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WAGE POSTING, NOMINAL RIGIDITY, AND CYCLICAL INEFFICIENCIES

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We consider a Burdett and Mortensen style wage posting model with aggregate shocks. We analyze the equilibrium under two alternative assumptions on wage setting in ongoing jobs: either fully flexible or downwardly rigid. In the model firms optimally pay only retention premiums. The equilibrium is characterized by a Taylor expansion. The model yields two simultaneous relations for wages and quits, of which the parameters are simple functions of three empirically observable arrival rates of: (i) jobs, (ii) lay offs, and (iii) aggregate shocks. Hence, there are overidentifying restrictions, which are supported remarkably well by the data. We find strong evidence for wage downward rigidity and inefficiently low job-to-job transitions during the downturn. Furthermore, we find evidence that firms pay only retention premiums, not hiring premiums. A model with wage rigidity in ongoing jobs and OJS is therefore a useful benchmark for a wage equation in macro models.

Wage posting, nominal rigidity, and cyclical inefficiencies*

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Abstract

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Keywords: Nominal wage rigidity, on-the-job search, job-to-job transitions

JEL Classification: J31, J63, J64

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1 Introduction

Are wages rigid, and if so, why? There is a large literature that suggests so and that provides insights into why this is the case, e.g. Baker et al. (1994), Campbell and Kamlani (1997), Card and Hyslop (1997), Bewley (1999), Shimer (2004, 2005) and Hart and Moore (2008). Wage rigidity for ongoing jobs implies that current wages are determined by the unemployment rate at the date of hiring. Bils (1985) reported that wages for new hires are indeed more cyclical then those of job stayers. Later on, Beaudry and DiNardo (1991) found that current wages are related to the lowest unemployment rate since the date of hiring rather than at the date of hiring itself. This suggests that wages are downwardly rigid, but can be increased when that is optimal given labour market conditions. The risk that workers might quit is an important argument for firms not to reduce wages during a recession, next to the negative effect of wage cuts on worker moral, see Campbell and Kamlani (1997). One would therefore expect that these findings had played a major role in models with On-the-Job Search (OJS) in the spirit Burdett and Mortensen (1998) celebrated model of wage posting, and its later offsprings like Bontemps et al. (2000), and Gautier et al. (2010). Wages can only be used as an effective instrument for retention when firms can either explicitly or implicitly commit to future wage payments: when receiving an outside offer, workers will not respond to the carrot of a retention premium if they expect firms to renege on its payment as soon as the outside offer has been rejected. This is exactly what the concept of a posted wage reflects: a commitment to pay a particular wage for the duration of a job. Nevertheless, papers written in this tradition have largely ignored this issue. We suspect that one of the reasons why the literature has largely avoided the analysis of OJS models with this type of inefficiency is their mathematical complexity. However, we shall present evidence that there are, in fact, considerable inefficiencies in job-to-job transitions.

The current paper fills this gap. We apply a random search model with OJS, where the distribution of match productivity is exogenous and where the economy switches between the up- and the downturn. The key question is then how wages in ongoing jobs respond to this cyclical variation. We consider two different assumptions. Under the first assumption, wages are fully flexible. The (implicit) contract offered by the firm stipulates how the wage will be adjusted in response to cyclical variations in the job offer arrival rate. Our model is then almost identical to a standard wage posting model.

Under the alternative assumption, wages can not be lowered. When labour market conditions improve, the firm can unilaterally raise its wage offer, since this is in the joint interest of the worker and the firm; it is in the worker's interest, because she receives a better pay; it is in the firm's interest, as it improves the trade off between the wage bill and the risk of the worker quitting. When the job offer arrival rate goes down, the firm would like to reduce its wage offer, as the risk of the worker quitting has decreased. However, if the firm deviates by violating the implicit contract and sets a lower wage, then workers are assumed to expect firms to pay the lowest wage forever afterwards, as in Coles (2001). The worker will then quit as soon as she receives another offer. The firm will then optimally not lower its wage offer, leading to downward rigidity, as in Beaudry and DiNardo (1991).

