UNIVERSITY OF CAMBRIDGE
Faculty of Economics

Institute for New Economic Thinking

Cambridge-INET Institute

Cambridge-INET Working Paper Series No: 2016/11

Cambridge Working Paper Economics: 1645

STEP AWAY FROM THE ZERO LOWER BOUND:

SMALL OPEN ECONOMIES IN A WORLD OF SECULAR STAGNATION

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(Revised - February 7, 2018)

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Step away from the zero lower bound: Small open economies in a world of secular stagnation

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Abstract

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Keywords: Monetary policy, zero lower bound, deflation, depreciation, beggar-thy-neighbour, capital controls

JEL-Codes: F41, E62

*Corsetti: Cambridge University, Cambridge-INET and CEPR. Mavroeidi: Bank of England. Thwaites: Bank of England. Wolf: University of Vienna. We thank our discussants John Leahy and Roberto Piazza, and the seminar participants in 2016 Hydra Conference, the 2016 Conference at the Bank of Italy, and 2nd International Conference in Applied Theory, Macro and Empirical Finance, May 6-7, 2016, University of Macedonia, the European Central Bank, the University of Cambridge and the University of Rome Tor Vergata. We thank Samuel Mann for superb research assistance. Corsetti gratefully acknowledges support by the Centre for Macroeconomics, and the Keynes Fellowship at Cambridge University. Wolf gratefully acknowledges financial support by the German Science Foundation (DFG) under the Priority Program 1578, as well as by the Graf Hardegg Research Foundation. The views expressed in this paper are our own, and do not reflect those of the Bank of England or its MPC, or any institution to which we are affiliated.

1 Introduction

Real interest rates fell to historic lows around the industrialised world in the two decades prior to the Global Financial Crisis of 2008, and fell even further thereafter. Unemployment rose during the crisis, and remains high in several major economies. Leading observers have raised concerns that this may reflect "secular stagnation" (Summers (2013), echoing Hansen (1938))—a durable, possibly permanent fall in the natural interest rate to levels too low to be attained with existing (low) inflation targets, hence giving rise to chronic unemployment (Eggertsson et al. (2017)). In the leading interpretation, secular stagnation is the result of an "asset shortage": because of financial distortions in the form of credit constraint on economic agents with the high marginal propensity to spend, the desired savings of the economy at full employment cannot be absorbed by the stores of value available in the domestic economy at positive interest rates.[1]

In this paper, we study the implications of global secular stagnation for macroeconomic dynamics and the trade-offs faced by policymakers in a small open economy that is integrated in the world goods and financial markets. Open economies can in principle alleviate asset shortage by acquiring foreign assets and depreciating their currencies. Eggertsson et al. (2016) (henceforth EMSS) and Caballero et al. (2016) (henceforth CFG) study a global equilibrium in which the negative spillovers from such developments can actually spread secular stagnation worldwide. Complementing these analyses, we study the domestic equilibrium and issues in economic stabilization from the vantage point of a country that simultaneously faces weak external demand due to the global slump and, as long as it remains financially open, an inefficiently high real interest rate, pinned down by the equilibrium conditions in the international capital markets. To carry out our analysis, we build an overlapping-generations framework that combines the features stressed by EMSS and CFG. In particular, we specify an economy that has some monopoly power over its terms of trade, but is otherwise small enough not to affect the policy behavior and the macroeconomic equilibrium in the rest of the world.

Our main contributions are as follows. First, we show that under plausible conditions a small open economy in a world of chronic deflation and deficient demand can be in either of two disjoint steady-state equilibria—an underemployment steady state symmetric to the rest of the world, and a full-employment equilibrium with inflation at target, trend nominal depreciation, a permanently weaker real exchange and the accumulation of net foreign assets. In the full-employment equilibrium, trend nominal depreciation is required to insulate domestic

[1] To the extent that the domestic monetary policy framework is unable to accommodate the fall in the real interest rate required to bring supply and demand for assets back into line when the economy operates at full employment, the margin of adjustment becomes a reduction in output.

inflation from the deflationary drift abroad (associated to the global secular stagnation). A depreciated real exchange rate is required to maintain a high foreign demand for domestic output and support full employment. Accumulation of net foreign assets is required to absorb higher national savings, to the extent that these are generated by income and balance sheet effects from escaping secular stagnation.

Second, we study welfare gains from escaping global secular stagnation as the result of two opposing forces. On the one hand, when pursuing domestic full employment, domestic incomes and lifetime utilities rise with increasing output at given terms of trade. On the other hand, for a given output path, real depreciation reduces purchasing power internationally and may exacerbate the financial constraints in the economy—in our baseline, these constraints limit the ability of the young generation to borrow and consume in anticipation of future income, whose value falls with real depreciation.[2] We find that the adverse income effects of real depreciation may be strong enough to reduce welfare overall. This is the case if trade elasticities are sufficiently low, as this increases the rate of real depreciation required to support full employment. Moreover, as further discussed below, beggar-thy-self effects also arise when countries attempt to escape stagnation relying on policies that address domestic distortions—and this independently of trade elasticities. Currency wars can therefore be *beggar thy self*, rather than *beggar thy neighbor*.[3]

Third, we characterize the macroeconomic dynamics along the transition from the underemployment to the full employment equilibrium. Since bringing the economy to full employment entails an increase in domestic inflation relative to the rest of the world, it follows that the nominal exchange rate must steadily depreciate—an echo of Svensson's "Foolproof Method" (Svensson (2001))—and therefore that escape is not possible without exchange rate flexibility. However, capital mobility means that the real interest rate is pinned down in international bond markets, and hence is the same (and too high) in both the stagnation *and* the full employment equilibrium. This is in contrast to theoretical and policy accounts of business-cycle stabilisation, stressing the need for policy makers to set the real interest rate at its "natural" level to achieve full employment.[4]

[2] In the case in which lifetime utilities rise when moving from secular stagnation to full employment, in each generation losses when young are more than compensated by gains when middle-aged or old. To provide further insight on the role of balance sheet effects of depreciation in driving net foreign asset accumulation, in an extension of the model, we explore the implication of a different specification of the borrowing constraint on the young.

[3] For an analysis of beggar-thy-self depreciation in open economy macroeconomics, see Corsetti and Pesenti (2001).

[4] The transition path to full employment is different from the escape from liquidity traps in the standard New-Keynesian models. As expectations shift to the full employment equilibrium, the real exchange rate tends immediately to fall towards its new equilibrium level. This switches demand to the home economy and leads to net capital outflows. As output starts to recover, both nominal and real interest rates immediately rise in

Three additional sets of results concern policy implications, with focus on capital controls, measures that address domestic financial distortions and hence raise asset supply, and monetary stabilization. Starting with the capital controls, it is well understood that by preventing cross-border asset trade, domestic policy makers regain control over domestic real interest rates, as these are no longer determined in international financial markets via international asset arbitrage. In global stagnation, however, control over the real rate may not helpful *per se*, because the natural rate under financial autarky may still be negative and unachievable by conventional monetary policy because of the zero lower bound on nominal rates. The question is whether, under such circumstances, the country is nonetheless able to achieve full employment by boosting its competitiveness and external demand via depreciation. We show that the answer is negative. Unless the financial autarky natural rate is sufficiently high, closing the economy financially and hence barring the possibility to accumulate foreign assets actually eliminates the full-employment steady state: there is no equilibrium in which the increase in external demand following real depreciation is sufficient to rebalance the secular stagnation distortions. This result provides an instructive caveat on the desirability of capital controls as a way to avoid secular underemployment. In a global secular stagnation, capital controls are effective only if complemented by policies that raise domestic asset supply, with sufficiently positive effects on the domestic natural rate.

Although measures that increase the supply of domestic assets are desirable under financial autarky, we show that domestic policymakers have no incentive to pursue them under financial integration. This is so not only because these measures are neutral in terms of domestic output and inflation—again, this follows from the fact that real rates are not sensitive to domestic savings imbalances, but determined internationally. More importantly, to the extent that they raise the saving-absorption capacity of the economy, they lead to capital inflows, and thus to lower external surpluses and a weaker real exchange rate. By the logic of the "transfer problem" in international economics (after Keynes (1929)), aggregate consumption falls, detrimental to national welfare. Hence under financial integration, measures to address domestic financial distortions turn out to be *beggar thy self*.

Similar considerations apply to policies attempting to counter low inflation and/or lackluster growth by relaxing the effective lower bound (ELB) on nominal interest rates.[5] We show that, in financially integrated small open economies, a lower ELB is generally not benefi-

anticipation of further nominal and real depreciation.

[5]The Danmarks Nationalbank (DN), the European Central Bank (ECB), the Sveriges Riksbank and the Swiss National Bank (SNB) all cut interest rates to below zero during the period from mid-2014 to early 2015. The Bank of Japan (BoJ) and Bank of England applied similar policies. See Bech and Malkhozov (2016)) and BoEMonetary Policy Summary, 4 August 2016, http://www.bankofengland.co.uk/publications/minutes/Documents /mpc/mps/2016/mpsaug.pdf

cial to welfare. The intuition resonates with the neo-Fisherian argument recently made by Schmitt-Grohé and Uribe (2017) and Cochrane (2017), among others. Namely, with real rates determined internationally, lower nominal rates must eventually entail trend exchange rate appreciation, lower inflation, and thereby a more negative output gap. Hence again, a measure aimed to adress domestic distortions turns out to be *beggar thy self* under financial integration. Yet, we also emphasize that, should the ELB become binding *along the escape path*, relaxing it can help to ease the transition to full employment.[6]

Two points of contact with the literature are worth highlighting here. In line with the recent literature reviving secular stagnation, we focus on the case where the world is *permanently* in a liquidity trap with negative natural rates. Yet, our analysis naturally relates to contributions that study open economies in a *temporary* liquidity trap, either using a two-country model (Cook and Devereux (2013) and Acharya and Bengui (2017)), or taking the perspective of a small open economy (Corsetti et al. (2017)). Exchange rate adjustment is key in all these studies and the present one to moderate the negative effects of a global liquidity trap for the domestic economy. In particular, the nominal exchange rate needs to depreciate persistently to allow domestic inflation to rise above the world deflationary drift. But in contrast to our analysis, in these frameworks the domestic real rate must fall to boost demand.[7]

As we focus on a small open economy, we do not discuss the cross-border spillovers that may stem from different domestic adjustment paths—amply discussed by Eggertsson et al. (2016) and Caballero et al. (2016) in their two-country frameworks. By no means do we intend to downplay these issues. In particular, if a sufficient number of small open economies pursue the escape path depreciating their currencies, their joint behavior is likely to have a first-order effect on the world real interest rate and allocation. In this case, domestic policymakers might need to focus on domestic stimulus rather than relying solely on the exchange rate channel, while support from other policies, including structural policies and asset-supply policies, might be necessary to lift global natural rates. It is precisely because, as we have shown here, each small country has an incentive to pursue an individual escape path from stagnation—but not to raise the supply of its assets by addressing domestic financial distortions—that international policy coordination may be highly beneficial.

The remainder of this paper is organised as follows. Section 2 outlines the two-country model used in our analysis. Section 3 establishes that a full-employment steady state exists under

[6]In an extension of our model we show that, along the escape path, the exchange rate overshoots its long run level on impact if trade volumes are initially slow to respond. In this case, the nominal rate remains stuck at zero for a number of periods, such that relaxing the ELB would ease the transition to full employment.

[7]Since we do not model capital, we abstract from the issues in portfolio allocation in a liquidity trap, whereby agents may prefer holding monetary assets over equities, creating an endogenous negative feedback effect on real activities, see Den Haan et al. (2017) and Bacchetta et al. (2016).

financial integration, but also that this steady state may be welfare inferior (beggar thy self) to the stagnation steady state. It also studies how the economy escapes from the secular stagnation. Sections 4 and 5 turn to policies and discuss, respectively, financial autarky and asset supply policies, and alternative monetary policies. Section 6 concludes. The appendix includes proofs as well as model extensions and robustness analysis.

