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NARROW IDENTITIES

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A person's social identity has many facets, involving language, personal interests, customs, religion, and ethnicity, among other attributes. Yet, all over the world many people seek to define themselves in exclusive terms. This paper develops a simple model of personal incentives and group interests to offer one possible explanation for the puzzle.

Narrow Identities

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Abstract

A person's social identity has many facets, involving language, personal interests, customs, religion, and ethnicity, among other attributes. Yet, all over the world many people seek to define themselves in exclusive terms. This paper develops a simple model of personal incentives and group interests to offer one possible explanation for the puzzle.

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1 Motivation

> *"Each individual's identity is made up of a number of elements ... they include allegiance to a religious tradition; to a nationality; ... to a profession, an institution, or a particular social milieu... A person may feel a more or less strong attachment to a province, a village, a neighbourhood, a clan, a professional team or one connected with sport, a group of friends, a union, a company, a parish, a community of people with the same passions, the same sexual preference, the same physical handicaps, or who have to deal with the same pollution or other nuisance... Of course, not all these allegiances are equally strong, at least at any given moment."*
> Amin Maalouf (2000: 10), On Identity (London: The Harvill Press), translated by Barbara Bray.

It is a near-truism today that identity is multi-dimensional. Although some are inherited (ethnicity, nationality), psychologists observe that there are aspects of a person's identity that are fluid and built on deliberative choices of the person herself and those of others (Tajfel and Turner, 1979; Tajfel, 1981; Deaux, 1996). Liberal cosmopolitans say that if we were to recognise identity to be primarily about the unfettered choices of individuals regarding where they belong, people would be seen as having multiple identities. Proponents of liberal cosmopolitanism thereby tell us to recognize humanity whenever and wherever it appears, and argue that it is deserving of our allegiance over and above the social identities people may have. Sen (2006) in particular has insisted that because people have multiple affiliations, claims for the sanctity of narrow social identities by those having them are unwarranted, even delusionary. And yet, all over the world we see people defining themselves in narrow, exclusive terms and being so regarded by others. Religion, language, more broadly, ethnicity, are salient features. But because there are markers for ascribing identity that are not inherited in quite the same way (membership of evangelical churches is a prominent contemporary example), narrowness in the way people often see themselves and are correspondingly seen by others requires explanation.

In their study of the diversity of human natures, evolutionary biologists have examined why we are disposed toward narrow identities. They have explained why our social horizons are often very restricted even when we have recognized that the benefits of collective enterprises would be most effectively realized if we admitted others into our group (Ehrlich, 2000). In this paper we develop a model of personal incentives and group interests that offers an explanation that complements the one provided by evolutionary biologists.

Our model is an adaptation of the one prominent in the theory of clubs. The adaptation moves us however in a different direction. In constructing our model we deliberately work with a lean notion of identity. The formulation neither pre-supposes nor pre-determines any of the many significant features of the notion that have motivated authors of the applied-theoretic works we review below. One can doubt that such a stripped-down model as ours could be of any use in understanding such an elusive notion as human identity. But the very richness of the notion suggests it may pay to distil it into an almost "pre-societal" form. The wide variety of ways in which people actually express their identity can then be seen as reflecting the constraints people face, the choices they make, and the loyalties they form. The view we take here is that the complexity of identity is an emergent feature of the fundamental human need to find a place in the social world. Our model's central assumptions, such as that a group's size brings positive benefits to its members, are based on broad empirical regularities to be found in identity groups (e.g. religious congregations). We then derive conclusions about the workings of our model that are consistent with a number of findings in the empirical literature.

We study a stripped-down model of individual incentives and group interests. We imagine that people choose a group (or groups) to join on the basis of the conditions attached to membership. Those conditions include the way benefits and burdens would be shared. Although we endogenize membership rules, we do not specify how the rules are reached and implemented. It could be that someone with an idea (an aspiring leader) draws up a constitution and invites people to join his group; or it could be that a group forms without the deliberate intention of its eventual members and only subsequently finds a common cause; and so on.

We assume that the net benefit (or payoff) a typical member of a group enjoys is an increasing function of the group's size. This is a key feature of our model. The justification for the assumption is that it meets a feature common among affiliations. A group could be one seeking funding for cultural activities; it may be an evangelical church offering members the opportunity to connect directly to God (Luhrman, 2008), a national research body aiming to promote fundamental research, a community in conflict with other communities over land; and so on. In each of these examples, size is an advantage.

A second element in our model is that a group's payoff depends not only on its own size (positively, as just mentioned), but also on the size of other groups. Negative inter-group externalities prevail when groups are in conflict over the allocation of such resources as land, public funds, and minerals. The chance of success in seizing the resources depends on the relative sizes of groups. In contrast, positive inter-group externalities occur among groups engaged in fundamental research, the externality being the spill-over of knowledge. We show below that inter-group externalities create the tensions that influence the choice between narrow and multiple social identities.

We follow Appiah (2006a, 2006b) in keeping other features of groups unspecified and identify a group by its label (X, Y, and so on). Ours is a minimalist conception of identity formation.

For simplicity of exposition we consider two potential groups (a generalization of the model to more than two is easily achieved). We first examine the case where rules of membership in groups are exogenously given. If payoffs increase with the size of one's group as well as the size of the other group (positive spill-over), multiple identities are in the interest of each group as well as that of society at large. Moreover, if the cost of joining a group is small, multiple identities are in the interest of each individual too. If, however, the costs of joining more than one group are substantial (learning different sets of rule and acting on them when required may prove difficult), there is a divergence between individual incentives and group interests. That's because individuals do not take into account the positive effects of group membership on the payoffs of group members. In that situation identities would be narrower than what is socially efficient.

We next study situations in which a group's payoff is an increasing function of its own size but a decreasing function of the size of the other group (negative spill-over). If an individual who belongs to one group also joins the other group, she generates a negative externality; the cost incurred by the former group is shared among its members. If membership costs are small, individuals would wish to belong to both groups - thus assuming multiple identities - even though every group would prefer that each individual commits herself to only one group (narrow identity).

We next endogenize membership rules. We imagine there are people (politicians, entrepreneurs, potential founders of mega churches) who are primarily interested in aggregate group payoffs. We may call them group leaders. In the presence of negative spill-over the tension between individual incentives and group interests leads group leaders to require that individuals join only one group; that is, they acquire narrow identities. The welfare consequences of rules requiring narrow identities are significant. Whether they are better or worse than the consequences of people assuming multiple identities depends, of course, on the strength of the negative spill-over. If spill-overs are large, their presence facilitates the attainment of socially efficient outcomes; if they are small, their presence leads to inefficiency.

This leads us to examine the nature of membership rules. We show that if leaders of all but one group impose the rule that their members are required to assume a narrow identity, the rule imposed by the remaining group does not materially alter the possibilities facing individuals. Thus, each group insisting on a narrow identity constitutes a social equilibrium.