The characterization of the equilibrium relies on two innovations. Firstly, similar to Coles and Mortensen (2016), we analyze a model where wages only affect retention and not hiring. Secondly, we approximate the loss of downward rigidity by a second order Taylor expansion. We consider both innovations in turn. Burdett and Mortensen (1998) assume that firms post binding wage offers before encountering a job seeker. In that context, it is optimal for the firm to pay both hiring and retention premiums. We follow Gautier et al. (2010) in making a distinction between hiring and retention premiums. Workers learn about the quality of a potential match before they actually apply for the job. An application comes at some epsilon cost in our model. Workers will then only apply if they expect the firm to make an acceptable offer. Hence, the firm has no incentive to pay a hiring premium.

The fact that firms pay only retention premiums greatly simplifies the analysis. Hiring premiums are backward looking: a firm must consider the whole history of wage setting by its competitors to infer the effect on hiring. Retention premiums, to the contrary, are forward looking: they depend on the job offer arrival rate only. Where our assumption of firms paying only retention premiums serves the purpose of analytical convenience, our framework provides a test of this assumption. If firms were to pay hiring premiums, they would have no incentive to raise wages after an increase in the job offer arrival. After hiring, the hiring premium is a waste of money. Hence, the hiring premium provides a buffer that should be depleted before the increase in the optimal retention premium justifies an increase in wages. Our evidence suggest that firms respond to even small increases in the job offer arrival rate by increasing their wage offers, suggesting that firms do not pay hiring premiums.

Even with this convenient assumption, characterizing the equilibrium is a hard problem. We use an idea by Akerlof and Yellen (1985), who apply a Taylor expansion for the characterization of the equilibrium. While the difference in wage offers between newly created jobs and jobs that started before the downturn of the economy is a first order phenomenon, the difference in profitability is only a second order phenomenon. The first order term is zero by the envelope theorem. With this trick the model allows for a convenient decomposition of the cyclical component of wages into three terms reflecting: (i) the change in the job offer arrival rate, (ii) the change in the value of output, and (iii) the mitigating effect of downward rigidity on wage increases during the upturn: when making a wage-offer during the upturn, the firm accounts for the risk of a future downturn. The first effect yields a unit elasticity of wages with respect to the job offer arrival rate, the second effect leads to an even higher elasticity, while the third effect leads to a reduction of the elasticity below unity. Empirically, the second effect is small, since the job offer arrival rate is orders of magnitude more volatile than the value of output. Hence, the model predicts an elasticity that is (substantially) smaller than unity.

Estimation of these relations requires a proxy for the match quality. Here, we use previous results discussed in Gottfries and Teulings (2016). Using their proxy, the model generates two simple log linear equations, one for wages and one for the quite rate. The coefficients of both equations are simple analytical functions of three arrival rates: (i) new jobs, (ii) layoffs, and (iii) aggregate shocks. All three can be directly estimated from the data. Applying these estimates, our two log linear equations yield five overidentifying restrictions. Both equations are estimated using data from the NLSY 79. We find strong evidence for wage rigidity and inefficiently low job-to-job transitions during the downturn. Furthermore, the overidentifying restrictions are supported remarkably well by the data, in particular so for the wage equation.

Recently, there has been a lot of discussion about the proper correction for match quality, see for example Kudlyak (2011), Hagedorn and Manovskii (2013), Bauer and Lochner (2016), Bellou and Kaymak (2016) and Galindo da Fonseca et al. (2016). Without a proper correction, Beaudry and DiNardo (1991)'s conclusion that wages in ongoing jobs are determined by the lowest unemployment rate since the date of hiring might be due to a misspecification. Hagedorn and Manovskii find that once a proxy for match quality is included, wages exhibit no history dependence. Using arguments discussed in

Gottfries and Teulings (2016) we reach the opposite conclusion, see Section 3 for details. Kudlyak and Bellou and Kaymak revisit the evidence of Hagedorn and Manovskii (2013) and find that in a more flexible specification, a model with history dependence fits the data best. Galindo da Fonseca et al. emphasize that an aggregate analysis misses substantial differences across occupations. In particular, the prevalence of performance pay is an important determinant of the cyclicality of wages.