2 A model of a small open economy in a global depression

We consider a model of a small open economy (SOE) under perfect foresight, integrated with the rest of the world (ROW) in goods and asset markets. The framework is one of overlapping generations (OLG), as in Eggertsson et al. (2016) and Eggertsson et al. (2017). The SOE and the ROW each specialize in the production of one good, but consume all goods. With home bias in demand, this leads to fluctuations in the terms of trade and the real exchange rate. Furthermore, international asset markets are incomplete. In both economies, households live for three periods: young, middle-aged, and old. All the income accrues to the middle aged, such that the young borrow to be able to consume subject to a borrowing constraint, whereas the old consume their savings from last period. Labor supply is exogenous, and wages are assumed to be downwardly sticky.

2.1 Households

Denoting $0 < \beta < 1$ the time-preference rate and $1/\rho > 0$ the intertemporal elasticity of substitution, domestic households maximize

$$\max_{\{C_t^y, C_{t+1}^m, C_{t+2}^o\}} \left\{ \frac{(C_t^y)^{1-\rho}}{1-\rho} + \beta \frac{(C_{t+1}^m)^{1-\rho}}{1-\rho} + \beta^2 \frac{(C_{t+2}^o)^{1-\rho}}{1-\rho} \right\}$$

subject to the sequence of constraints

$$\begin{aligned} P_t C_t^y &= -B_t^y \\ P_{t+1} C_{t+1}^m &= P_{H,t+1} Y_{t+1} + (1+i_t) B_t^y - B_{t+1}^m \\ P_{t+2} C_{t+2}^o &= (1+i_{t+1}) B_{t+1}^m \\ -(1+i_t) B_t^y &\leq D_t P_{H,t+1}. \end{aligned}$$

Here, P_t is the consumer price index (CPI—the price of domestic consumption), $P_{H,t}$ is the producer price index (PPI—the price of domestic output), i_t is the nominal interest rate, and D_t is the borrowing constraint faced by the young. Furthermore, $C_t^i, i \in \{y, m, o\}$, represent consumption by the young, middle-aged, and old, respectively, and $B_t^i, i \in \{y, m\}$ are bond holdings/savings by the young and middle-aged, respectively. We assume that bond holdings consist of three assets $B_t^i = B_t^{n,i} + P_t B_t^{r,i} + \mathcal{E}_t B_t^{x,i}$, where $\mathcal{E}_t$ is the nominal exchange rate (the price of foreign currency in terms of domestic currency). Hence households face a domestic-currency bond, a CPI-indexed (real) bond and a foreign-currency bond.

Without loss of generality, we will focus on equilibria where only the domestic-currency bond is traded ($B_t^i = B_t^{n,i}$),[8] and where the young borrow all the way up their borrowing constraint. This implies

$$C_t^y = -B_t^y \frac{1}{P_t} = \frac{D_t}{1+i_t}\frac{P_{H,t+1}}{P_t} = \frac{D_t}{1+r_t}\frac{P_{H,t+1}}{P_{t+1}}, \tag{1}$$

where $(1+r_t) \equiv (1+i_t)P_t/P_{t+1}$ is the domestic consumption-based real interest rate (see below). Note that a rise in the CPI relative to the price of domestic income exacerbates the borrowing constraint on the young. This is because we define the borrowing limit in units of output—i.e., the nominal value of the constraint is deflated by PPI (equivalent to the GDP deflator in our specification) rather than with the CPI. We motivate this by noting that the collateral the young can possibly pledge when borrowing is the output they will produce when middle aged. In the Appendix A.5, nonetheless, we discuss a variant of the model where the borrowing limit is defined in consumption units.

Because the middle-aged may trade three different assets, they must satisfy three Euler equations. The Euler equation for the CPI-indexed bond is

$$(C_t^m)^{-\rho} = \beta(1+r_t)(C_{t+1}^o)^{-\rho}.$$

The Euler equation for the domestic nominal bond, once combined with the Euler equation for the CPI-indexed bond, implies the Fisher equation

$$(1+r_t) = (1+i_t)\frac{P_t}{P_{t+1}}. \tag{2}$$

By the same token, the Euler equation for the foreign-currency bond implies the uncovered interest parity (UIP) condition

$$(1+i_t) = (1+i_t^*)\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}. \tag{3}$$

The UIP condition reflects a pricing equilibrium condition in international asset markets, because foreign-currency bonds are traded internationally at interest rate $(1+i_t^*)$.

In contrast, the old consume all their savings from last period, $C_t^o = (1+r_{t-1})(B_{t-1}^m/P_{t-1})$. By combining the previous equations, we obtain the equilibrium bond holdings of the middle-aged generation (stated in real terms)

$$\frac{B_t^m}{P_t} = \frac{1}{1+[\beta(1+r_t)^{1-\rho}]^{-\frac{1}{\rho}}}\left[\frac{P_{H,t}}{P_t}Y_t - \frac{P_{H,t}}{P_t}D_{t-1}\right], \tag{4}$$

[8] As we assume one period debt and perfect foresight, the denomination of debt is not relevant for the equilibrium allocation. For example, valuation effects of the exchange rate on foreign-currency-denominated debt are anticipated and priced in the cost of debt.

which coincide with the gross savings of the economy.

2.2 Goods market integration

We assume the domestic consumption basket is made up of domestically-produced and ROW-produced goods as follows

$$C_t^i = \left[(1-\omega)(C_{H,t}^i)^{\frac{\sigma-1}{\sigma}} + \omega(C_{F,t}^i)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \quad i \in \{y, m, o\},$$

where $C_{H,t}$ is demand for the domestically-produced, $C_{F,t}$ is demand for the ROW-produced good, where $0 < 1-\omega \leq 1$ is the degree of home-bias in consumption and $\sigma > 0$ the intratemporal (trade) elasticity of substitution.[9] Expenditure minimization leads to a relationship between the CPI and the PPI as follows

$$P_t = [(1-\omega)(P_{H,t})^{1-\sigma} + \omega(P_{F,t})^{1-\sigma}]^{\frac{1}{1-\sigma}}, \tag{5}$$

where $P_{F,t}$ is the price of the ROW-produced good (expressed in terms of domestic currency). We assume the law of one price holds at the level of each good

$$P_{H,t} = \mathcal{E}_t P_{H,t}^* \quad P_{F,t} = \mathcal{E}_t P_{F,t}^*,$$

where an asterisk indicates variables in the ROW (here: the price of the domestic and the ROW-produced good, expressed in terms of foreign currency). Then, the domestic terms of trade (the price of imports in terms of exports) are given by

$$S_t = \frac{P_{F,t}}{P_{H,t}}, \tag{6}$$

whereas the real exchange rate (the price of ROW-consumption in terms of domestic consumption) obtains from

$$Q_t = \frac{\mathcal{E}_t P_t^*}{P_t}. \tag{7}$$

2.3 Firms and labor market

Output is produced using labor from residents of the respective country, according to

$$Y_t = L_t^{\alpha},$$

where $0 < \alpha \leq 1$. Optimality requires $W_t = P_{H,t}\alpha L_t^{\alpha-1}$. As in Eggertsson et al. (2017), we assume that nominal wages are downwardly sticky of degree $0 \leq \gamma \leq 1$

$$W_t = \max\{\tilde{W}_t, W_t^{flex}\},$$

[9] Under $\omega = 1$, domestic households would consume exclusively foreign goods, such that the real exchange rate would be one at all times. A value for $\omega < 1$ is sufficient to insure equilibrium real depreciation, as long as the trade elasticity σ is smaller than plus infinity.

where $W_t^{flex} \equiv P_{H,t}\alpha(L^f)^{\alpha-1}$ (here, $L^f > 0$ denotes full employment), and $\tilde{W}_t$ is a wage norm which solves

$$\tilde{W}_t = \gamma W_{t-1} + (1-\gamma)W_t^{flex}.$$

2.4 Phillips curve and monetary policy

Following Eggertsson et al. (2017), we use the production function together with the wage norm to derive a Phillips curve of the form

$$Y_t = \left[\gamma \frac{Y_{t-1}^{\frac{\alpha-1}{\alpha}}}{\Pi_{H,t}} + (1-\gamma)(Y^f)^{\frac{\alpha-1}{\alpha}}\right]^{\frac{\alpha}{\alpha-1}} \tag{8}$$

whenever $\Pi_{H,t}$ $(:= P_{H,t}/P_{H,t-1}) < (Y^f/Y_{t-1})^{\frac{1-\alpha}{\alpha}}$, and

$$Y_t = Y^f \tag{9}$$

else, where $Y^f = (L^f)^\alpha$ is output under full employment. Notice that the Phillips curve is a function of current, not expected, inflation.

We posit that the monetary authorities target PPI inflation subject to a zero-bound constraint on nominal rates:

$$\Pi_{H,t} = \bar{\Pi} \ \text{ subject to } \ i_t \geq 0. \tag{10}$$

If inflation falls below target, nominal rates fall to zero:

$$i_t = 0 \ \text{ if } \ \Pi_{H,t} < \bar{\Pi}. \tag{11}$$

So, once the economy is in stagnation and nominal rates are at the zero lower bound, they are inelastic to current as well as target inflation: a change in the inflation target per se does not affect agents' expectations about the current and future monetary stance.

2.5 Market clearing

Because the domestic economy is small, the CPI in the ROW will correspond to the price of foreign output, $P_t^* = P_{F,t}^*$, from which follows

$$Q_t = \frac{P_t^* \mathcal{E}_t}{P_t} = \frac{P_{F,t}}{P_t} = S_t \frac{P_{H,t}}{P_t}. \tag{12}$$

Domestic goods market clearing is then given by

$$Y_t = (1-\omega)\left(\frac{P_t}{P_{H,t}}\right)^\sigma C_t + \omega S_t^\sigma Y_t^*, \tag{13}$$

where $C_t := C_t^y + C_t^m + C_t^o$ denotes aggregate domestic consumption. Furthermore, asset market clearing (stated in real terms) requires

$$\frac{NFA_t}{P_t} = -\frac{D_t}{1+r_t}\frac{P_{H,t+1}}{P_{t+1}} + \frac{1}{1+[\beta(1+r_t)^{1-\rho}]^{-\frac{1}{\rho}}}\left[\frac{P_{H,t}}{P_t}Y_t - \frac{P_{H,t}}{P_t}D_{t-1}\right], \tag{14}$$

where NFA_t is the domestic country's net foreign asset position. In each period, this is the difference between the savings of the middle-aged generation and the borrowing by the young (compare equations (1) and (4)). From the households' budget constraint, the flow budget of the domestic country is

$$C_t + \frac{NFA_t}{P_t} = (1+r_{t-1})\frac{NFA_{t-1}}{P_{t-1}} + \frac{P_{H,t}}{P_t}Y_t. \tag{15}$$

2.6 The domestic natural real rate

In a financially integrated small open economy, as we will see further below, the real interest rate is determined in the international bond markets, irrespective of whether there are nominal rigidities at national level. Hence, under full financial integration, there is no distinct measure of the country-specific *natural* real interest rate, customarily defined as the real rate which prevails when prices and wages are perfectly flexible. However, for future reference, it is convenient to define a natural rate for a small open economy operating under financial autarky. To do so, assume the economy is closed to trade in financial assets such that the net foreign asset position equals zero at all times. We may then deduce the real rate that clears the asset market from equation (15)

$$\left(\frac{[(1-\omega)+\omega S_t^{1-\sigma}]}{[(1-\omega)+\omega S_{t+1}^{1-\sigma}]}\right)^{\frac{1}{1-\sigma}} \frac{D_t}{1+r_t^{\text{nat}}} = \left(\frac{1}{1+[\beta(1+r_t^{\text{nat}})^{1-\rho}]^{-\frac{1}{\rho}}}\right)\left[Y^f - D_{t-1}\right], \tag{16}$$

where we have used that, absent nominal rigidities, the economy operates at potential at all times ($Y_t = Y^f$). Note that the term multiplying the borrowing by the young is a function of the expected change in the terms of trade and therefore drops out in steady state. As a result, in steady state, the domestic natural rate is a decreasing function in domestic full-employment output as well as in the tightness of the domestic financial constraint.[10]

2.7 Rest of the world: secular stagnation at global level

In modeling the rest of the world, we assume symmetry in economic structure with the small open economy, except for size and policy parameters, including those potentially affecting the tightness of the borrowing constraints on the young.

