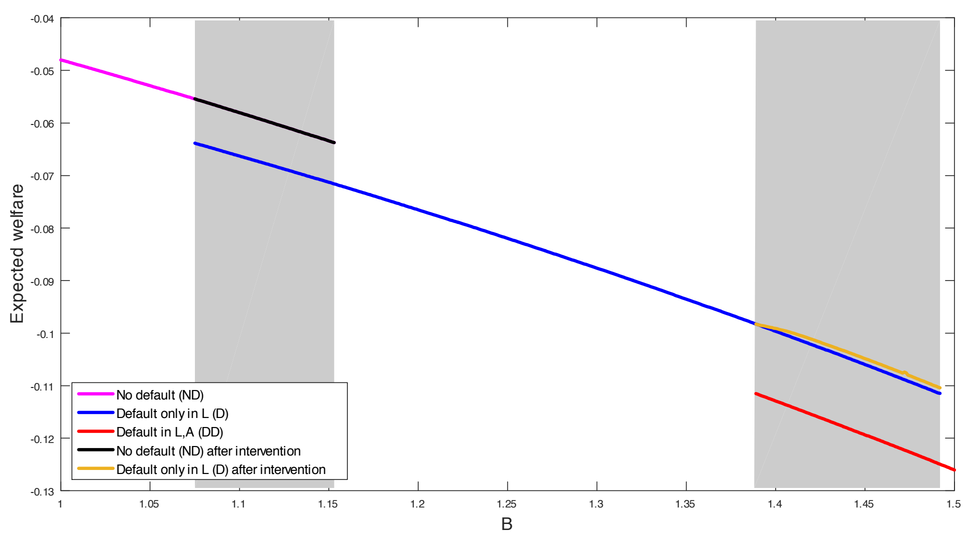

Figure 3a
Minimum Interventions required to eliminate the non fundamental equilibrium
Case of budget separation with a binding central bank constraint

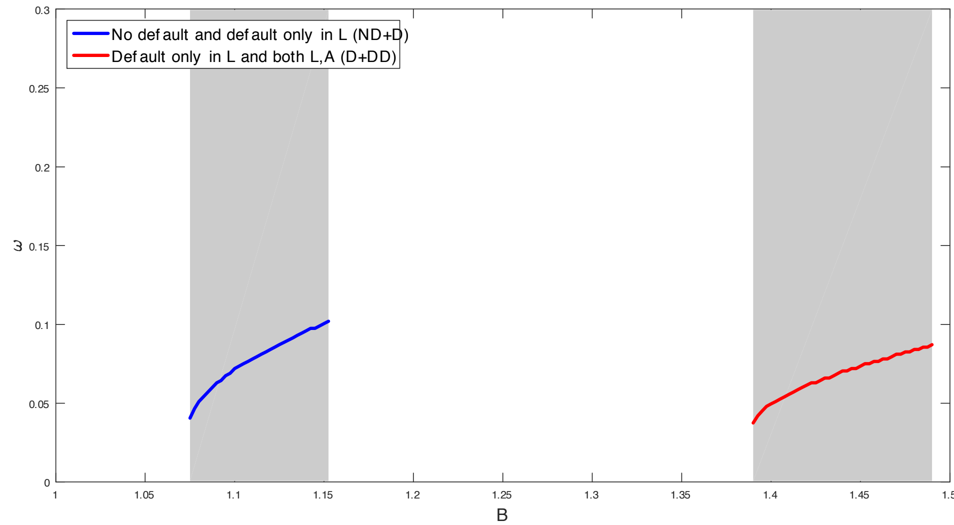

Figure 3b
Welfare with no backstop and with a minimum-intervention backstop
Case of budget separation with a binding central bank constraint