A number of other papers include business cycle variation into a model with OJS, for example Menzio and Shi (2011) for a model with directed search, Moscarini and Postel-Vinay (2013) and Coles and Mortensen (2016) for a model with wage posting and Lise and Robin (2017) for a model with the sequential auctions. Our model relates most closely to Moscarini and Postel-Vinay and Coles and Mortensen. Our innovations compared to these papers is the inclusion of wage rigidity in a tractable way. Wage rigidity has been introduced before in search models without OJS in the tradition of Mortensen and Pissarides (1994), see for example Shimer (2004) and Hall (2005) and in models with OJS without an endogenous response of the quit rate to the wage (Carlsson and Westermark, 2016; Gertler et al., 2016). In these models, inefficiencies may arise due to excess layoffs or violations of the Hosios condition. In our model, the job destruction rate is constant, ruling our inefficient layoffs by assumption. Our paper features full wage flexibility for new jobs. Inefficiencies arise in our model due to excessively low job-to-job transitions, which can only occur in models with OJS. In turn, a low level of job-to-job transition rates in recessions makes it less profitable to open vacancies then. Focusing on quits rather than layoffs is consistent with the evidence by Elsby et al. (2009), Fujita and Ramey (2009) and Shimer (2012) that job finding explains most of the variation in unemployment. Our empirical evidence provides strong support for an inefficiently low quit rate in recessions. Moreover, the degree of wage flexibility for new jobs observed in the data is consistent with the predictions of our model. Hence, we expect a model with wage rigidity in ongoing jobs and OJS to provide a useful benchmark for macro-economics.

The structure of this paper is as follows. Section 2 develops the theoretical model and characterizes its equilibrium for both versions, with flexible and downwardly rigid wages. Section 3 lays out our methodology for testing the model and presents our regression results. Section 4 concludes.

2 The model

2.1 Assumptions

Our point of departure is a wage posting model with OJS. The production structure is similar to the circular model by Marimon and Zilibotti (1999). Both jobs and workers are points distributed uniformly on the circumference of a circle with length 2. The quality of a potential match between a worker and a firm is measured by the distance F between their positions on the circumference of the circle: shorter the distance, the higher the productivity X_t of the match. By construction, F is uniformly distributed on the unit interval; the best possible match has $F = 0$ and the worst possible match has $F = 1$; we shall therefore refer to F as the mismatch indicator; F is also equal to the probability that a random job offer has lower mismatch than the current job. Let $X_t(F)$ be productivity of a match with mismatch F, where t is time; we assume $X_t(F)$ to be strictly decreasing and twice differentiable. The reservation wage for unemployed job seekers is B_t. Job seekers, either employed or unemployed, receive job offers at a rate λ_t.[1] We allow $\lambda_t, X_t(F)$ and B_t to vary over time. More specifically, there are two states of the economy, the upturn and the downturn. The economy switches back and forth between both states at a rate θ. Each state is characterized by different values for $\lambda_t, X_t(F)$ and B_t.

$$\begin{aligned} \lambda_d &= \lambda\left(1-\tfrac{\Delta}{2}\right), & \lambda_u &= \lambda\left(1+\tfrac{\Delta}{2}\right), \\ X_d(F) &= X(F)\left(1-x\tfrac{\Delta}{2}\right), & X_u(F) &= X(F)\left(1+x\tfrac{\Delta}{2}\right), \\ B_d &= B\left(1-x\tfrac{\Delta}{2}\right), & B_u &= B\left(1+x\tfrac{\Delta}{2}\right), \end{aligned}$$

with $t \in \{u, d\}$, where u is the upturn and d the downturn, where $\Delta > 0$ and $x \geq 0$ are parameters. This modelling of the effect of cyclical variations is *rank preserving*: the rank ordering of productivity $X_t(F)$ for different degrees of mismatch F and the reservation wage of unemployed job seekers remains the same when the economy transits between the up- and the downturn. The parameter x is the ratio of the coefficients of variation of $X_t(F)$ and λ_t. Empirically, the coefficient of variation of the job offer arrival rate is orders of magnitude greater than that of the value of output, hence $x \ll 1$.