[10] This is easiest to see in the case of log-utility, $\rho = 1$, where $1/(1+[\beta(1+r_t^{\text{nat}})^{1-\rho}]^{-1/\rho})$ becomes $\beta/(1+\beta)$. In this case $(1+r^{\text{nat}})$ is easily seen to be decreasing in Y^f, but increasing in D. A tighter domestic financial constraint means a lower D, i.e., D being closer to zero.

Assumption 1. *The small open economy and the rest of the world have the same degree of risk aversion, time preference, curvature of the production function, full-employment (per capita) output and degree of downward stickiness.*

Since the domestic economy is small, the rest of the world effectively is a closed economy, so we can draw on the main results by Eggertsson et al. (2017). Specifically, we posit that D^*, the borrowing limit imposed on the young, is sufficiently tight that the ROW is in a secular stagnation steady state. Inflation and output (Π^*, Y^*) then jointly solve

$$D^*\Pi^* = \left(\frac{1}{1+[\beta(\Pi^*)^{-(1-\rho)}]^{-\frac{1}{\rho}}}\right)[Y^* - D^*]$$

as well as

$$Y^* = Y^f\left(\frac{1-\gamma/\Pi^*}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}},$$

where we have used the fact that $i^* = 0$ such that $(1+r^*) = (\Pi^*)^{-1}$— the ROW counterpart of Fisher equation (2). It is easy to show that, in this steady state, both $\Pi^* < 1$ and $Y^* < Y^f$ and therefore that

$$(1+r^*) = (\Pi^*)^{-1} > 1.$$

Hence, in secular stagnation, the market rate is positive and therefore strictly higher than the natural real rate. As in equation (16), the latter is the real rate that would prevail absent nominal rigidities, implicitly defined in

$$\frac{D^*}{1+(r^{\text{nat}})^*} = \left(\frac{1}{1+[\beta(1+(r^{\text{nat}})^*)^{1-\rho}]^{-\frac{1}{\rho}}}\right)\left[Y^f - D^*\right].$$

The mechanism by which the economy finds itself trapped in stagnation is the same as in the analysis by Eggertsson et al. (2017). At the core of the problem is a tight budget constraint D^* for the young that drives down the natural real rate clearing the market for saving by the middle-aged to negative territory. A negative natural rate is not necessarily a problem if the central bank chooses a high enough inflation target. In this case, if markets coordinate expectations on the (high) inflation target, there is enough room for setting nominal policy rates sufficiently low, while still in positive territory, for the bond market to clear at the natural rate. Yet, even with the high enough target, markets could still coordinate expectations on the low inflation rate associated with stagnation, causing nominal policy rates to be constrained at their zero lower bound. In this equilibrium, the monetary stance is endogenously contractionary, causing the output gap to remain open on a permanent basis—thereby validating the adverse private sector expectations.[11]

[11] It is worth stressing that, in our model specification, prices are fully flexible. With competitive firms,

2.8 Equilibrium definition for the small open economy

For initial conditions $\{Y_{-1}, NFA_{-1}, P^*_{-1}, P_{-1}\}$ and exogenous $\{D_t\}$, and for given variables in the rest of the world $\{D^*, r^*, Y^*, \Pi^*, P^*_t = \Pi^* P^*_{t-1}, i^* = 0\}$, an equilibrium in the domestic economy is a set of sequences $\{C_t, Y_t, Q_t, S_t, P_{H,t}, P_{F,t}, P_t, \Pi_{H,t}, \Pi_t, \mathcal{E}_t, i_t, r_t, NFA_t\}$ that solve equations (2)-(3), equations (5)-(15) and the definitions for PPI and CPI inflation $\Pi_{H,t} \equiv P_{H,t}/P_{H,t-1}$ and $\Pi_t \equiv P_t/P_{t-1}$.

Throughout the paper, we will posit that the small open economy faces a global secular stagnation, a situation in which the world economy is permanently trapped in an equilibrium with deflation, zero nominal interest rates, and inefficient output gaps. From the perspective of the domestic economy, this means stagnating external demand, and inefficiently high real rates, via the equilibrium conditions in the international bond markets. We will explore how a small open economy can cope with this situation, first looking at its steady states, then at transition dynamics, then at the effects of domestic asset supply and monetary policies.

3 Escaping a global secular stagnation

In our analysis, we explicitly focus on a situation in which the ROW is permanently depressed. We now show that, despite the resulting low demand and inefficiently high real rates internationally, a small open economy may nonetheless operate at full employment.

In this section, for ease of exposition, we present this result conditional on the domestic economy and the ROW sharing the same parameters in terms of policy, including the inflation target $\bar{\Pi}$ and, more importantly, the borrowing constraint on the young D_t—in addition to our assumed symmetry in terms of the underlying technology (Assumption 1). However, we stress that this assumption is by no means necessary to derive our multiplicity results that follow: in Sections 4-5 we explore the effects of systematically departing from our assumption of a symmetric conduct of policy.[12]

thus, prices are equal to marginal costs—firms are on their labor demand schedule. In contrast, because of nominal wage rigidities, workers are not necessarily on their (vertical) labor supply. An insufficient demand in the good market at the equilibrium real interest rate reduces the demand for labor below full employment. An important theoretical property of the model is that, given the Phillips curve above, wage flexibility *per se* would not help addressing underemployment. As long as a response of current wages to low demand does not affect expected inflation, scaling down labor costs (and thus prices) proportionally today and in the future cannot have real effects in equilibrium.

[12] For example, this implies that the domestic economy may find itself in full employment even when its own distortions are tighter than in the ROW, hence potentially conducive to secular stagnation. At the opposite extreme, as we emphasize in Section 4, with integrated goods and capital markets, a small open economy may find itself trapped in stagnation in spite of relatively sound policy fundamentals, i.e., mild financial distortions and a high inflation target.

3.1 Two steady states

Under our symmetry assumptions, as the ROW is in secular stagnation, there exists a symmetric steady state in which the small open economy is also in stagnation. But, as the following proposition states, with integrated world goods and bond markets, such an equilibrium for the home economy is not unique. There is also a second steady state, characterized by full employment, inflation on target, and a strictly positive nominal interest rate.

Proposition 1. *The small open economy can find itself in one of two steady states, one with stagnation, the other with full employment. Under our symmetry assumptions, the stagnation steady state is characterized by purchasing power parity $Q = S = 1$ and zero net foreign assets $NFA = 0$. Conversely, the full employment steady state has $Y = Y^f$, inflation on target, $(1 + i) = \bar{\Pi}/\Pi^* > 1$, and is associated with trend nominal depreciation, permanently depreciated real exchange rates, $Q > 1$ and $S > 1$, and a larger (strictly positive) stock of net foreign assets $NFA > 0$.*

Proof. See the Appendix A.2.[13] □

Proposition 1 states that a small open economy *can* find itself in a full employment steady state even if the rest of the world is not. Intuitively, it can "export the problem", in that domestic full employment requires a weaker real exchange rate, so to increase demand for the country's output. Open capital markets allows the country to export its excess saving, and build up net foreign assets.

3.2 Full employment requires depreciation and foreign asset accumulation

To dig deeper into the economics of our first proposition, observe that, by the Phillips curve (8), full employment is not attainable at a negative inflation rate. In the full employment steady state, domestic inflation must necessarily be higher than in the secular stagnation equilibrium. From purchasing power parity (7) and the fact that the real exchange rate must be constant in steady state, this also implies that the small open economy must experience trend nominal depreciation, at a rate determined by the ratio of the domestic inflation target and the negative inflationary drift abroad:

$$\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} = \bar{\Pi}/\Pi^* > 1. \tag{17}$$

It follows that a full employment steady state is feasible only under exchange rate flexibility. If the domestic monetary authorities pursue a fixed exchange rate target, they must accept

[13] The existence proof contains the general case of asymmetry in the policy parameters which, as explained above, is explored in detail in Sections 4-5.

an inflation rate identical to the international one, which is negative and associated with stagnation.

However, a higher inflation rate does not imply a lower real interest rate. With integrated financial markets, the real interest rate in the domestic economy is pinned down by its ROW counterpart up to (expected) changes in the real exchange rate

$$(1 + r_t) = (1 + r^*)\frac{Q_{t+1}}{Q_t}, \tag{18}$$

where we have combined the domestic Fisher equation (2) and its foreign counterpart, the UIP condition (3) and purchasing power parity (7). Because in steady state the real exchange rate must be constant, it follows that $(1 + r) = (1 + r^*)$. As a result, from the perspective of the small domestic economy, the steady state domestic real rate is exogenous and therefore the *same* in both the employment and stagnation steady state.

This has two key implications. First, since the real rate cannot decline in the small domestic economy, full employment is not achievable without a boost in demand for domestically produced goods, which under imperfect goods' substitutability requires real depreciation. Second, because the real rate is identical in the two steady states, but the rate of inflation is strictly higher in the full employment than in the stagnation steady state, the domestic nominal rate must lift off zero, from Fisher equation (2).

The country accumulates net foreign assets as the flip-side of exporting its full employment domestic production. In steady state we have

$$[(1-\omega) + \omega S^{1-\sigma}]^{\frac{1}{1-\sigma}}\frac{NFA}{P} = -\frac{D}{1+r^*} + \frac{1}{1+[\beta(1+r^*)^{1-\rho}]^{-\frac{1}{\rho}}}\left[Y^f - D\right], \tag{19}$$

where we have used that $(1 + r) = (1 + r^*)$ and re-written the relative price P/P_H by using equation (5) and the definition of the terms of trade (6). The right hand side of this expression defines the "excess savings" of the domestic economy under full employment, which are a function of exogenous fundamentals and policy parameters, but independent of international relative prices (the terms of trade or, equivalently, the real exchange rate). As the left hand side shows, these excess savings are absorbed by a positive net foreign asset position. Note the size of the required net asset adjustment crucially depends on the equilibrium real depreciation, which in turn is a function of trade elasticities. In general, under a high trade elasticity, the real exchange rate may move by only little to have significant effects on external demand: the change in the net asset position will be the main margin of adjustment. For lower trade elasticities, the real exchange rate movements will be larger: the adjustment in net foreign asset will correspondingly be smaller.

3.3 The beggar-thy-self effects of real depreciation

The real exchange rate depreciation associated with full employment produces contrasting effects on demand and welfare. On the one hand, it makes domestically produced goods cheaper, redirecting demand towards them. This is what raises the level of domestic production to potential. On the other hand, for any given output, it reduces the international value of domestic production. Everything else equal, this makes domestic residents poorer. With incomplete markets, the income effects from depreciation are not mitigated by risk diversification via financial markets. They can actually become stronger than the substitution effects (see Corsetti et al. (2008b) and Corsetti et al. (2008a)).

Depending on the extent of real depreciation that is required to bring the economy to potential, it is possible that the full employment steady state characterized in Proposition 1 is welfare deteriorating. We state this in the following Proposition.