The rest of the paper is organized as follows. In section 2 we discuss the related literature. In Section 3 we present the formal model and in Sections 4-5 we develop our main results. Section 6 presents an extension to the model that allows for a more transparent mapping between theory and applications. Section 7 concludes.

2 Background Literature

Identity is a source of value. The social philosopher Kwame Anthony Appiah (2005: 24-26) observed that it can be an integral part of the specification of one's satisfactions and enjoyments. Identity thereby motivates and gives meaning to acts of supererogatory kindness and generosity. Appiah noted also that the presence of an identity concept in the specification of one's personhood may be part of what explains why the person has an identity at all. Identity can therefore involve deeply held self-conceptions. We may call that, personal identity.

But someone's projects and purposes can often be furthered if she were to assume a social identity, involving collective intentions that can range from religious practices requiring the coordinated involvement of fellow worshippers to norms of conduct in professional associations. No doubt there is no sharp distinction between personal and social identities. People necessarily depend on others to develop into persons; and because we have an innate desire to relate to certain others or be seen as being related to those others, even social identity can be an end in itself. Possession of a social identity is not only valuable as a means to one's ends, but serves the person's emotional needs as well.

2.1 Identities as Labels

Appiah (2006b: 16) offers a stripped-down formulation of social identities by considering the labels, or names, that are attached to them. Taking some arbitrary identity-label, X, he observes that X will have four defining features: (i) X has criteria of ascription; that is, X possesses the properties on the basis of which we are able to sort people into those we do and those we don't call X's. (ii) Some people identify as X's; that is, they think like and feel like X's and act as X's in relevant ways. (iii) Some people treat others as X's. (iv) X has norms of identification; that is, we can often make predictions of someone's intentions and behaviour on the basis that the person is an X.

Appiah's formulation is overarching because it includes the idea of social identity not only when it serves as an end in itself (as in the case of deeply-held religious beliefs) but also when it has a purely instrumental value (as with membership in a professional association). By extension it includes the multitude of cases where social identity serves both purposes. The formulation is overarching also because it is lean: it neither pre-supposes nor pre-determines any of the many characteristics social identity can take. It does not say whether X is inherited or acquired; whether once someone is an X it is hard for her to shake the label off (because others make it hard for her to do so or because psychologically she becomes tethered to it); whether a person can simultaneously have more than one X; whether X is imposed by others or is acquired willingly; whether social and cultural differences give rise to different X's or whether they are the other way round; whether narrow identities are less costly for those having them (psychologically or otherwise) than multiple ones; or whether being an X incurs the disdain of those who are not X's or whether it elicits admiration.

The social historian David Hollinger has identified features of identity that can be viewed as "solidarity." He writes: "To share an identity with other people is to feel solidarity with them; we owe them something special, and we believe we can count on them in ways that we cannot count on the rest of the population." (Hollinger, 2006: 23). By way of illustration Hollinger observes, "Feminism is a solidarity, but womanhood is not." Previously Sunstein and Ullmann-Margalit (2001) had noted that there are cases where people may not even care for some of the activities their solidarity groups are engaged in (taking part in protest marches, choice of clothing, adopting particular eating habits, and so on), but nevertheless join them so as to express solidarity. Reasonably, the authors had named the goods people use to express solidarity, "solidarity goods."

2.2 Social Preferences

A simple but instructive way to study the effects of solidarity on resource allocation is to postulate people as having social preferences. The reciprocity that is practised within a group could be intrinsic. Alternatively, it could be that a person's social preferences represent a reduced form, embodying (instrumental) motives familiar in the theory of repeated games. In their well-known work on the economics of identity, Akerlof and Kranton (2000) offered an explanation for a number of widely held social assumptions and beliefs by postulating that a person's well-being is a function not only of her consumption of goods and services but also her identity. In turn a person's identity was taken to be a function of (a) the social category to which she has been assigned (her caste or class), (b) her personal characteristics (her human capital, preferences, and so on), (c) the specification of norms of behaviour in different social categories (who is expected to do what under which circumstances), and (d) the choices made by all (purchase of goods, acts of generosity and so forth), including her own. An individual's identity was thus taken to be a socially constructed reference point. In Akerlof and Kranton (2010) the authors used their formulation to explain a variety of otherwise puzzling features of the education sector and the work place.

Sobel (2005) is a comprehensive guide to the way social preferences (he calls them "interdependent preferences") explain the reciprocity and a sense of the self that have been displayed by subjects in laboratory experiments such as the ultimatum game. The formulation of social preferences he has proposed has a wide reach, including the Akerlof-Kranton model of identity. More particularly, it includes cases where people desire to engage in communal activities and coordinate on goods that serve as the basis for those activities. Donati (2011) calls the latter, "relational goods." Religious expenditures are built around relational goods (Iannaccone, 1990, 1998; Olson and Perl, 2005; Ysseldyk, Matheson, and Anisman, 2010). Food, clothing, and even reading practice would appear to be driven at least in part by the human desire to belong (Sahlins, 1968; Bourdieu, 1984; Douglas and Isherwood, 1996; Warde, 1997; Pratt and Rafaeli, 1997; Warde and Martens, 2000). The conformity that emerges when members of a group have social preferences would seem to play a role in reproductive behaviour as well (Dasgupta, 1993; Bongaarts and Watkins, 1996; Jensen and Oster, 2009). Social preferences include cases where people join in activities so as to show solidarity with their group (Sunstein and Ullmann-Margalit, 2001).

2.3 Narrow Identities and Conflicts

In a pioneering work Schelling (1978) showed that if the domain of one's social preferences is limited to the activities of neighbours, society can fragment into groups. And although deeply felt solidarity is compatible with cosmopolitanism (Hollinger, 2006), scholars have given particular attention to the social conflicts that narrow social identities can give rise to. For example, Alesina and La Ferrara (2005) have found evidence of a lower supply of public goods in more fragmented societies. Previously Easterly and Levine (1997) had found ethnic fragmentation to be a factor in political instability and the prevalence of economic policies that hinder development. The authors identified rent-seeking activities among contending groups as a source of bad governance.

In view of the costs people have to bear in engaging in conflicts, ethnic fragmentation per se isn't enough to precipitate them. Extreme fragmentation could render the chance of success by any one group to be so low that no group finds it in its interests to start a conflict (Horowitz, 1985). Moreover, the presence of a dominant group could, and there is evidence that it does, forestall conflict even if the group were exploitative of others.

The possibility of violence, even armed conflict, arises when large groups compete for scarce resources. In a study of civil wars fought between 1945 and 1999, Fearon and Laitin (2003) found that the main factors determining both secular trends and cross-national variations are not ethnic or religious diversity per se, but conditions that favour insurgency. Factors that make for those conditions include poverty, political instability, rough geographical terrain (which make it possible for insurgents to hide), and large populations. Because the authors sought to identify conditions favourable for groups to express their grievances, their finding doesn't deny that when circumstances enable them to do so, the avenue through which people express their grievances is their ethnicity. When governance is weak, groups are able to fan hatred against rival groups by spreading false rumours (Glaeser, 2005). Looting and the destruction of capital follow.