[1] In our empirical application we allow for the job offer arrival rate to differ between employed and unemployed job seekers. In the theoretical model, this yields complications since job-seekers give up part of the option value of search by accepting a job when on- is less efficient than off-the-job search. We avoid these complications in our theoretical analysis.

This rank preserving specification implies that the match quality F is not affected when the economy transits between the up- and the downturn; only the productivity $X_t(F)$ changes from $X_u(F)$ to $X_d(F)$ or the other way around. We assume that the surplus of employment is positive in all states, even if we compare the output in the worst match during the downturn to the value of leisure in the upturn $X_d(1) \geq B_u$. Jobs end at an exogenous job destruction rate δ, which we assume to be constant over time. Several authors have analyzed what share of the cyclical variation of the unemployment rate is due to variation in job finding rate versus variation in the separation rate rate, see Elsby et al. (2009), Fujita and Ramey (2009) and Shimer (2012). The estimates suggest that the job finding rate is more important than the job destruction rate. The estimated contribution is centered around 2/3 but falls somewhat when the participation margin in included (Elsby et al., 2015). Hence, a model that treats the job destruction rate as a time-invariant constant still captures most of the variation. Both workers and firms discount the future at a rate ρ. It is convenient to define χ as the sum of the job destruction rate, the interest rate and the arrival rate of aggregate shocks $\chi \equiv \delta + \rho + \theta$.

Let $W_t(F)$ be the wage offer as a function of the mismatch F, and let $F_t(W)$ be the fraction of firms that a pay a wage strictly higher than W. Hence, $F_t(\cdot)$ is the inverse function of $W_t(\cdot)$. Assuming $W_t(F)$ to be strictly decreasing, these definitions imply for all $F \in [0, 1]$

$$\begin{aligned} F_t[W_t(F)] &= F, \\ f_t[W_t(F)] &= W_t'(F)^{-1}, \\ f_t'[W_t(F)] &= -W_t'(F)^{-3} W_t''(F), \end{aligned} \tag{1}$$

where $f_t[\cdot]$ is the derivative of $F_t[\cdot]$. When a firm creates a vacancy it sends a signal about its position on the circle. When a job seeker meets a vacancy, she observes this signal. By comparing this signal to her own position on the circle, she can infer the mismatch F of that match. Based on this information, she decides whether or not she will apply for the job. An application comes at an infinitesimal small cost $\varepsilon \to 0$ to the job seeker. Only after application, the firm observes the position of the job seeker on the circle. It can then infer the mismatch F. However, this information is non-observable by the job seeker. Unlike in the standard models by Burdett and Mortensen (1998) and Bontemps et al. (2000), firms can therefore not commit to a binding wage offer contingent on the mismatch F prior to the application of the worker.

The firm could always renege on this posted wage offer by claiming that the match is of lower quality. Hence, it can post a wage only after the worker and the firm have met.

Since a job-seeker can infer the match quality from the signal sent by the firm, she only applies for jobs where she expects her asset value to be higher than in her current job (unemployed job seekers will apply to any job, since $X_t(1) \geq B_t$ for all $t \in \{u, d\}$). The firm can infer from the job-seeker's costly application that she expects the value of the new job to be greater than the value of the job she currently holds. Hence, as long as all other firms do not pay hiring premiums, the firm can make an offer without a hiring premium because the worker applied for the job anyway and she did not expect the firm to pay a hiring premium in the first place. Hence, this is a sub-game perfect equilibrium. The firm does not have to worry about the job-seeker rejecting its offer since in equilibrium only those job seekers apply who will accept the offer anyway. There are other equilibria, for example where all firms paying hiring premiums. Then, job seekers expect firms to pay hiring premiums. Hence, some applicants will reject offers that do not include a hiring premium, which makes such an offer not best response of the firm. However, the paper focuses on the equilibrium without hiring premiums.