Proposition 2. *Under our symmetry assumptions, for a trade elasticity that is sufficiently low—below approximately $\sigma < 1/(2-\omega)$—the full employment steady state produces lower levels of domestic consumption despite higher levels of domestic production. The escape to full employment in this case is beggar thy self.*

Proof. See the Appendix A.4. □

The equilibrium rate of real depreciation depends on the degree of substitutability between domestic and foreign production, as captured by the trade elasticity. Intuitively, as domestically produced and imported goods become less substitutable, larger swings in relative prices are required in order to redirect demand from one to the other. Hence with a lower trade elasticity, the extent of real depreciation in the escape increases. At the same time, real depreciation, all else equal, reduces the purchasing power of domestic households internationally. In the case of symmetric policy parameters, Proposition 2 states that below the threshold of $\sigma < 1/(2-\omega)$, this effect is strong enough for domestic consumption to actually *fall*—notwithstanding the rise in domestic production.

The main lesson from our analysis is apparent. While a small open economy can escape global secular stagnation via real depreciation, strong income effects from the adverse movements in the terms of trade reduce the net benefit of the escape—in principle, welfare may even fall when moving to the full employment steady state. Unless there are large utility gains from preventing involuntary unemployment, of the kind advocated by Schmitt-Grohé and Uribe (2016), a "currency war" can then be beggar thy self.

3.4 Transition dynamics: the path to full employment

We have established the existence of a full-employment steady state for the small domestic economy. In this subsection, we rely on numerical analysis to study how the domestic economy escapes stagnation by solving explicitly for the transition path to full employment. We design our exercise taking advantage of the fact that the equilibrium in the model is globally indeterminate, in that the dynamics can be affected by "sunspot shocks". Starting from the stagnation steady state, we posit that a change in beliefs triggers transition to the full employment steady state.[14] The sunspot shock is modeled as once-and-for-all non-anticipated shock, after which there is no further uncertainty. Here we consider the escape of a country that is not at the risk of beggar thy self, as trade elasticities are sufficiently high.

The transition to full employment may be summarized as follows. The country first experiences a large upfront nominal and real depreciation, and more gradual depreciation thereafter, as well as a temporary rise in both nominal and real interest rates relative to their initially depressed steady states. As the economy gradually recovers, it runs current account surpluses, in the process accumulating net foreign assets. In the long run, the country finds itself in the new steady state characterized by the *same* real interest rate as under stagnation, however by a permanently depreciated real exchange rate, trend nominal depreciation, as well as a strictly larger stock of net foreign assets.

As the economy recovers, consumption of the young actually *declines*, while the consumption of the middle-aged and old increases. The latter reflects that, because trade elasticities are high, the national income valued at international prices increases along the escape. In contrast, the young are negatively affected by the escape, for two reasons. First, the real exchange rate depreciation deteriorates their balance sheet: the collateral against which the young borrow becomes less valuable in terms of their consumption. Second, the real interest rate rises, such that their borrowing becomes more expensive. Still, for each generation, the contraction in consumption when young is more than compensated by the gain in consumption when middle-aged and old: a "jam-tomorrow" result by which there are net gains over their life-cycle, and overall the escape is welfare improving.[15,16]

[14] For a discussion of global indeterminacy, see earlier models of inflation targeting under a zero lower bound constraint (for example Benhabib et al. (2001)). Benhabib et al. (2002) discuss how global indeterminacy can be ruled out in a closed economy setting with a zero lower bound. Cochrane (2011) critically reviews papers trying to resolve global indeterminacy.

[15] Note that this result also depends on our modeling utility as deriving only from consumption. In contrast, if the higher labor supply associated with higher production at full employment carries utility losses, this result may be overturned. In contrast, if there are utility gains from avoiding involuntary unemployment, of the kind advocated in Schmitt-Grohé and Uribe (2016), this result may be further strengthened.

[16] In one key dimension the escape path discussed is reminiscent of Svensson's "Foolproof Method" to escape a liquidity trap in an open economy: what jumps-start the economy is an increase in inflation expectations associated with a weakening of the nominal and real exchange rate (Svensson (2001) and Svensson (2003)).

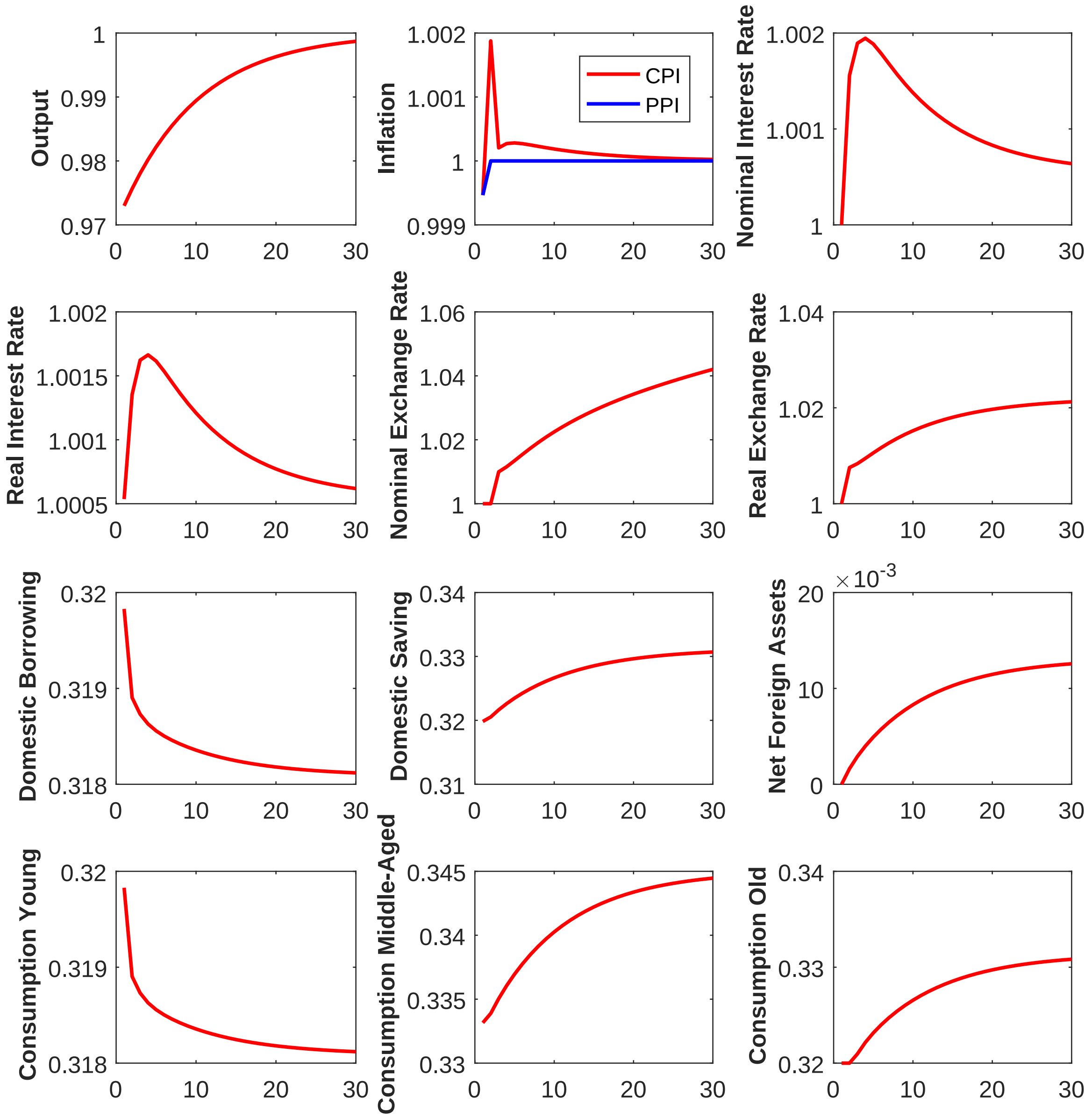

Figure 1: Transition to full employment, w/o beggar thy self. The parameters are taken from Eggertsson et al. (2017); we are using $\beta = 0.96, \alpha = 0.85, \gamma = 0.9, \rho = 1, \bar{L} = 1, D = 0.32, \bar{\Pi} = 1$. For the parameters that are not in their study, we are using an import share of $\omega = 0.2$, as well as a trade elasticity of $\sigma = 1$.

In this context, the escape path is different when the economy is affected by beggar thy self. In the Appendix A.4 we show the equivalent of Figure 1 in such an economy. The negative welfare implications of the escape are illustrated by the fact that the consumption of all generations drops along the transition. Interestingly, in this case, the savings by the middle-aged contract along with the borrowing by the young. Yet, capital outflows are still positive because the contraction of domestic borrowing outpaces the contraction of domestic saving. In the Appendix A.4, we also show the escape for an economy in which trade elasticities are initially low, but rising along the transition to the new steady state.[17] An intriguing novel result is that, with trade elasticities changing over time, the exchange rate *overshoots* in the initial phase of the escape. As a result, the lift-off of the nominal interest rate from zero is not immediate, but it remains stuck at its zero-bound for a number of periods.

3.5 Equilibrium determinacy

A notable result in our analysis is that the the full employment steady state is locally determinate, whereas the stagnation steady state is locally indeterminate. This last result appears at odds with the claim by EMSS that the stagnation steady state is locally determinate when both economies are symmetrically constrained by their zero lower bound. We verify that there is no inconsistency: in the Appendix A.3 we include a discussion of the local stability properties of the symmetric stagnation steady state.

We now turn to the policy analysis. We thereby move away from our baseline specification assuming symmetry in policy parameters, and consider a host of measures concerning segmenting domestic from global financial markets, removing financial distortions hinging on the credit constraint on the young, and/or reforming monetary policy by either raising the inflation target or reducing the effective lower bound on nominal interest rates.

4 Capital flows and asset supply

In this section we analyze two issues related to capital account liberalization and financial regulation policies. First, we ask whether a country may be better off by closing down its

However, Svensson focuses on the escape from a temporary liquidity trap, assuming a unique steady state. In this context, because of the presence of nominal rigidities, a boost in demand and temporary depreciation requires a *fall* in real interest rates. In Figure 1, instead, depreciation and inflation are associated with a temporary *rise* in real rates—a common result in contributions modeling the escape from a liquidity trap as a transition between two, stagnation and full employment, steady states—see, e.g., Cochrane (2011) or Schmitt-Grohé and Uribe (2017).

[17] The motivation for this is the empirical evidence that trade elasticities may vary at different horizons, see for example Ruhl (2008) and Crucini and Davis (2016). More generally, going out of secular stagnation may be associated with a redefinition of the country's patterns of trade, which may be reflected in a changing trade elasticity.

border to trade in assets. This is tantamount to ask whether trade openness may help a country escape secular stagnation without changing its external position. Second, we study the effects of loosening domestic credit conditions via policies that foster access to financial resources by constrained agents.

4.1 Restricting capital mobility: financial autarky

Our first question is whether the country may be better off by giving up financial integration. A potential benefit is that absent trade in cross-border assets, the UIP condition (3) does not need to hold in equilibrium. As a result, also the UIP condition in real terms (18) is not an equilibrium condition, such that domestic policy makers re-gain control over the domestic real rate of interest. Hence the country may expect to raise the demand for its output up to potential via a combination of lower real rates (below the international level) and a depreciated real exchange rate—without any need to run current account surpluses. Our model yields a striking negative result, stated in relation to our baseline symmetric economy.

Proposition 3. *Under our symmetry assumptions, consider the case in which the small open economy is in financial autarky such that it trades goods internationally but capital markets remain closed and hence $NFA_t = 0$ at all times, implying that the UIP condition* (3) *does not need to hold in equilibrium. The secular stagnation steady state is now unique: the multiplicity of equilibria characterized in Proposition 1 disappears.*

Proof. Consider equation (14) *and impose that $NFA_t = 0$. In steady state, this equation can be written as*

$$\frac{D}{1+r} = \frac{1}{1 + [\beta(1+r)^{1-\rho}]^{-\frac{1}{\rho}}}(Y - D).$$

Imposing full employment $Y = Y^f$, this equation is solved by $r = r_{nat}$—see equation (16)*. Because of our symmetry assumptions with the ROW, r^{nat} is sufficiently negative by assumption, such that $r = r^{nat}$ is incompatible with the inflation target and the zero lower bound constraint $i \geq 0$.* □

In financial autarky, escape from secular stagnation via depreciation is a balancing act between raising foreign demand and containing internal balance sheet effects. On the one hand, higher foreign demand moves the economy to full employment, which raises domestic savings. On the other hand, balance sheet effects on the young reduce the domestic borrowing. We have seen above that, in an economy open to asset trade, the consequence are capital outflows. In contrast, in a financially closed economy, the consequence is an equilibrium drop in the domestic real interest rate. Yet, as demonstrated in the proof of Proposition 3, for this drop in the real interest rate to restore full employment, its size would have to be so large that the implied nominal interest rate, under the given inflation target, would violate the zero lower bound constraint. Hence in our baseline specification, the balancing act from real depreciation resolves against full employment.