In an important and interesting research programme, Esteban and Ray (2008a-b, 2011a-b) have explored the idea that the existence of a small number of large-sized groups with opposing interests is a favourable pre-condition for conflict, other things being equal of course. The authors interpret "polarization" to be the extent to which a population is clustered round a small number of distant poles. The "extent" is the greater the more homogeneous is each contending group and the more different the groups happen to be. Esteban and Ray (1994) had previously constructed a numerical measure of polarization which they then used to uncover the salience of ethnicity and religion, rather than class, in civil conflicts. In Esteban and Ray (2008a) the authors studied a world in which groups that form compete for a share of public resources. They showed that the individual desire for resources leads to the formation of polarized groups (or coalitions). In their model people have multiple identities ex-ante, but find having to choose between two mutually exclusive identities. The authors constructed a model in which it is more attractive for the rich and the poor in a community, rather than the rich in two communities, to form a group.

The famous Robbers Cave experiment - on the social construction of identity - exposed the way identities can be created ab initio and then give rise, if circumstances call for it, to cultural differences and conflict (Sherif et al, 1988). The experiment revealed also that, again if circumstances call for it, groups that were previously hostile toward one another can change stances and cooperate. We draw on that work to imagine that people ex ante are not tied to any group.

3 A simple model

Suppose that there are $N = \{1, 2, ...n\}$, $n \geq 2$ individuals and they each choose to belong to any subset of groups $M = \{A, B\}$. The payoff to a group depends on its own size as well as the size of the other group. We will also assume that, within a group, the group payoff is equally divided among all the members.

All players simultaneously choose their membership strategy. For player i let her strategy $s_i \in S = \{\{A\}, \{B\}, \{A, B\}\}$. Let $s = \{s_1, s_2, ..., s_n\}$ be the strategy profile and let $\mathcal{S} = \prod_{i \in N} S$, be the set of all strategy profiles. Define $K_A(s)$ as the number of players who choose A and $K_B(s)$ as the number of players who choose B in profile s. The membership of each group ranges from 0 to n. For any allocation of individuals between the two groups $\{x, y\}$, $R(x, y)$ is the aggregate surplus of a group with x members when the other group has y members. We turn next to a player's payoffs. Given a strategy profile s a player i's payoffs are given by:

$$\Pi_i(s_i, s_{-i}) = \frac{\mathbf{1}_{s_i(A)}}{F(K_A(s))} R(K_A(s), K_B(s)) + \frac{\mathbf{1}_{s_i(B)}}{F(K_B(s))} R(K_B(s), K_A(s)) \tag{1}$$

where $\mathbf{1}_{s_i(A)}$ is the indicator function for membership in group A under strategy s_i, and $\mathbf{1}_{s_i(B)}$ is the indicator function for membership in group B. The function $F(.)$ reflects the rules of payoff division within a group. We will focus on two polar cases: one, equal division within the group, $F(K_i(s)) = K_i(s)$, and two, pure public good, $F(K_i(s)) = 1$.

A strategy profile s^* is a Nash equilibrium if for each player i, $\Pi_i(s_i^*, s_{-i}^*) \geq \Pi_i(s_i, s_{-i}^*)$, for all $s_i \in S$. An outcome is said to exhibit *narrow* identities if all individuals join a single group, while it is said to exhibit *multiple* identities if some individuals join both groups.[1]

Assume that $R(.,.)$ is (weakly) increasing in the first argument. Also assume that $R(0, y) = 0$, for all $y \in \mathcal{Z}$, where $\mathcal{Z}$ is the set of integers. A strategy profile s yields a configuration (x, y). Define the total social surplus as $S(x, y) = R(x, y) + R(y, x)$. A configuration (x, y) is said to be efficient if it yields the highest social surplus, $S(x, y) \geq S(x', y')$, for all $(x, y) \in \mathcal{Z}^{\in}$.

Observe that in our formulation individuals can seamlessly belong to one or multiple groups. Moreover, we are abstracting from the issue of level of individual effort or loyalty. This is an important simplification; loyalty will be compromised in multiple membership situations. We do not give due weight to it in the current formulation. Turning to the issue of efforts, in some settings, such as example 2 below, efforts across groups will be strategic complements and introducing efforts will probably reinforce our results. But it is also clear that in other settings, such as Example 3, introducing effort will bring in offsetting factors and that this will significantly complicate the analysis. From the view of applications with conflict, therefore our formulation is perhaps more appropriate in a context where the level of effort is not a central issue: an instance where this is plausible is when membership takes the form of 'voting'. This application is further discussed in the extension to the basic model developed in section 6.

3.1 Examples

The following examples illustrate the range of situations which are covered by our model.

Example 1 *Independent groups*

When $R(x, y) = R(x)$ for $x, y \in \mathcal{Z}$, the payoffs to a group are independent of the membership in the other group. ■

[1] In Section 7, we present a slight variation on the model of identity choice: individuals possess two traits, one trait is shared with all other members of society, while a second trait has two types, and every individual has one of these two types. We study identity choice and equilibrium and develop conditions under which there is a pressure toward narrow identities (that are socially inefficient).

Example 2 *Positive spillovers: Promotion of Fundamental Research*

A group is a research organization promoting fundamental research, e.g., a scientific laboratory or a science foundation. Each member who joins the group brings a special skill to the group and the combination of the skills gives rise to inventions. Suppose that there is knowledge spillover across groups. The payoffs to a group A faced with memberships (K_A, K_B) are given by:

$$R(K_A, K_B) = \begin{cases} \frac{[D+K_A+\beta K_B]^2}{4} & \text{if } K_A > 0 \\ 0 & \text{if} K_A = 0. \end{cases}$$

where $D > 0$ is a positive parameter and $\beta > 0$ reflects the magnitude of spillover. Clearly, a group's payoff is increasing in own as well as other group's membership. Moreover, marginal payoffs increase are also increasing in own group as well as other group size.

In this setting, an individual gains by joining a second group, his current groups gains by dual membership due to positive spillovers and so social, group and individual interests all point toward universal multiple membership. Notice that in this example, there is no cost to joining a group. If joining a group entails personal costs while the returns are shared with the group then we face a familiar situation of positive externalities and individuals will typically provide too little membership.

Example 3 *Negative spillovers: conflict over land, minerals and public money*

There is a fixed resource and groups compete for a share of this resource. The resource may be land or minerals and the groups are tribes, ethnicities or religions. Alternatively the resource may be public funds for cultural activities, while the groups are language groups. The resource may be market share and the group may be an alliance of firms seeking an innovation.