In equilibrium, firms do not have an incentive to signal another than their true position on the circle, since by mis-signaling some job seekers do not apply who would have accepted the firm's wage offer, while some other job seekers do apply, who will in the end reject the firm's wage offer, since it is lower than she had expected.

When workers decide on whether or not to accept an outside offer, they have to form beliefs about the firm's future wage policy. Since workers do not observe the match quality F, they have to base their beliefs about this future wage policy on the current wage offered by the firm.[2] We consider two different assumptions about workers' beliefs the first, as in Coles and Mortensen (2016), leading to full flexibility of wages in ongoing jobs in response to aggregate shocks and the second leading to downward wage rigidity. In both cases all future wage expectations change in response to a change in the wage.

In the first version, workers expect firms to continue paying the current

[2]The worker could also base it beliefs on the signal on match quality provided by the firm. We assume that if this signal and the actual wage policy are inconsistent, then the worker bases its beliefs on the actual wage only and ignore the firm's prior signal.

wage as long as the aggregate state of the labour market remains the same. As soon as the aggregate state changes, workers expect firms to reoptimize. After this reoptimization, workers again expect firms to continue paying the new reoptimized wage until the aggregate state changes again, when firms are expected to reoptimize again.

In the second version, workers expect firms not to reduce the current wage at any time in the future. If a firm deviates from this belief by reducing its wage offer today, the worker expects the firm to deviate in the future by paying only the reservation wage of an unemployed job seeker, B_t. Hence, the worker will accept any outside offer that comes in today that pays slightly more than B_t. Since the worker accepts any outside offer that pays more than B_t after a slight downward deviation of the firm today anyway, it does not make sense for the firm to pay more than B_t after today. Hence, the worker's and the firm's out-of-equilibrium beliefs form a consistent set of best responses. Given these beliefs, the firm does not find it attractive to reoptimize its wage offer when the economy enters the downturn since it would undermine the credibility of retention premium. Hence, wages are downwardly rigid.

In this paper, the firm can continuously adjusts the wage but workers are unable to observe mismatch. Contrary to these assumption, Gottfries (2017) considers a model where workers do observe match quality F but wages are infrequently renegotiated. In that case, a deviation of the firm only changes the expected wage for the duration of the contract as the worker rationally expects the firm to use the actual value of F when deciding on its optimal offer. Hence, if workers were to observe F, firms would only be able to pay retention premiums by using a wage contract as commitment device. That is also their optimal strategy (Gottfries, 2017).

The empirical evidence provided by Campbell and Kamlani (1997) and Bewley (1999) suggests that firms are indeed reluctant to reduce wages during to the downturn to avoid a weakening of worker moral. Several other theoretical models predict this type of response of firms. Hart and Moore (2008) argue that workers do not have to quit the firm for retaliation after a disputed wage cut. They can retaliate at zero cost by working to rule. In a world with fully flexible wages, workers have to form beliefs about the firm's future offers. Coles (2001) considers a model where a slight deviation drastically changes workers beliefs and hence, the equilibrium does not unravel, in contrast to Gottfries (2017) where beliefs only change during the contract length and the equilibrium unravels.

The next two subsections characterize the wage setting policy and the asset values of the firm and the worker for both versions of the model.