This line of arguments suggests that the equilibrium could be sensitive to how responsive the households' balance sheet is to real depreciation. To provide insights on this, in the Appendix A.5 we specify the credit constraint on the young in terms of consumption rather than in terms of output units. In the so-modified model specification, the borrowing by the young hence the savings-absorption capacity of the economy does not fall with a weakening currency. As a result, the real interest rate needs to fall by less (not all the way to the natural rate) to bring asset demand back in line with asset supply, such that the implied nominal interest rate may remain positive. Notably, in this case the escape from stagnation becomes more likely when the income effects from depreciation are strong, since, as the economy recovers from stagnation, the desired savings by the middle-aged rise by little or may even fall (recall the discussion in Section 3.4).

Proposition 3 is derived under symmetry of economic conditions in our SOE and the ROW. By the logic of this proposition, however, it should be clear that closing the economy financially can be instrumental to maintain full employment if the small open economy operates in the parameter space outside the secular stagnation, namely, if D_t is high enough that the domestic natural rate is non-negative (see also the next subsection).

Under this asymmetric model specification, the domestic economy would be fundamentally sound, yet, because the stagnation steady state still exists (see the proof of Proposition 1 in the Appendix A.2), at risk of *importing* global secular stagnation under financial integration. As long as the domestic natural rate is non-negative, imposing capital controls can therefore shield the economy from being drawn into stagnation alongside the ROW. Intuitively, by de-linking domestic from international financial markets, with enough domestic saving-absorption capacity at full employment, monetary policy is able to stabilize the economy at positive nominal interest rates.

4.2 Relaxing the domestic financial constraints under financial integration

Our discussion above suggests that, under financial autarky, the domestic policymakers have a strong incentive to ease the distortions in the domestic credit conditions, as a way to raise the domestic natural rate. Indeed, in line with closed economy analysis of secular stagnation (Eggertsson et al. (2017)), under financial autarky policies that boost the amount of lending to the young not only increase their consumption demand, but also reduce the desired savings by the middle-aged.[18]

This is in sharp contrast with the effects of such policies under financial integration. As we have argued above, in this case domestic real rates are determined internationally via equation (18) and are therefore insensitive to domestic savings imbalances. As a consequence, a policy of financial deepening cannot but be neutral in terms of domestic real rates in steady state. Then, from the Fisher equation (2), its ROW counterpart, and the fact that both economies are at the zero lower bound, the rates of deflation in the two economies must be synchronized in steady state, too. In turn, from the Phillips curve (8), the steady state output gaps in the two economies must also be perfectly aligned.

From this does not follow, however, that financial deepening is neutral in terms of the equilibrium allocation. Precisely, a rise in D_t raises domestic real borrowing B_t^y/P_t via equation (1), reduces domestic real saving B_t^m/P_t via equation (4), and hence reduces the net foreign asset position. Intuitively, as the young borrow more while the middle-aged save less, the economy experiences capital inflows: domestic households end up holding less foreign assets. As a consequence, income generated from foreign assets is lower in the new steady state, such that the real exchange rate depreciates and domestic consumption falls (in the Appendix A.6 we show these results analytically).

These effects are best understood in light of the "transfer problem", after Keynes (1929). The main issue concerns the effects on relative prices of transferring purchasing power from one country to another: in our model, a lower net foreign asset position in steady state can be interpreted as a domestic transfer to the ROW. It is well known that, with home bias in consumption, the country that transfers resources experiences a fall in demand for its own output (as foreigners will demand proportionally more of theirs). Equilibrium then requires a weaker real exchange rate. Moreover in the present context, because output is unchanged with the transfer (as argued earlier on), only the income effect from the depreciation remains, such that the depreciation—even for high trade elasticities—unambiguously reduces domestic consumption. Therefore, paradoxically, even if neutral with respect to aggregate output, an increase in the supply of domestic assets ends up being *beggar thy self*.[19],[20]

[18]The latter follows from the fact that, if the household accumulates greater debts while young, it has fewer assets to save from when middle-aged.

[19]By the logic of the transfer problem, increasing D_t has beggar-thy-self effects also conditional on the full employment equilibrium under financial integration. A subtle and unsettling result from our analysis is thus that, under financial integration, the looser is the credit limit on the young (the higher is D_t), the lower consumption is in *both* the full employment and the stagnation steady state. So, in neither steady state the SOE has any incentive to introduce policies loosening domestic financial distortions and supply more assets.

[20]The transfer problem also applies in the case of financial autarky. In this case, it can be shown that the beggar-thy-self effects of an increase in D_t depend on the trade elasticity as long as the country is below full employment (when D_t is not sufficiently loose). In particular, unless the elasticity is very low, the gains for consumption arising from the higher domestic output more than offset the loss of purchasing power

4.3 Implications for deregulation and liberalization

From the analysis above, we conclude that from the perspective of individual countries, relaxing the domestic borrowing constraints so as to raise the domestic natural rate thereby pursue full employment is effective only if complemented by capital control measures, insuring a high degree of financial autarky. For domestic stabilization purposes, deregulation and capital controls are complements.[21]

But while counterproductive from a domestic vantage point under full integration, measures that raise the supply of domestic assets are always desirable from a global perspective (see CFG and EMSS). Unfortunately, our results suggest that individual countries have the incentives to pursue asset supply policies only under financial autarky, in which case the increased domestic asset supply does not fuel the pool of globally available assets. Hence, the right incentives are at work in precisely the wrong conditions.

5 Reforming the monetary policy framework

A key result of the secular stagnation literature is that, with policy rates stuck at their lower bound, real interest rates endogenously rise above the market clearing level in steady state, depressing demand and generating persistent negative output gaps.

In a closed economy context, there is therefore an argument for either increasing the target inflation, or for lowering the effective lower bound (ELB) constraint, i.e. set negative policy rates, in an attempt to enhance the ability of the monetary authorities to stimulate demand. In this section, we discuss the effectiveness of either measure in a small open economy that is financially integrated in world capital markets in turn.

5.1 A higher inflation target

In a closed economy, raising the inflation target can be helpful to the extent that the target is set above the negative of the natural real rate. In this case, the full employment steady state necessarily exists—and is characterized by a strictly positive nominal interest rate. However, as discussed in Eggertsson et al. (2017), a revision of the target by itself does not rule out the stagnation steady state. In this sense, the measure is less effective as a tool for forward guidance than in economies where the duration of the zero lower episode is finite (Eggertsson

internationally due to the real depreciation.

[21] The effects of conventional fiscal policy are amply discussed in other contributions on secular stagnation (for example EMSS and Eggertsson et al. (2017)). While we do not discuss fiscal policy explictly, we note that, to the extent that such policies aim at increasing the domestic natural rate, their effects can be expected to be similar as those from asset supply policies.

and Woodford 2003). Building on our analysis in the previous section, it is easy to show that the same result extends to our small open economy in financial autarky.

Under financial integration, however, the policy's benefits are subtler. A higher target by itself does not affect the existence of the two steady states: the full employment steady state exists even for inflation targets that are sufficiently low for the stagnation steady state to be unique if the economy was financially closed. Moreover, the mechanism by which the full employment steady state exists does not rely on increasing the inflation target by enough to enable the monetary authorities to match the (negative) domestic natural real rate. Indeed, the domestic real rate *exceeds* the natural real rate even in the full employment steady state—and this is so independently of (any positive) domestic inflation target given that all asset prices are determined internationally.[22]

What *can be* altered via a change in the inflation target is the transition to full employment discussed in Section 3.4. Recall Figure 1 where, in the escape, per effect of shifting expectations, domestic policy makers hit the inflation target instantly, as the zero lower bound ceases to bind in the period of the lift-off. Thus with a higher inflation target, the recovery is accompanied by higher levels of inflation, with two key implications. On the supply side, from the Phillips curve (8), output recovers more quickly by relaxing the downward constraint on nominal wages at a faster pace.[23] On the demand side, the transition is fostered by faster depreciation towards the new steady state of the exchange rate.

5.2 A lower effective lower bound (ELB)

In a closed economy (or under financial autarky), lowering the effective lower bound (ELB) on policy rates helps alleviate secular stagnation. This is because, as the nominal rate falls, the real rate falls alongside the nominal rate thereby moving closer to the (negative) natural rate rate. As the gap between the real rate and its natural counterpart is reduced, the output gap narrows.

Under financial integration, this policy has no such effect, but is instead counterproductive. This is so by the very logic of the Neo-Fisherian argument proposed by, e.g., Schmitt-Grohé and Uribe (2017) and Cochrane (2017). For a given real rate, lower rates make inflation more negative, which exacerbates the underemployment problem. Formally, when the ELB

[22] A negative inflation target would not work, as the Phillips curve (8) in this case would imply a negative output gap. However, for any positive target, the size of the target itself is entirely neutral for the allocation. Namely, a higher target would merely be reflected in a higher nominal interest rate, and in a higher rate of trend nominal depreciation by condition (17).

[23] By repeating the experiment from Figure 1, but with a slightly higher inflation target, we note that output recovers more quickly and at some point jumps to potential straight away, in the period where the wage rigidity ceases to bind. This occurs when $\bar{\Pi} > (Y^f/Y_{t-1})^{(1-\alpha)/\alpha}$, recall equations (8) and (9).

is lowered to negative territory, (10) and (11) change to

$$\Pi_{H,t} = \bar{\Pi} \quad \text{subject to} \quad i_t \geq -\bar{i}_{\text{ZLB}},$$

along with

$$i_t = -\bar{i}_{\text{ZLB}} \quad \text{if} \quad \Pi_{H,t} < \bar{\Pi},$$

where $\bar{i}_{\text{ZLB}} > 0$ is the degree of effective relaxation of the zero bound constraint. From the Fisher equation (2) and its ROW counterpart, it follows that domestic inflation falls below inflation in ROW as $i = 0$ falls to $i = -\bar{i}_{\text{ZLB}}$, again because domestic (long run) real rates are determined internationally and hence not affected by domestic policy. That is, as $\bar{i}_{\text{ZLB}} > 0$ and the economy is in the stagnation steady state, the economy must experience a higher rate of deflation than ROW, which, from Phillips curve (8), implies an even deeper recession—in fact, deeper than the recession in the rest of the world.

Hence again, and in line with the arguments made in the previous section, a policy that has an overall favorable effect when the economy is (financially or entirely) closed, turns out to be *beggar thy self* when the economy is financially open.

In concluding this subsection we observe that, under appropriate circumstances, reducing the ELB may be beneficial along the escape transitional dynamics. This is not the case in Figure 1, since the nominal interest rate is strictly positive at every point in time, but it may be useful in economies where the nominal interest rate remains at its lower bound in the initial phase of the transition. This possibility is illustrated in Figure 3 in the Appendix A.4—the version of the model where along the escape, trade elasticities are changing over time, as discussed in Section 3.4. In this context, a lower ELB allows a faster recovery in the conventional way: it shortens the length of time in which monetary policy is constrained at the beginning of the escape.