Individuals provide ideas and labor; so a larger group is at an advantage while a larger opponent group is a disadvantage. The value of the resource is 1 and conflict entails a cost $c > 0$. We use the Tullock (1980) contest function and say that the payoffs to group A are:

$$R(K_A, K_B) = \begin{cases} 1 & \text{if } K_A > 0, K_B = 0 \\ \frac{K_A^\alpha}{K_A^\alpha + K_B^\alpha}(1-c) & \text{if } K_A > 0, K_B > 0 \\ 0 & \text{if } K_A = 0. \end{cases}$$

The payoffs to group B are analogous. The payoffs are increasing in own members and decreasing in membership of other group. We will refer to α as the technology of conflict parameter. Observe that when $\alpha = 0$ the group sizes do not matter for the probability of winning, while if α is very large then the large group wins almost for sure.

Recall, the payoff to player i under a strategy profile $s = (s_i, s_{-i})$ are given by:

$$\Pi_i(s_i, s_{-i}) = \frac{\mathbf{1}_{s_i(A)}}{K_A} R(K_A, K_B) + \frac{\mathbf{1}_{s_i(B)}}{K_B} R(K_B, K_A) \tag{2}$$

What are the incentives of an individual player? Let us start with the case where all players join group A, and we now ask if player i would gain by also joining group B. The payoff to player i from being in a group with n players, while the other group is empty is:

$$\Pi_i(s_i, s_{-i}) = \frac{1}{n} \tag{3}$$

In this situation, the payoff to player i if he joins group B, in addition to A is:

$$\Pi_i(s_i, s_{-i}) = \frac{1-c}{n} \frac{n^\alpha}{n^\alpha + 1} + (1-c) \frac{1}{n^\alpha + 1} \tag{4}$$

The incentives to join two groups thus depend on the value of α and c. It follows that for large α, there is no incentive to join a second group. On the other hand, for small α there is a strict gain in payoffs from joining the second group. More generally, there is a threshold $\alpha^* > 1$ such that for $\alpha < \alpha^*$, it is profitable to join both groups, while for $\alpha > \alpha^*$ it is better to stay with one group. Thus for $\alpha > \alpha^*$ a single group with universal membership is an equilibrium outcome.

Next consider the situation in which all players have joined both groups. The payoffs to a player are:

$$\Pi_i(s_i, s_{-i}) = 2 \frac{1-c}{n} \frac{1}{2} \tag{5}$$

Exiting from one group and remaining member of only one group leads to the following individual payoff:

$$\Pi_i(s_i, s_{-i}) = \frac{1-c}{n} \frac{n^\alpha}{n^\alpha + (n-1)^\alpha} \tag{6}$$

Thus, if everyone is a member of both groups then it is always strictly better to remain in both groups (so long as $\alpha < \infty$). Thus universal membership of both groups is always an equilibrium!

Social welfare exhibits the following ranking: $S(n,0) = S(0,n) = 1 > 1-c = S(x,y)$, for $x, y > 0$.

From above, we know that for $\alpha < \alpha^*$ the single group outcome cannot be sustained in equilibrium. When an individual becomes a member of a second group there is a loss due to conflict that is shared with current members of the current group. On the other hand, the gain to joining a new group accrues fully to this player. So this player underestimates the costs of dual membership. This gives rise to excessive membership in equilibrium. However, for $\alpha > \alpha^*$, social optimum is compatible with equilibrium. This suggests that the incentives for excessive membership are negatively related to the returns from group size advantage.

■

Example 4 *Indirect negative spillovers: private provision of local public goods*

A group consists of individuals who provide time and effort for a group specific local public good. Individuals have a fixed budget and allocate resources equally across the groups they join. Let n_A be the number of individuals who join group A solely, n_B be the number of individuals who join group B solely, and n_{AB} be the number of individuals who join both groups. The payoffs to group A are:

$$\hat{R}(s) = f(n_A + \frac{1}{2}n_{AB}) \tag{7}$$

where $f(.)$ is increasing. The payoff to player i under a strategy profile $s = (s_i, s_{-i})$ is:

$$\Pi_i(s_i, s_{-i}) = \hat{R}_A(s)1_{s_i(A)} + \hat{R}_B(s)1_{s_i(B)}. \tag{8}$$

So when a person switches to the other group the group loses 1 unit of contribution, while if a person moves from sole membership to dual membership then the group loses 1/2. Thus the payoff of a group is negatively affected by the size of the other group. This model is similar to the model of religious sects developed in Iannaconi (1992).

Finally observe that this example corresponds to the case where $F(K_i(s)) = 1$, and the payoff takes the form of a pure local public good, i.e., there is no congestion.

4 Identity: multiple *vs* narrow

This section studies the relation between socially efficient and Nash equilibrium construction of social identity. We focus on the case where $F(K_i(s)) = K_i(s)$.[2]

It is useful to start with the benchmark case in which one group's payoffs are independent of the size of the other group. In this case $R(x, y) = R(x)$, for all y; recall that payoffs are increasing in own membership $R(x+1) \geq R(x)$, for all $x \in \{0, 1, ..., n\}$. The following result is then immediate.

Proposition 1 *Suppose $R(x, y) = R(x)$ and $R(x+1) \geq R(x)$. Then universal multiple identities is socially optimal. This group formation is also an equilibrium. If $R(x)$ is strictly increasing in x then universal multiple identities is uniquely efficient and also the unique equilibrium.*

While group reward is increasing in own membership we do not have any restrictions on the per capita payoffs; they may be rising or falling or indeed the maximum may be attained at an intermediate point. Thus while there may be no conflict between social and individual incentives, there could still be a conflict between the insiders in a group and players outside the group who want to join it.

We now turn to the more interesting case where payoffs of a group are related to the size of both groups. Consider examples 2 and 3. There are two aspects of the examples we want to bring out: one, the relation between aggregate payoffs and groups sizes and two, the relation between marginal payoffs and group sizes. With regard to the former relation we note that in example 2, group payoffs are increasing in own size and in the size of the other group, while in example 3 group payoffs are increasing in own group size, but are falling in other group size. With regard to marginal returns, in example 2 we note that marginal payoff to a new member is increasing in the size of the other group, while in example 3, the marginal gains from a new

[2]The case of pure public goods, $F(K_i(s)) = 1$, for all $K_i(s)$ is of independent interest and is taken up in section 8.

member are falling with an increase in other group size. The concepts of positive and negative spillovers and strategic complements and substitutes capture these different effects.

Definition 1 *Formally, positive spillover obtains if for all* $(x, y) \in \mathcal{Z}_+^2$*,* $R(x, y+1) \geq R(x, y)$*, while negative spillover obtains if for all* $(x, y) \in \mathcal{Z}_+^2$*,* $R(x, y+1) \leq R(x, y)$*.*

The following result summarizes our analysis of games with positive spillovers.

Proposition 2 *If group payoffs are increasing in own group size and exhibit positive spillover then universal multiple identities is socially efficient as well as an equilibrium.*

Proof: Consider social efficiency. Starting at any $(x, y) < (n, n)$ the payoffs of both groups can be raised by increasing identities of any group. It follows that universal multiple identities is socially optimal. Similarly, starting at any $(x, y) < (n, n)$, a player i has an incentive to join both groups, and so universal identities is an equilibrium. ■

In Proposition 2 it is assumed that costs of joining groups are zero. If costs of joining different groups are significant then the positive spillovers could create a divergence between individual and collective interests: an individual does not take into account the positive effects of his membership on the payoffs of group members and this leads to narrow identities in excess of what is socially desirable. Groups seek to address this problem by offering subsidies to its members for joining other groups. This result offers a simple explanation for the research fellowships offered by research foundations to individuals to visit other counties or groups. It also explains why these fellowships often come with the obligation that the individual has to return to the sponsoring group for a minimum period of time.