2.2 Flexible wages

Since firms reoptimize their wage offer after a change in the aggregate state of the labour market, $t \in \{u, d\}$, the optimal wage offer of other firms is a function of t. The asset value of a filled job for the firm, $\Pi_t^o(F, W)$, as a function of the wage and the worker, $V_t^o(W)$, read

$$\begin{aligned} [\chi + \lambda_t F_t(W)] \Pi_t^o(F, W) &= X_t(F) - W + \theta \Pi_{-t}[F_t(W)], \\ \chi V_t^o(W) &= W + EV_t^o(W) + \theta V_{-t}[F_t(W)], \end{aligned} \tag{2}$$

for $\forall F \in [0,1], \forall W \in [0, \infty]$ and $t \in \{u, d\}$, and where

$$EV_t^o(W) \equiv \lambda_t \int_W^\infty [V_t^o(w) - V_t^o(W)] \, dF_t(w)$$

denotes the option vale of finding a better paying job. Since the firm's wage offer maximizes the value of a filled job, it satisfies

$$W_t(F) = \arg\max_W \Pi_t^o(F, W). \tag{3}$$

The equilibrium asset values of the worker and the firm are defined as

$$\begin{aligned} \Pi_t(F) &\equiv \Pi_t^o[F, W_t(F)], \\ V_t(F) &\equiv V_t^o[W_t(F)], \\ EV_t(F) &\equiv EV_t^o[W_t(F)]. \end{aligned}$$

We use equation (2) and (3) to characterize the market equilibrium.

Proposition 1 *Characterization of the equilibrium for flexible wages*

$$\begin{aligned} (\chi + \lambda_t F) \Pi_t(F) &= X_t(F) - W_t(F) + \theta \Pi_{-t}(F), \\ \chi V_t(F) &= W_t(F) + EV_t(F) + \theta V_{-t}(F), \\ -W_t'(F) &= \lambda_t \Pi_t(F), \\ W_t(1) &= B_t, \\ q_t(F) &= \lambda_t F, \end{aligned}$$

for $\forall F \in [0,1]$ *and* $t, -t \in \{u, d\}, t \neq -t$. $\Pi_t(F)$ *and* $V_t(F)$ *are the asset values of a firm and the worker;* $q_t(F)$ *is the quit rate from a job with mismatch* F.

$$EV_t(F) \equiv \lambda_t \int_0^F [V_t(f) - V_t(F)]\, df.$$

The proof of this proposition follows standard arguments in the literature. The first two equations in Proposition 1 are the Bellman equations for the asset value of the firm and the worker respectively. The only special feature in these equations is their final term which captures the effect of a transition between the up- and the downturn of the economy; in that case, the firm reoptimizes its wage offer, which then changes from $W_t(F)$ to $W_{-t}(F)$. The third equation is the first order condition associated with equation (3). It is a differential equation. The fourth equation is the initial condition for this differential equation: the wage in the least productive job with mismatch $F = 1$ is equal to the reservation wage of an unemployed.

We make four observations regarding this equilibrium, all well understood in the literature on this type of models

1. All job transitions in this model are efficient, that is, workers exploit all available opportunities to move to more productive jobs;
2. Workers accept a wage offer if and only if it is higher than their current wage;
3. The wage offer distribution can be calculated without reference to the asset value of the worker;
4. The wage offer distribution can be calculated separately for each state of aggregate labour market conditions.

2.3 Downwardly rigid wages

Downwardly rigid wages lead to a slight change in the conditions for a market equilibrium. As before, when the economy transits from the down- to the upturn, firms reoptimize their wage offer. However, downward rigidity of wages does not allow firms to reduce their offer when the economy transits back from the up- to the downturn. Hence, the wage is the maximum of the current wage and the ideal reoptimized wage. For the sake of simplicity, we focus on the case where $\Delta = (\lambda_u - \lambda_d)/\lambda$ is small, so that $W_u(F) \leq X_d(F)$ for all F and the continuation of the employment relation remains profitable for all firms, even when wages are downwardly rigid. Proposition 2 below can be adapted to allow for the case $W_u(F) > X_d(F)$ for some F, but that would complicate the model.