6 Conclusion

This paper shows that small open economies in a world of secular stagnation can attain full employment by depreciating their currencies and acquiring net foreign assets. Yet, weakening terms of trade can reduce domestic welfare even though output rises to full employment. While the adjustment path can involve a temporary hike in domestic real rates, in the long run, these remain anchored at the level determined in international asset markets. In nominal terms, instead, domestic rates rise above zero also in the long run, commensurate with the higher inflation and nominal depreciation at full employment.

Our relatively simple model illustrates that cross-border trade in goods and assets offers important adjustment margins and opportunities to avoid stagnation. Under full financial market integration, nonetheless, policy prescriptions that are effective in closed economies in temporary downturns—i.e., in temporary liquidity traps in standard New Keynesian models—do not necessarily work in open economies in permanently depressed environments. To the extent that arbitrage in capital markets constrains real rates, lowering the policy rate below zero can be harmful rather than helpful, as is the case in closed economy models or models of temporary downturns. Similarly, relaxing the domestic frictions that constrain domestic borrowing is neutral in terms of output, yet beggar thy self in terms of consumption when asset markets are internationally integrated.

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A Appendices

A.1 Model equations

We define the relative price $\Phi_t \equiv P_t/P_{H,t}$. We also denote the real stock of net foreign assets $NFA_t^r \equiv NFA_t/P_t$. The home economy can be summarized by the following equations. The stock equation for the *net foreign asset position*

$$NFA_t^r = -\frac{D_t}{1+r_t}\frac{1}{\Phi_{t+1}} + \frac{1}{1+[\beta(1+r_t)^{1-\rho}]^{-\frac{1}{\rho}}}\frac{1}{\Phi_t}(Y_t - D_{t-1})$$

as well as the flow equation

$$NFA_t^r = (1+r_{t-1})NFA_{t-1}^r + \frac{Y_t}{\Phi_t} - C_t.$$

The *aggregate demand equation*

$$Y_t = \Phi_t^\sigma\left((1-\omega)C_t + \omega Q_t^\sigma Y^*\right)$$

The *price indexes, Fisher and UIP equations*

$$\begin{aligned}
Q_t &= \frac{S_t}{\Phi_t} \\
S_t &= \frac{\mathcal{E}_t P_t^*}{P_{H,t}} \\
\Phi_t &= [(1-\omega) + \omega(S_t)^{1-\sigma}]^{\frac{1}{1-\sigma}} \\
1+r_t &= (1+r^*)\frac{Q_{t+1}}{Q_t} \\
1+i_t &= \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t},
\end{aligned}$$

where we have used that $1+i^* = 1$. The *aggregate supply* and *policy rule* given by

$$\begin{aligned}
Y_t &= \left[\gamma\frac{Y_{t-1}^{\frac{\alpha-1}{\alpha}}}{\Pi_{H,t}} + (1-\gamma)(Y^f)^{\frac{\alpha-1}{\alpha}}\right]^{\frac{\alpha}{\alpha-1}} \\
1+i_t &= 1
\end{aligned}$$

around the unemployment steady state (as long as $\Pi_{H,t} < (Y^f/Y_{t-1})^{\frac{1-\alpha}{\alpha}}$), and by

$$\begin{aligned}
Y_t &= Y^f \\
\Pi_H &= \bar{\Pi}
\end{aligned}$$

around the full employment steady state.

A.2 Existence of steady states

The domestic steady state can be summarized by the following set of equations:

$$NFA^r = -\frac{D}{1+r}\frac{1}{\Phi} + \frac{1}{1+[\beta(1+r)^{1-\rho}]^{-\frac{1}{\rho}}}\frac{1}{\Phi}(Y-D) \tag{A.1}$$

$$NFA^r = (1+r)NFA^r + \frac{Y}{\Phi} - C \tag{A.2}$$

$$Y = \Phi^\sigma\left((1-\omega)C + \omega Q^\sigma Y^*\right) \tag{A.3}$$

$$Q = \frac{S}{\Phi} \tag{A.4}$$

$$\Delta S = \frac{\Delta\varepsilon\Pi^*}{\Pi_H} \tag{A.5}$$

$$\Phi = [(1-\omega) + \omega(S)^{1-\sigma}]^{\frac{1}{1-\sigma}} \tag{A.6}$$

$$(1+r) = (1+r^*)\Delta Q \tag{A.7}$$

$$(1+i) = \Delta\varepsilon \tag{A.8}$$

$$Y = \{ \begin{matrix} Y^f\left[\frac{(1-\gamma/\Pi_H)}{(1-\gamma)}\right]^{\frac{\alpha}{\alpha-1}} & \text{if } \Pi_H < \bar{\Pi} \text{ and } 1+i=1 \\ Y^f \text{ if } \Pi_H = \bar{\Pi} \text{ and } 1+i \geq 1 \end{matrix} \tag{A.9}$$

Relative prices must be constant in steady state, such that $\Delta Q = \Delta S = \Delta\Phi = 1$. As a result, the real interest rate in the domestic economy is the same as in ROW, from (A.7). So, from the steady state value of the real interest rate in the ROW and from equation (A.7), we have $(1+r) = (1+r^*) = 1/\Pi^*$ in steady state.

A.2.1 Existence of stagnation steady state

In this case the zero lower bound is binding such that $1+i=1$, implying $\Delta\epsilon = 1$ from (A.8). Then, $\Pi_H = \Pi^*$ and $Y = Y^*$ from (A.5) and (A.9) respectively. We proceed by combining equations (A.1), (A.2) and (A.3) as

$$Y = \Phi^\sigma[(1-\omega)(r^*NFA^r + Y\Phi^{-1}) + \omega Q^\sigma Y^*]$$
$$\Leftrightarrow \; Y(1-\Phi^{\sigma-1}(1-\omega)) = \Phi^{\sigma-1}(1-\omega)\frac{r^*}{1+r^*}\frac{D^*-D}{Y^*-D^*}Y^* + \omega\Phi^\sigma Q^\sigma Y^*$$

where we have used that

$$\frac{D^*}{1+r^*} = \frac{1}{1+[\beta(1+r^*)^{1-\rho}]^{-\frac{1}{\rho}}}(Y^*-D^*),$$

see the allocation in ROW described in Section 2.7. If we further combine (A.4) and (A.6) to $\Phi = ((1-\omega)/(1-\omega Q^{1-\sigma})^{1/(1-\sigma)}$ we can re-write this as

$$Y(1-(1-\omega Q^{1-\sigma})) = (1-\omega Q^{1-\sigma})\frac{r^*}{1+r^*}\frac{D^*-D}{Y^*-D^*}Y^* + \omega(\frac{1-\omega}{1-\omega Q^{1-\sigma}})^{\frac{\sigma}{1-\sigma}}Q^\sigma Y^*. \quad \text{(A.10)}$$

Once we recognize that under stagnation, $Y = Y^*$, we note that equation (A.10) can be seen as determining Q for any given D. The existence of the stagnation steady state therefore depends on the existence of a Q which solves this equation.

First note that, in the case of *symmetric* $D = D^*$, the term in the middle drops out. In this case, equation (A.10) can be easily seen to be solved by $Q = 1$, implying $\Phi = S = 1$ from equations (A.6) and (A.4), in turn implying $C = Y^*$ from equation (A.3) and hence $NFA^r = 0$ from equation (A.2). This is the first part of Proposition 1.

In the general case $D \neq D^*$ we need to refer to the Intermediate Value Theorem. Subtracting the right hand side in equation (A.10) we restate the condition for existence as $f(Q) = 0$. First, it is easy to see that $f(1) > 0$ for $D > D^*$, and $f(1) < 0$ for $D < D^*$.[24] Let us make this case distinction more explicit.

Case $D > D^*$: When $\sigma > 1$, we see that f tends to minus infinity as Q tends to plus infinity, as long as condition $r^*/(1-r^*)(D^*-D)/(Y^*-D^*) > -1$ is satisfied. This condition holds unless D is unreasonably large, given that r^* is close to zero. Because, as we have argued above, $f(1) > 0$, we conclude that there must exist a $1 < Q < \infty$ such that $f(Q) = 0$.[25] Instead, when $\sigma < 1$, we exploit that f has a pole at $Q = \omega^{1/(\sigma-1)} > 1$. Namely, as can be seen in equation (A.10), the last term on the right hand side tends to plus infinity as Q tends to the pole from below. This implies that f tends to minus infinity as Q tends to the pole from below. Again, because of $f(1) > 0$, we conclude that there must exist a $1 < Q < \omega^{1/(\sigma-1)}$ such that $f(Q) = 0$.

Case $D < D^*$: When $\sigma > 1$, f tends to $Y^* > 0$ as Q tends to $Q = \omega^{1/(\sigma-1)} < 1$. Because, as we have argued above, $f(1) < 0$, there must exist an $\omega^{1/(\sigma-1)} < Q < 1$ such that $f(Q) = 0$. Finally, when $\sigma < 1$, it turns out that σ must be high enough for an equilibrium to exist (that is, σ can not be too close to zero). Moreover, the threshold value for σ depends on the degree of distortion, i.e. on the exact size of $D < D^*$. Here we merely state a sufficient condition. Assume that $\omega(1+\omega) < 1$.[26] In this case, $f(\omega) > 0$ whenever $(1-\omega^{2-\sigma})(1+\omega/(1-\omega^2) + r^*/(1-r^*)(D^*-D)/(Y^*-D^*)) < 1$.[27] This condition holds for σ large enough and D^*-D

[24] This follows because $f(1) = 0$ under $D = D^*$ as argued earlier above.
[25] f is easily seen to be continuous on this interval.
[26] In our calibration, $\omega = 0.2$ such that this expression is $0.24 < 1$.
[27] To see this, note that $f(\omega) = Y^*\{1-(1-\omega^{2-\sigma})(1+\omega(1-\omega)^{\sigma/(1-\sigma)}\omega^\sigma(1-\omega^{2-\sigma})^{1/(\sigma-1)} + r^*/(1-r^*)(D^*-D)/(Y^*-D^*))\} > Y^*\{1-(1-\omega^{2-\sigma})(1+\omega/(1-\omega^2)+r^*/(1-r^*)(D^*-D)/(Y^*-D^*))\}$, where the last inequality holds as long as $\omega(1+\omega) < 1$.

small enough (for example, it never holds for σ close to zero). Again, because $f(1) < 0$, there must exist an $\omega < Q < 1$ such that $f(Q) = 0$.
Notice that, once Q is determined, the remaining endogenous variables follow. First we obtain a Φ from $\Phi = ((1-\omega)/(1-\omega Q^{1-\sigma})^{1/(1-\sigma)}$, which implies an S from (A.6), a C from (A.3) and, finally, a NFA^r from (A.2).

A.2.2 Existence of full employment steady state

Let us now prove existence of the full employment steady state. First, note that under full employment $Y = Y^f$ at all times. Inflation must be on target from (A.9) such that $\Pi_H = \bar{\Pi}$, which from (A.5) implies that $\Delta\epsilon = \bar{\Pi}/\Pi^* > 1$, such that from (A.8) $1 + i > 1$. Furthermore, notice that equation (A.10) holds also in the full employment steady state, with the difference that $Y = Y^f$ instead of $Y = Y^*$. The proof follows therefore the same steps as the existence proof of the stagnation steady state: for given D, it is required to obtain a Q such that equation (A.10) under $Y = Y^f$ is satisfied. Let us start with the symmetry case. When $D = D^*$, $f(1) > 0$ because, as we have shown above $f(1) = 0$ under $Y = Y^*$ and $Y^f > Y^*$. Hence, the existence proof is the same as the case $D > D^*$ in the previous subsection: there must exist a $Q > 1$ such that $f(Q) = 0$. This immediately implies that $S > 1$ and $\Phi > 1$ in this steady state, from equations (A.4) and (A.6). To show that net foreign assets must be strictly positive, rewrite (A.1) as

$$NFA^r * \Phi = -\frac{D^*}{1+r^*} + \frac{1}{1+[\beta(1+r^*)^{1-\rho}]^{-\frac{1}{\rho}}}(Y - D^*).$$

We know that that right hand side is zero under $Y = Y^*$. Hence, because $Y^f > Y^*$, it must be greater than zero under $Y = Y^f$. For the left hand side to be greater than zero, it is required that $NFA^r > 0$. This completes the proof of Proposition 1.
The asymmetric case proceeds as above. When $D > D^*$, $f(1) > 0$ (even more positive than under $D = D^*$), for the reasons spelled out in the existence proof of the stagnation steady state. Hence, there must exist a $Q > 1$ such that $f(Q) = 0$ (this Q must therefore be greater than under symmetry, i.e., a higher D implies an even more depreciated equilibrium real exchange rate). Conversely, when $D < D^*$ it can be the case that either $f(1) < 0$ or $f(1) > 0$. In either case, we proceed as in the existence proof of the stagnation steady state above.