We now turn to the case where the spillover across groups is negative. We will present two results. The first result covers the case when the marginal returns to increase in own group size exceed the fall in payoff of the other group.

Proposition 3 *Suppose group payoff is increasing in own group size and exhibits negative spillover. If* $R(x+1, y) - R(x, y) \geq R(y, x) - R(y, x+1)$*, for all* $x < n, y \leq n$*, then universal multiple identities is socially efficient and also an equilibrium. If this inequality is strict then universal multiple memberships is the unique equilibrium and the socially efficient outcome.*

Proof: Suppose that some $(x, y) \neq (n, n)$ is socially optimal. Suppose, without loss of generality, that $x < n$ and consider the configuration $(x + 1, y)$.

$$S(x + 1, y) = R(x + 1, y) + R(y, x + 1) \tag{9}$$

The hypothesis that $R(x + 1, y) - R(x, y) \geq R(y, x) - R(y, x + 1)$ implies that $S(x + 1, y) \geq S(x, y)$. Iterating on the argument yields the first part of the result.

Consider next the equilibrium statement. Start with the configuration (n, n); Individual payoff is given by $2R(n, n)/n$. Next consider deviations by an individual; suppose that a player withdraws from one of the two groups. Then his payoff is given by $R(n, n - 1)/n$. Note that $S(n, n) = 2R(n, n) \geq S(x, y)$, for all $(x, y) \leq (n, n)$. In particular then $S(n, n) = 2R(n, n) \geq R(n, n - 1)$. No individual has an incentive to deviate from the configuration (n, n).

We now take up the uniqueness part. First observe that unique efficient outcome follows from the argument above plus the strictness of the inequality. Next consider the equilibrium result.

We have already shown that (n, n) is an equilibrium. We now establish uniqueness. Suppose there is an equilibrium configuration $(x, y) \neq (n, n)$. First suppose that $x = n$ while $y < n$. The payoff to a player who is a member of one group only is $R(n, y)/n$. A deviation by a player which involves joining both groups yields this player:

$$\frac{1}{n} R(n, y + 1) + \frac{1}{y + 1} R(y + 1, n) \tag{10}$$

A deviation is strictly profitable if

$$\frac{1}{n} R(n, y + 1) + \frac{1}{y + 1} R(y + 1, n) > \frac{1}{n} R(n, y) \tag{11}$$

which is true if:

$$\frac{1}{y + 1} R(y + 1, n) > \frac{1}{n} [R(n, y) - R(n, y + 1)]. \tag{12}$$

Since $y + 1 \leq n$, this inequality is satisfied if:

$$\frac{1}{n} R(y + 1, n) > \frac{1}{n} [R(n, y) - R(n, y + 1)] \tag{13}$$

This last inequality is satisfied since $R(y+1,n)+R(n,y+1) > R(n,y)+R(y,n)$ and $R(x,y) \geq 0$, for all $0 \leq x, y \leq n$.
We now take up the case where $x, y < n$. Without loss of generality suppose $x \geq y$. The case where $x \geq y+1$ can be proved using a variation of the argument above. We turn to the case $x = y$.

A deviation in which a player in group A also joins group B is profitable if:

$$\frac{1}{y+1}R(y+1,x) > \frac{1}{x}[R(x,y) - R(x,y+1)]. \tag{14}$$

Noting that $x = y$ and rewriting, we get

$$xR(y+1,x) > (x+1)[R(x,y) - R(x,y+1)]. \tag{15}$$

Rearranging terms we get:

$$x[R(y+1,x) + R(x,y+1)] > xR(x,y) + [R(x,y) - R(x,y+1)]. \tag{16}$$

which is satisfied since $x[R(y+1,x)+R(x,y+1)] > x[R(x,y)+R(y,x)]$ under our hypothesis, $R(y,x) = R(x,y)$, and $R(x,y+1) \geq 0$.

∎

This result suggests that if negative spillovers are modest relative to the positive effects of own group size effects then social efficiency dictates multiple identities and this is also the unique equilibrium. While there is congruence between aggregate social and individual incentives, there may well be a tension between group incentives and individual incentives. This is because a group always wants the other group to be smaller due to negative spillovers. We discuss this tension further in section 5 below.

We now turn our attention to games in which positive own group size effects may be weaker than negative effects on other group. In some contexts, it is reasonable to suppose that competition among groups grows in intensity and wastefulness as groups get more equal. This situation is reflected in the following assumption: positive own size effects dominate negative spillover on other group if the growing group is larger. The following result considers socially efficient networks.

Proposition 4 *Suppose group payoff is increasing in own group size and exhibits negative spillover. Suppose that* $R(x+1,y)-R(x,y) \geq R(y,x)-R(y,x+1)$ *if* $x \geq y$ *and* $R(x+1,y)-R(x,y) < R(y,x)-R(y,x+1)$ *if* $x < y$, *then narrow identities with all players joining one group is socially efficient. If the inequality in payoffs is strict then it is the unique socially optimal outcome.*

Proof: Start with any profile (x, y) and set $x \geq y$, without loss of generality. Suppose to start that $x < n$. Then it follows from the hypothesis $R(x+1,y)-R(x,y) \geq R(y,x)-R(y,x+1)$ for $x \geq y$ that $S(x+1,y) \geq S(x,y)$. Iterate on this argument and we arrive at $S(n,y)$, where at each step social welfare increases weakly. Note that by hypothesis $n \geq y$. Next lower y, and note by the hypothesis $R(x+1,y)-R(x,y) \geq R(y,x)-R(y,x+1)$ if and only if $x \geq y$; so $R(y,x)-R(y-1,x) < R(n,y-1)-R(n,y)$ which implies that $S(n,y-1) > S(n,y)$. Iterate on this and we arrive at $S(n,0)$, where at each step we have increased the payoff weakly. This completes the argument. ■

A comparison of Propositions 3 and 4 is worthwhile as it brings out the role of negative spillovers nicely. The hypothesis in Proposition 3 says that the gain from an extra member is greater than the negative externality on the other group imposed by this move. In this case, social surplus grows as individuals subscribe to both groups. However, in Proposition 4 this inequality only holds if and only if the group gaining new members is the larger one: so to raise social surplus, we make a large group still larger by taking away individuals from the smaller group.