A.3 Is the stagnation steady state locally determinate?

Here we demonstrate that, in our model, the stagnation steady state is locally indeterminate. We do so, because in Eggertsson et al. (2016) the steady state where both economies are symmetrically are their zero bound is found to be *determinate.* For simplicity, we proceed

with the case of full symmetry only, where in particular $D_t = D^*$. Lower case letters denote log-deviation of upper case letters, absolute deviation in the case of NFA_t^r.
Using that $NFA^r = 0$, $\Phi = 1$, $Y = Y^*$ and $r_t = r^*$ we linearize the stock equation for net foreign assets as

$$nfa_t^r = \frac{D^*}{1+r^*}\left((1+\xi)r_t + \phi_{t+1} - \phi_t + \frac{Y^*}{Y^* - D^*}y_t\right),$$

where we define $\xi \equiv \frac{1-\rho}{\rho}\frac{[\beta(1+r^*)^{1-\rho}]^{-1/\rho}}{1+[\beta(1+r^*)^{1-\rho}]^{-1/\rho}}$. Using that $C = Y^*$, the asset flow equation can be linearized to

$$nfa_t^r = (1+r^*)nfa_{t-1}^r + Y^*(y_t - \phi_t - c_t).$$

The aggregate demand curve is given by

$$y_t = \sigma\phi_t + (1-\omega)c_t + \omega\sigma q_t,$$

the price indexes are

$$\begin{aligned} q_t &= s_t - \phi_t \\ s_t &= e_t - p_{H,t} \\ \phi_t &= \omega s_t, \end{aligned}$$

the asset price equations are

$$\begin{aligned} r_t &= \Delta q_{t+1} \\ 0 &= \Delta e_{t+1} \end{aligned} \tag{A.11}$$

where we have used that $i_t = 0$, and finally, the Phillips curve is given by

$$y_t = (\gamma/\Pi^*)(y_{t-1} + \frac{\alpha}{1-\alpha}\pi_{H,t}).$$

Using the price indexes and the flow equation for net foreign assets we may rewrite the demand equation as

$$\begin{aligned} y_t &= \sigma\frac{\omega(2-\omega)}{1-\omega}q_t + (1-\omega)c_t \\ &= \sigma\frac{\omega(2-\omega)}{1-\omega}q_t + (1-\omega)(y_t - \frac{\omega}{1-\omega}q_t - \frac{1}{Y^*}(nfa_t^r - (1+r^*)nfa_{t-1}^r)). \end{aligned}$$

Solving for y_t yields

$$y_t = \frac{1}{1-\omega}(\sigma(2-\omega) - (1-\omega))q_t - \frac{1-\omega}{\omega}\frac{1}{Y^*}(nfa_t^r - (1+r^*)nfa_{t-1}^r). \tag{A.12}$$

Furthermore, we may write the Phillips curve as

$$y_t = (\gamma/\Pi^*)(y_{t-1} - \frac{\alpha}{1-\alpha}(\frac{1}{1-\omega}\Delta q_t - \Delta e_t)), \tag{A.13}$$

where $\gamma/\Pi^* < 1$, see Section 2.7. Finally, from the first equation and the real version of the UIP condition

$$nfa_t^r = \frac{D^*}{1+r^*}\left(\left(1+\xi+\frac{\omega}{1-\omega}\right)\Delta q_{t+1} + \frac{Y^*}{Y^*-D^*}y_t\right). \tag{A.14}$$

The four equations (A.11)-(A.14) in $\{y_t, q_t, nfa_t^r, \Delta e_t\}$ pin down the determinacy properties of the system. We find numerically that the system is indeterminate as long as the condition $\sigma(2-\omega)-(1-\omega)>0$ (the factor multiplying q_t in equation (A.11)) is satisfied.[28] We may assume that this condition holds, as it implies a very low threshold value $\sigma = (1-\omega)/(2-\omega) < 0.5$ below which the dynamic properties of the system change.

In particular, this condition includes the case of full substitutability between foreign and domestic consumption considered in Eggertsson et al. (2016), which is $\sigma = \infty$. Substituting this in, we obtain $q_t = 0$ from (A.12) such that the system reduces to

$$\begin{aligned} y_t &= (\gamma/\Pi^*)(y_{t-1} + \frac{\alpha}{1-\alpha}\Delta e_t) \\ 0 &= \Delta e_{t+1}, \end{aligned}$$

with nfa_t^r being residually determined in $nfa_t^r = (D^*/(1+r^*))(Y^*/(Y^*-D^*))y_t$. This system is clearly seen to be indeterminate.[29]

A.4 Beggar-thy-self escape to full employment

One main result of the paper is that, for low enough trade elasticities, the escape from stagnation to full employment may be beggar thy self—reducing consumption while eliminating underemployment. Here we compute the approximate threshold for the elasticity σ below which the escape is beggar thy self in the case of symmetry, stated in Proposition 2. We also show how the beggar-thy-self property changes transitional dynamics.

[28] Checking this analytically requires the computation of eigenvalues of a five by five matrix.

[29] The reason Eggertsson et al. (2016) obtain a different result is that they are deriving the determinacy properties of the *world* economy in the symmetric secular stagnation. Specifically, once the domestic country has zero mass, their proof in Proposition 4 verifies that output and inflation in the foreign economy (the ROW in our context) are locally determinate around their stagnation steady states. This is true also in our analysis: because the ROW is effectively a closed economy, the analysis from Eggertsson et al. (2017), which implies local determinacy under stagnation, directly applies. In contrast, here we have shown that variables in the *small domestic* economy are locally indeterminate under symmetric stagnation.

A.4.1 Derivation of threshold

We show that, once σ falls below some (approximate) threshold, the full employment steady state has a lower level for aggregate consumption than the stagnation steady state. We focus on consumption, because in our model, welfare and consumption are directly related (compare the households' objective in Section 2).

Start with combining the equations for the asset flow (A.2), aggregate demand (A.3), and price indexes (A.4) and (A.6) from Appendix A.1 to obtain

$$Y_t(1-(1-\omega Q_t^{1-\sigma})) = \left(\frac{1-\omega}{1-\omega Q_t^{1-\sigma}}\right)^{\frac{\sigma}{1-\sigma}} \left[(1-\omega)((1+r_{t-1})NFA_{t-1}^r - NFA_t^r) + \omega Q_t^\sigma Y^*\right],$$

which determines Q_t for given Y_t and NFA_t^r. Log-linearizing this equation around the stagnation steady state (characterized by $Q = 1$ and $NFA^r = 0$, see above) gives

$$q_t = \left(\frac{2-\omega}{1-\omega}\sigma - 1\right)^{-1} y_t =: \kappa y_t. \tag{A.15}$$

Up to first order, equation (A.15) determines the required real depreciation necessary as the economy moves to the full employment steady state (i.e. the required depreciation is κ times the increase in output). Note in particular that κ falls in σ, such that the required real depreciation is smaller as the trade elasticity increases.

Second, log-linearizing the asset flow from Appendix Section A.1 around the stagnation steady state yields

$$c_t = y_t - \frac{\omega}{1-\omega} q_t. \tag{A.16}$$

Now combine equations (A.15) and (A.16) to obtain

$$c_t = \left(1 - \frac{\omega}{1-\omega}\kappa\right) y_t, \tag{A.17}$$

such that, up to first order, moving to the full employment steady state raises consumption if $\omega\kappa/(1-\omega) < 1$. Recalling the definition of κ and rearranging yields

$$\sigma > \frac{1}{2-\omega}, \tag{A.18}$$

which is the condition stated in the main text.

A.4.2 Escape path under beggar thy self

The transition for the case of low trade elasticity is depicted in Figure 2, where we use the same parameter values as in Section 3.4, except for the trade elasticity which now is $\sigma = 0.5$. As explained in the main text, with a low trade elasticity, the income effects from depreciation are strong: note that the rate of depreciation is higher than in Figure 1. As a result, domestic consumption falls on impact, driven by a contraction in income valued at international prices (i.e., $P_{H,t}/P_t$ falls faster than Y_t rises), and keeps falling towards a new, lower, steady state level. The domestic savings fall along with the consumption of the middle-aged, again due to the strong income effects from depreciation. Still, because the domestic borrowing contracts faster than the saving—due to the borrowing constraint of the young becoming tighter as a result of the depreciation—the net foreign asset position increases in the escape. Overall, the large adverse terms of trade movements have a negative effect on consumption for all generations, such that the escape is beggar thy self.

A.4.3 Escape under time-varying trade elasticities

In what follows, we construct a numerical example showing the implications for the escape path of letting elasticities differ as a function of time. In Figure 3, we set the elasticity at 0.5 at the beginning of the escape path, and increase it gradually to 3 as time passes. In turn, the other parameter values are as in Figure 1 in the main text. A number of striking results emerge. The exchange rate needs to adjust in nominal and real terms much more at the beginning of the transition, so does inflation. The real exchange rate actually overshoots its long-run value upfront, and appreciates during the transition. By implication, the real interest rate falls, implying that domestic saving temporarily rises. Most remarkably, despite the return of inflation to its target level, the nominal interest rate remains at the zero lower bound for some periods.

Our exercise suggests that, with time-varying elasticities along the escape route, the interest rate in a small open economy may remain at its zero-bound on impact even when the economy starts to recover. It may take some time before the improvement in macroeconomic conditions and persistent inflation allows the policy rate to return to positive territory. During the zero-bound spell, the economy actually experiences some exchange rate appreciation, after an initial overshooting of the currency's new equilibrium (depreciated) level.

A.5 Alternative borrowing constraint

Following up on Section 4.1, here we consider an alternative specification for the model where the borrowing constraint is expressed in terms of domestic consumption rather than output units. That is, we replace equation (1) by $C_t^y = -(1/P_t)B_t^y = D_t/(1+r_t)$. As a result, the flow net foreign asset condition (14) changes to

$$\frac{NFA_t}{P_t} = -\frac{D_t}{1+r_t} + \frac{1}{1+[\beta(1+r_t)^{1-\rho}]^{-\frac{1}{\rho}}}\left[\frac{P_{H,t}}{P_t}Y_t - D_{t-1}\right] \tag{A.19}$$

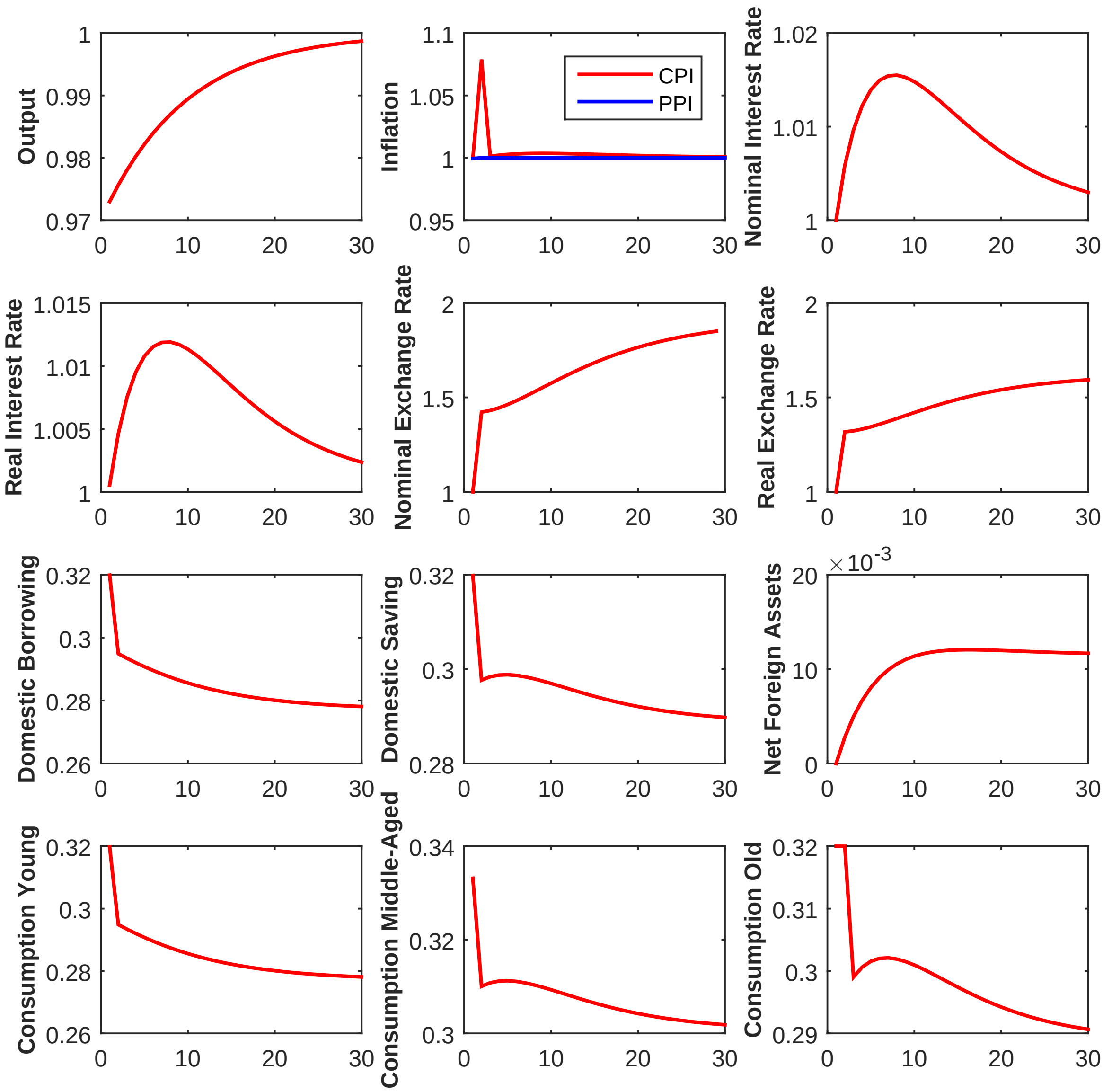

Figure 2: Transition to full employment, w/ beggar thy self escape. Parameters are as in Figure 1, except for the trade elasticity which we set to $\sigma = 0.5$.

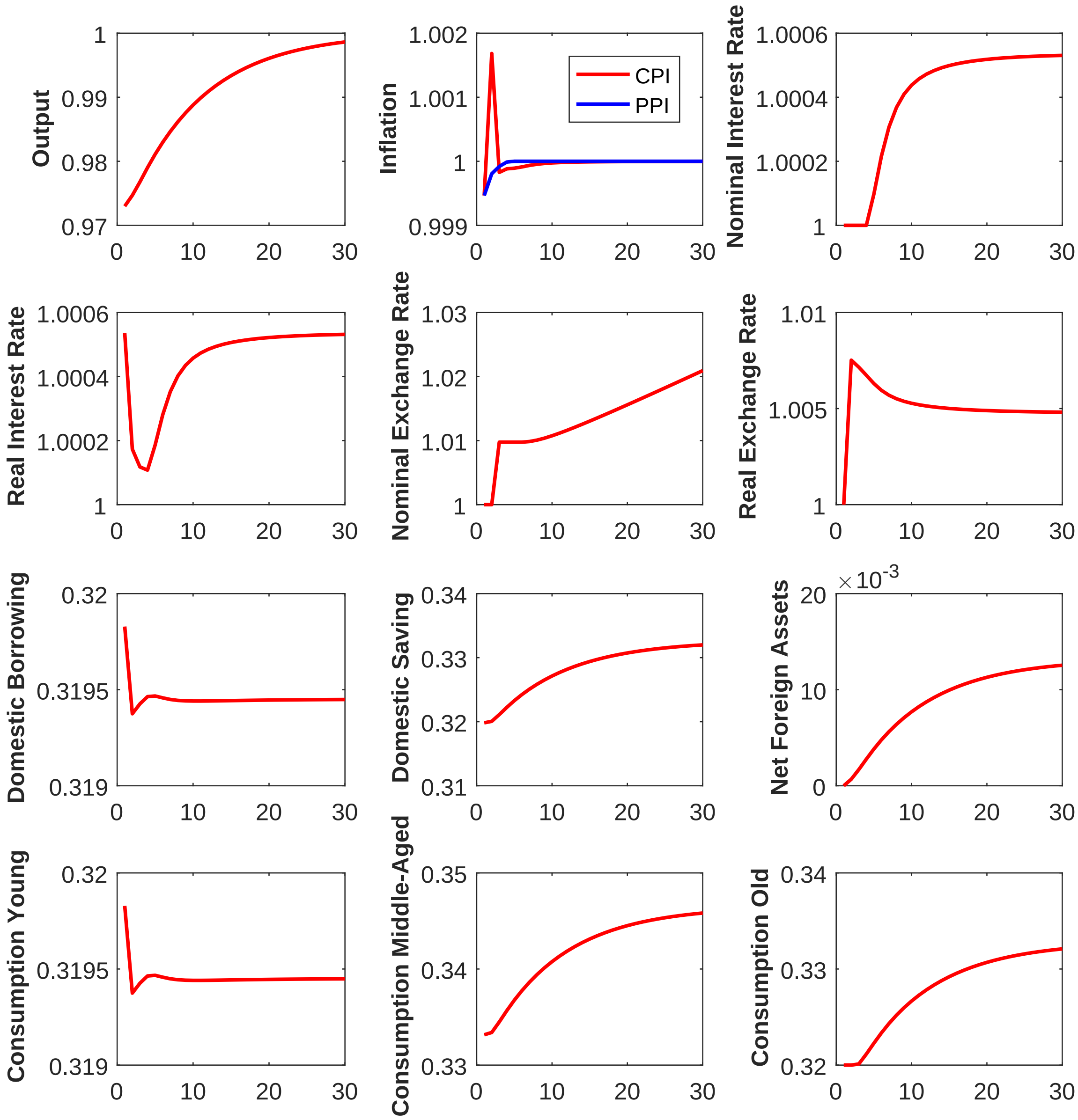

Figure 3: Transition to full employment, case of changing trade elasticity. Parameters as in Figure 1, except for the trade elasticity which gradually changes from $\sigma = 0.5$ to $\sigma = 3$.

while all other equations remain as summarized in Appendix Section A.1—that is, the rest of the model remains unaffected by this adjustment. As can be seen from equation (A.19), the difference with respect to the baseline model is that the borrowing constraint by the young now does not tighten per effect of the real depreciation. This has two key implications.

First, in the region where σ is sufficiently low for the escape to be beggar thy self, the escape may actually be accompanied by capital *inflows*, rather than capital outflows. Intuitively, this is because the savings-absorption capacity of the economy is unchanged with the depreciation, while the savings of the middle-aged are compressed per effect of the depreciation. Second, as a result, escape to full employment under financial autarky may actually be possible under this alternative specification (as mentioned in Section 4.1). This is because, again from equation (A.19), the real depreciation which raises output to full employment simultaneously reduces the drop in the domestic real rate that is required to solve equation (A.19) under $NFA_t = 0$. In intuitive terms, because the real depreciation directly reduces domestic excess savings, escape without accumulating external assets may be possible even with very small changes in domestic real interest rates.

A.6 Asset supply policies

Following up on Section 4.2, here we show that, by relaxing the domestic borrowing constraints while the economy finds itself in stagnation (i.e., raising $D > D^*$), this results in a steady state real depreciation, a fall in net foreign assets, and a drop in aggregate consumption.[30] To do so, first recall that changing D leaves the amount of domestic output $Y = Y^* < Y^f$ and the real interest rate $(1 + r) = (1 + r^*)$ unaffected.

The following three equations from the model's steady state summarized in Appendix Section A.2 are required for this question

$$\begin{aligned} NFA^r &= -\frac{D}{1+r^*}\frac{1}{\Phi} + \frac{1}{1+[\beta(1+r^*)^{1-\rho}]^{-\frac{1}{\rho}}}\frac{1}{\Phi}(Y^* - D) \\ NFA^r &= (1+r^*)NFA^r + \frac{Y^*}{\Phi} - C \\ Y^* &= \Phi^\sigma\left((1-\omega)C + \omega Q^\sigma Y^*\right) \end{aligned}$$

as well as $\Phi = (\frac{1-\omega}{1-\omega Q^{1-\sigma}})^{1/(1-\sigma)}$. This is a three-by-three system in the variables (C, NFA^r, Φ) for given exogenous D.

We conduct a comparative statics exercise. We change D to a higher level, and study whether the implied (C, NFA^r, Φ) in the new steady state rise or decline. To do so, we consider a log-linearization (linearization for NFA^r) of the previous conditions around $Q = \Phi = 1$, $NFA = 0$ and $C = Y^*$, which are the conditions in the initial (perfectly symmetric with ROW) stagnation steady state. Using small-case letters with a tilde to denote deviations

[30] These results can, in principle, also be inferred from the existence proof of the stagnation steady state under asymmetry, presented in Section A.2. Here we provide an alternative proof to underline our argument.

from the initial steady state we obtain

$$\begin{aligned}
n\tilde{f}a^r &= -1/(1+r^*)\ Y^*/(Y^*-D)\ \tilde{d} \\
n\tilde{f}a^r &= (1+r^*)n\tilde{f}a^r - Y^*(\tilde{\phi}+\tilde{c}) \\
0 &= (1-\omega)\tilde{c} + (2-\omega)\sigma\tilde{\phi}
\end{aligned}$$

where in the last equation we have used that $\tilde{q} = (1-\omega)/\omega\ \tilde{\phi}$. The first equation reveals that $n\tilde{f}a^r < 0$ as $\tilde{d} > 0$. That is, relaxing the domestic borrowing constraint unambiguously induces capital inflows. In turn, the second and third equation determine how those capital inflows impact on the real exchange rate and consumption.

Solving for the change in consumption $\tilde{c}$ by using the third equation yields

$$\tilde{c} = -\sigma\frac{2-\omega}{1-\omega}\tilde{\phi}, \tag{A.20}$$

such that consumption drops as the real exchange rate depreciates. Solving for the net foreign asset position from the second equation then gives

$$r^* n\tilde{f}a^r = -Y^*(\sigma\frac{2-\omega}{1-\omega} - 1)\ \tilde{\phi}, \tag{A.21}$$

such that, as long as $\sigma > (1-\omega)/(2-\omega)$, the net foreign asset position falls as the real exchange rate depreciates. Notice that this is the condition we had encountered in the proof of local determinacy of the stagnation steady state in Section A.3: there we had assumed this condition to hold, as it implies a very low threshold elasticity.

Because $n\tilde{f}a^r < 0$ falls as $\tilde{d} > 0$ rises, we have established that the real exchange rate depreciates ($\tilde{\phi} > 0$ and hence $\tilde{q} > 0$) from equation (A.21), and that consumption drops ($\tilde{c} < 0$) from equation (A.20), following a relaxation in the domestic borrowing limit.