We now turn to individual incentives. Recall, that payoffs to a player i given profile s are:

$$\Pi_i(s_i, s_{-i}) = \frac{\mathbf{1}_{s_i(A)}}{K_A(s)} R(K_A, K_B) + \frac{\mathbf{1}_{s_i(B)}}{K_B(s)} R(K_B, K_A). \tag{17}$$

Start with the case of $(n, 0)$. In this configuration a player earns $R(n, 0)/n$. If she deviates and joins both groups then she will earn,

$$\frac{1}{n} R(n,1) + R(1,n). \tag{18}$$

Thus $(n, 0)$ is an equilibrium if and only if

$$\frac{1}{n} R(n,1) + R(1,n) \leq \frac{R(n,0)}{n}. \tag{19}$$

This can be rewritten as

$$\frac{1}{n}[R(n,0) - R(n,1)] \geq R(1,n) \tag{20}$$

Similarly, we can check that (n, n) is an equilibrium if and only if,

$$R(n,n) \geq \frac{1}{2}R(n,n-1). \tag{21}$$

Let us now compare aggregate returns and individual incentives in games with negative spillovers and in cases where the hypotheses of Proposition 4 hold. In this case, $(n, 0)$ is socially optimal and so $R(1, n) \leq [R(n, 0) - R(n, 1)]$. The divergence between social and private incentives is clear: when a player joins a second group, she shares the loss with the existing group but gets the full share of the gain from the new group. This creates incentives for multiple membership in excess of what is socially desirable.

Next we ask if the converse is possible? Suppose universal multiple identities is socially optimal; do players always have an incentive to join both groups? The payoff to a player in the (n, n) configuration is $2R(n, n)/n$. The only deviation involves withdrawing from one group and remaining a member of only one group and the payoff from this deviation is $R(n, n-1)/n$. For this to be optimal it must be the case that $2R(n, n) < R(n, n-1)$. This however contradicts the social optimality of (n, n). Thus (n, n) is an equilibrium outcome as well. We have thus shown that if (n, n) is socially optimal then it is also an equilibrium. The following result summarizes this discussion.

Proposition 5 *If universal multiple identities is socially efficient then it is also an equilibrium. Suppose hypotheses of Proposition 4 hold. There are excessive incentives for multiple identities relative to what is socially optimal if* $\frac{1}{n}[R(n, 0) - R(n, 1)] < R(1, n) \leq [R(n, 0) - R(n, 1)]$.

Let us briefly consider example 3 again. In this example if $D^2 + 2D > 1$ and $D < 1/2$ then the hypotheses of Proposition 4 are satisfied but $(2, 2)$ is an equilibrium. The intuition turns on the negative externality generated by dual identity for existing group members.

5 The incentives of groups

This section studies the relation between individual incentives and the interests of groups. To fix ideas, we will suppose that groups wish to maximize aggregate group payoff. This assumption is motivated by the observation that groups are often defined by political leaders and cultural entrepreneurs whose utility is an increasing function of the aggregate group payoff.[3]

In a game with positive spillovers, a group gains from having more members and also from a member joining the other group. Proposition 2 shows that this is also in the interests of the individual. Thus, in games with positive spillovers multiple identities are appealing to the group as well as the individual.

We turn next to games with negative spillovers. To get a first impression of the tension between individual and social incentives here, we revisit example 3. In this example, with $n = 2$, the unique equilibrium is $(2, 2)$; however, in this equilibrium, the payoff is lower than the payoff in the configuration $(2, 0)$. Recall, a group realizes that its members have an incentive to join the other group as the negative effect on the group is shared among the current members, while the benefits are not. A natural response of the group would be to impose an *exclusive membership* rule. What is the equilibrium if both groups use this rule? There are two equilibria: $(2, 0)$ and $(0, 2)$. In these equilibria, individual players as well the active group fare better than in the non-exclusive rules equilibrium. Moreover, if the equilibria are equally likely then a group gains in an ex-ante sense, as well. This illustrates how exclusive membership rules can lead to "better" outcomes.

We now examine the scope of such exclusive membership rules, more generally.

Our first observation is that if a group imposes an exclusive membership rule then the choice of rule for the other group – whether it is exclusive or not – is immaterial: if all groups except one choose exclusive membership rules then the choice of this last group makes no difference to the options available to the individuals. Hence,

Proposition 6 *All groups imposing an exclusive membership rule is a Nash equilibrium in a game of membership rules.*

What are the welfare and equilibrium implications of exclusive membership rules? In games which satisfy the hypotheses of Proposition 3, universal multiple membership is attractive in

[3]We have also studied the case where the objective of a group is to maximize the per capita payoffs of a member. Our main findings on the attraction of exclusive and narrow membership rules also obtain in this case. This happens when multiple memberships lower surpluses for all groups. The details of these arguments and computations are available from the authors.

the aggregate as well as incentive compatible. How about the interests of the group? Due to negative spillovers, a group would like larger own size and smaller size for the other group(s); in particular, a group would like all players to be its members exclusively. What happens in equilibrium? The following result covers the case where returns to own group size are convex.

Proposition 7 *Suppose group payoff is increasing and convex in x and exhibits strict negative spillovers. Then there exist two equilibria under exclusive membership rules corresponding to the configurations, $(0, n)$ and $(n, 0)$.*

Proof: We first show that $(n, 0)$ is an equilibrium. The payoff to a player in the $(n, 0)$ configuration is $R(n, 0)/n$. The payoff from a deviation, which entails switching to the other group, is $R(1, n-1)$. From negative spillovers, we know that $R(1, n-1) < R(1, 0)$. Since $R(0, 0) = 0$, from convexity of $R(x, y)$ in x we know that $nR(1, 0) < R(n, 0)$. Thus $(n, 0)$ [and $(0, n)$] are equilibria of the membership game under the exclusive membership rule.

We next want to show that they are the only equilibria in this game. Consider a configuration (x, y) and suppose without loss of generality that $x \geq y$. We will show that $(x, y) \neq (n, 0)$ is not an equilibrium. First consider the case $x = y$. Payoffs to a player in group A are $R(x, x)/x$, while payoffs to deviating to other group are $R(x+1, x-1)/x+1$. Note that

$$\frac{1}{x}R(x, x) \leq \frac{1}{x+1}R(x+1, x) < \frac{1}{x+1}R(x+1, x-1), \tag{22}$$

where the first inequality is due to increasing and convex returns in x, while the second inequality is due to strict negative spillover. Next consider the case $x = y + 1$. Note that

$$\frac{1}{x+1}R(x+1, y-1) \geq \frac{1}{x}R(x, y-1) > \frac{1}{x}R(x, y) > \frac{1}{y}R(y, y) \geq \frac{1}{y}R(y, x). \tag{23}$$

where the first inequality follows from hypotheses that payoffs are increasing and convex in own group size, the second inequality follows from strict negative spillover, the third inequality follows from convexity in own group size, while the last inequality follows from strict negative spillover. Next, consider the case $x > y+1$. Again, a variant of the above argument, applying the hypothesis that payoffs are increasing and convex in own group size, and negative spillover leads to the conclusion that players find it profitable to deviate from smaller group with y players to the group with x players. Thus the only equilibrium with $x \geq y$ is $(n, 0)$. Analogous arguments show that $(0, n)$ is the only other equilibrium with $x \leq y$. ■

Proposition 7 covers the case where payoffs are convex in own group size. We turn next to games in which payoffs are concave in own group size. An examination of the proof of Proposition 7 reveals that under exclusive membership rules, convexity of returns in own group size implies that an individual always prefers to join the larger group. The negative spillover effect goes in the same direction as well: a smaller opponent group is better news as an individual switches to a larger group. If the payoffs to a group are concave in own group size then the concavity of returns and negative spillovers press in opposite directions. If diminishing marginal returns dominate then two equal groups, $(n/2, n/2)$ arise in equilibrium. On the other hand, if negative spillovers of opponent group size dominate then the outcome will involve a single active group, $(n, 0)$ or $(0, n)$.

Propositions 3, 7, and the above discussion on concave payoffs, identify a class of games in which free membership rules lead to universal multiple identities, while exclusive membership rules lead to a single group outcome, i.e., either $(n, 0)$ and $(0, n)$. Under the hypotheses of Proposition 3, social efficiency dictates universal multiple identities. We argued earlier that exclusive memberships is an equilibrium in a game of rules among groups. Putting together these observations suggests that *societies may function with exclusive membership rules even when it is individually and collectively inefficient.*

We finally turn to the games covered by Proposition 4. Recall that in such games, we know from Proposition 5 that if $\frac{1}{n}[R(n,0)-R(n,1)] \leq R(1,n) \leq [R(n,0)-R(n,1)]$, then the socially optimal arrangement is not sustainable in equilibrium. We next note that if $R(n,0)/n > R(1, n-1)$ then, under the *exclusive membership* rule, the configurations $(n, 0)$ and $(0, n)$ are sustainable in equilibrium. Since these configurations are not sustainable in the free membership scenario, this implies that a policy of exclusive membership can *facilitate* the attainment of higher payoffs in some environments. These observations are summarized in the following result.

Proposition 8 *Suppose group payoff is increasing in own group size and exhibits negative spillover. Suppose that* $R(x+1,y) - R(x,y) \geq R(y,x) - R(y,x+1)$ *if* $x \geq y$ *and* $R(x+1,y) - R(x,y) < R(y,x) - R(y,x+1)$ *if* $x < y$*. If, in addition,* $R(1,n-1) < R(n,0)/n < R(n,1)/n + R(1,n)$*, then the outcomes* $(n,0)$ *and* $(0,n)$ *are socially efficient, sustainable under exclusive membership rules, but they are* ***not*** *sustainable under free membership rules.*

We conclude by noting that in example 3, with $n = 2$, the game satisfies the hypotheses of Proposition 8, so long as $D < 1/2$.

6 A Variation on the Model

In the model the choice is between being a member of one or two groups. Being a member of one group represents 'narrow' identity and being a member of both groups represents 'multiple' identity. In some contexts, individuals share common features but differ on others. In such settings the choice is between focusing on the dimension that separate individuals versus the dimension that is common across them. In this section, following a suggestion from one of the referees, we extend our basic model with the aim of drawing a clearer connection between such settings and the theoretical model.

Suppose that, in a country, there is a shared cultural feature (such as religion) but that there is also a feature (such as language) that distinguishes individuals. Individuals choose on whether to identify themselves on the exclusive or the shared characteristic. We ask: when will they choose to separate themselves and when will they choose the shared identity?

To study this question, we modify our model as follows. There are two *narrow* identities, A and B and there is one *broad* identity, which we denote AB. A person of type A can either choose to stay with the broad identity AB *or* to identify herself with her type A; similarly for someone with type B. Individuals cannot join multiple groups. Suppose that individuals of type A constitute a majority.

To fix ideas, consider the setting of Example 3: groups in conflict. For simplicity, suppose that conflict takes place between three groups if there are individuals that join each of the groups, A, B, and AB. The payoffs are now defined via a natural generalization of the payoffs in the basic model to allow for three groups. We study individual choices and discuss equilibrium.

First, consider the case where α is large: in other words, size confers a decisive advantage on the larger group. The outcome with everyone in the group AB constitutes an equilibrium. In this outcome the individual payoff is $1/n$. If an individual of type A deviates to group A, she will earn something close to 0, because her singleton group has a negligible probability of winning against the AB group with $n-1$ individuals. The outcome with members of the majority group A all choosing to join group A is also an equilibrium. The payoff to a majority individual is $(1-c)/K_A$. No one in the majority group has an incentive to deviate and join the group AB so long as $K_A - 1 > K_B + 1$.

Second, consider the case with small α. When α is close to zero, group size offers no advantage in the conflict. Now the outcome with everyone in group AB is less likely. To see this note that the individual payoff is $1/n$. A single individual can deviate to her type group and guarantee herself a payoff close to $(1-c)/2$. For large n and modest c it is clearly profitable to deviate away from the broad group. More generally, for small α, individuals in the minority group have an incentive to separate themselves into a group define by their type. In particular, if the minority is less than half the size of the majority group, then in equilibrium it must be the case that all members of the minority will choose group B, and the majority will divide equally in two groups A and AB equally. An instance of small α would be a country in which all cultural groups get an equal share of a public resource as a matter of principle (i.e., irrespective of the group size).

To summarize, an outcome with everyone in group AB is possible only if α is large, while the outcome with individuals separating into distinct groups (that include A and B) obtains for under large and small α. The outcome in which everyone joins group AB is efficient. Individuals choosing type based groups leads to an inefficient outcome (as it entails a cost c).

7 Concluding remarks

Personal identity has many facets, as it involves race, language, personal interests, religion, and ethnicity, among other attributes. Yet all over the world we see individuals defining themselves in narrow and exclusive terms. In this paper, our aim is to provide an explanation for this puzzle using a simple model of individual incentives and collective interests.

We take the view that, in day to day life, the different aspects of an individual personality remain latent and retain the feature of "perpetual possibilities". Social and economic context present a background against which individuals choose to retain these different possibilities or to commit to one of these possibilities and to renounce the others.

In a context of inter-group conflict, a larger opponent group lowers the payoffs. So a group gains strictly by prohibiting its current members from joining the other group. On the other hand, individual choice creates a negative externality: when a player joins a second group his 'original' group incurs a cost in terms of lost competitiveness, but the individual shares this cost with other group members. Thus individuals prefer to have rich/multiple identities in excess of what groups desire. This tension is a reason for widely observed narrow and exclusive personal identities.

8 Appendix: The case of pure public good

The appendix considers a model of pure public goods. Formally, $F(K_i(s)) = 1$, for all $K_i(s)$. We will use example 4 from section 3 to study pure public goods. The analysis highlights the tension between individual incentives and group incentives and illustrates the role of exclusive membership rules as a response to these tensions. We also identify the key role of public good returns function: if it is increasing and convex (concave) in group size then exclusive membership rules facilitate (prevent) socially optimal outcomes.

We first analyze the case of increasing and convex returns from group size.

Proposition 9 *Suppose the group produces a pure public good and returns function is increasing and convex. Then a single active group is socially optimal. Single active groups constitute an equilibrium, but universal multiple memberships is also an equilibrium.*

Proof: The first step in the proof is to observe that $S(n,0) = nf(n) > 2nf(n/2) = S(n/2, n/2)$, since $f(.)$ is increasing and convex. We next observe that any partition of population $(a, n-a)$, where $a < n$, is socially dominated by the $(n,0)$ configuration. Observe that

$$S(a, n-a) = af(a) + (n-a)f(n-a) \tag{24}$$

Suppose without loss generality that $a \geq n - a$, then it follows from convexity of f that $S(a+1, n-a-1) > S(a, n-a)$, and the proof follows by iterating on this step until we reach $a = n$.

Now examine a multiple membership configuration (n_A, n_B, n_{AB}) and suppose that $0 < n_{AB}$. We wish to show that any $n_{AB} > 0$ is dominated by $n_A = n$ ($n_B = n$). Start from (n_A, n_B, n_{AB}) and suppose without loss of generality that $n_A \geq n_B$. We have already shown that $n_{AB} = n$ is socially dominated by $n_{AB} = 0$. So we consider $n_{AB} < n$. So there is at least one person who belongs only to group A. Consider the effects of moving a person from multiple membership to single membership. There are two possibilities: one, it increases social payoffs and two, it lowers payoffs. In the former case:

$$\begin{aligned}(n_A + n_{AB})f(n_A + \frac{n_{AB}}{2} + \frac{1}{2}) + (n_B + n_{AB} - 1)f(n_B + \frac{n_{AB}}{2} - \frac{1}{2}) \\ \geq (n_A + n_{AB})f(n_A + \frac{n_{AB}}{2}) + (n_B + n_{AB})f(n_B + \frac{n_{AB}}{2})\end{aligned} \tag{25}$$

This can be re-written as follows:

$$
\begin{aligned}
&(n_A + n_{AB})[f(n_A + \frac{n_{AB}}{2} + \frac{1}{2}) - f(n_A + \frac{n_{AB}}{2})] \\
\geq\ &(n_B + n_{AB})f(n_B + \frac{n_{AB}}{2}) - (n_B + n_{AB} - 1)f(n_B + \frac{n_{AB}}{2} - \frac{1}{2})
\end{aligned}
\tag{26}
$$

Consider moving the next person from n_{AB} to single membership n_A. The social payoff is given by:

$$
(n_A + n_{AB})f(n_A + \frac{n_{AB}}{2} + 1) + (n_B + n_{AB} - 2)f(n_B + \frac{n_{AB}}{2} - 1) \tag{27}
$$

This social payoff is higher than the configuration $S(n_A + 1, n_B, n_{AB} - 1)$ if

$$
\begin{aligned}
&(n_A + n_{AB})f(n_A + \frac{n_{AB}}{2} + 1) + (n_B + n_{AB} - 2)f(n_B + \frac{n_{AB}}{2} - 1) \\
>\ &(n_A + n_{AB})f(n_A + \frac{n_{AB}}{2} + \frac{1}{2}) + (n_B + n_{AB} - 1)f(n_B + \frac{n_{AB}}{2} - \frac{1}{2})
\end{aligned}
\tag{28}
$$

Rearranging terms, noting that $f(.)$ is increasing and convex and using equation (26) leads us to infer that (28) holds. So, iterating on this step we eventually arrive at a exclusive membership configuration $(n_A + n_{AB}, n_B, 0)$, and at each step we are raising social payoffs. However, we have already shown that any such exclusive membership configuration is dominated by $(n, 0)$ and $(0, n)$.

We next consider latter case, where moving a person from exclusive A membership to multiple membership raises social payoffs. Then we exploit convexity and show that it is socially strictly better to move to universal multiple memberships. But we have already shown that $n_{AB} = n$ is socially dominated by $(n, 0)$ and $(0, n)$. This proves that $(n, 0)$ and $(0, n)$ are socially efficient.

Single active groups are an equilibrium outcome: the payoff to player i from a group of size n is $f(n)$. The payoff from multiple membership is $f(n - 1/2) + f(1/2)$; $f(.) = 0$ and convexity of $f(.)$ implies that such a deviation is not profitable. Similarly, moving to a new group is not profitable. Thus $(n, 0)$ is an equilibrium. Analogous argument applies in the case $(0, n)$.

Finally, consider the universal multiple membership outcome $(n/2, n/2)$. Individual payoff is given by $2f(n/2)$. The payoff from a single group membership is $f(n/2 + 1/2)$. So universal multiple memberships is an equilibrium so long as $2f(n/2) \geq f(n/2+1/2)$. This condition is satisfied for example if $f(x) = x^2$ and $n \geq 3$.

■

If returns to group size are convex, and two groups are active then it is better to move a person from the smaller group to the larger group. This raises returns from the larger group more than the decline in the returns in the smaller group; moreover, the pure public good assumption reinforces this effect as the larger group has more people enjoying the larger public good. The proof extends this simple intuition to cover multiple memberships. If everyone is in one group then an individual has a strict incentive to also be a member of this group, due to increasing and convex returns from group size. However, if everyone is a member of two groups, then multiple memberships yields a payoff $2f(n/2)$ while single membership of large group yields a payoff of $f(n/2+1/2)$; under plausible conditions the former is larger than the latter (e.g., if $f(.)$ is a quadratic function of group size). In other words, universal multiple memberships constitutes an equilibrium. Under exclusive membership rules, an individual has an incentive to join the larger group, and so there exist two equilibria, both of which involve only one active group.

Proposition 10 *Suppose the group produces a pure public good and returns function is increasing and convex. Then exclusive membership rules ensure socially optimal outcomes.*

We next examine the case where returns are increasing and concave in group size. Here the efficient outcomes are harder to characterize as negative spillover effects and concavity on own group size press in opposite directions. The following result points to the benefits of multiple memberships.

Proposition 11 *Suppose the group produces a pure public good and returns function is increasing and concave. Then universal multiple memberships are socially better than single active group outcomes and also constitute an equilibrium.*

In a situation with increasing returns, exclusive memberships will lead individuals to move to the larger group, and so only single active groups are possible in equilibrium. From Proposition 11 these outcomes are inefficient. So we have shown: *with increasing and concave returns, exclusive membership rules lead to single active group outcomes which are socially inefficient.*
Proof: Universal multiple memberships are an equilibrium outcome: the payoff to a player in such a configuration is $2f(n/2)$. The payoff from a deviation to a single group membership is $f(n/2+1/2)$. Observe that:

$$f(\frac{n}{2}+\frac{1}{2}) \leq f(\frac{n}{2})+f(\frac{1}{2}) < f(\frac{n}{2})+f(\frac{n}{2}) = 2f(\frac{n}{2}), \tag{29}$$

where we have used concavity of $f(.)$ to derive the first inequality and strictly increasing f and $n \geq 2$ to derive second inequality. Thus universal multiple memberships is an equilibrium. ■

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