Proposition 2 *Characterization of the market equilibrium for downwardly rigid wages.*

$$
\begin{aligned}
(\chi + \lambda_d F)\,\Pi_d(F) &= X_d(F) - W_d(F) + \theta\Pi_u(F), \\
(\chi + \lambda_u F)\,\Pi_u(F) &= X_u(F) - W_u(F) + \theta\Pi_d^o\,[F, W_u(F)], \\
-W_d'(F) &= \lambda_d\Pi_d(F), \\
-W_u'(F) &= \lambda_u\Pi_u(F) - \theta\,\partial\Pi_d^o(F,W)/\partial W|_{W=W_u(F)}\,W_u'(F), \\
W_t(1) &= B_t, \\
q_t(F) &= \lambda_t F,
\end{aligned}
\tag{4}
$$

for $\forall F \in [0,1]$, $t \in \{u,d\}$ *and* $\forall W \in [0,\infty)$, *where* $V_{d+}^o(W)$ *is the asset value of a worker holding a job during the downturn while the wage has been set in the upturn;* $q_{d+}(F)$ *is the quit rate from that job.*

$$
\begin{aligned}
V_{d+}^o\,[W_{d+}(W)] &= V_d^o(W), \\
W_{d+}(W) - W &= \theta\left(V_u^o\,[W_{ud}(W)] - V_u^o\,[W_{d+}(W)]\right), \\
W_{d+}(W) - W &= [\chi + \lambda_d F_d(W)]\,[\Pi_d^o(F,W) - \Pi_d^o\,[F, W_{d+}(W)]], \\
q_{d+}(F) &= \lambda_d F_d\left[W_{d+}^{-1}(W_u)\right], \\
W_{ud}\,[W_d(F)] &\equiv W_u(F),
\end{aligned}
\tag{5}
$$

The functions $V_t^o(W)$ *and* $\Pi_d^o(F,W)$ *are defined in equation (2).*

The proof of this proposition follows from the arguments discussed below. The equations (4) are a straightforward extension of Proposition 1. The first two equations are the firms' asset values in the down- and the upturn respectively. The equation for the downturn is the same as in Proposition 1. The equation for the upturn differs since wages are downwardly rigid. The asset value $\Pi_{d+}(F)$ in the term for the transition to the downturn accounts for this wage rigidity. The third and fourth equations characterize the firm's wage setting policies. Again, the equation for the downturn is the same as in Proposition 1. The equation for the upturn differs, as firms account for the risk that the economy may transit to downturn; in that case, downward rigidity prevents them from re-optimizing their wage policy. The fifth equation is again the initial condition for the differential equations characterizing the wage setting policies, while the sixth equation describes the quite rate.

There are two missing links in equation (4), which are dealt with in equation (5). For the first missing link, consider a worker who holds a job in which the wage has been set during the upturn while the economy has now shifted to the downturn. This wage is bound by downward rigidity. Suppose that this worker receives an outside offer W. What is the maximum value of her current wage $W_{d+}(W)$ for which she would accept this offer? This wage $W_{d+}(W)$ satisfies the condition in the first line of (5): the asset value in the new job should be equal to the asset value in her current job. The worker will accept any outside offer that pays more than her current wage. However, the worker will also accept an outside offer that pays slightly less than her current job because when the economy transits back to the upturn, the worker would not receive a wage increase in her current job since that wage was been set in the previous upturn, so there is no need for upward adjustment. In the new job, she will get a wage increase, from W to $W_{ud}(W) > W$. Hence $W_{d+}(W) > W$. Analogous to equation (2), $V_{d+}^{o}[W_{d+}(W)]$ satisfies

$$\chi V_{d+}^{o}[W_{d+}(W)] = W_{d+}(W) + EV_{d}^{o}(W) + \theta V_{u}^{o}[W_{d+}(W)]. \tag{6}$$

This equation differs from equation (2) in two aspects. First, the term for the change in the asset value when the economy transits to the upswing accounts for the fact that the wage will not be increased in that case. Second, the option value term $EV_{d}^{o}(W)$ accounts for the fact that a worker will accept jobs that offer a wage higher than W rather than higher than $W_{d+}(W) > W$, as we argued before. Combining the Bellman equations (2) and (6) and using $V_{u}[F_{d}(W)] = V_{u}^{o}[W_{ud}(W)]$ yields the second equation in (5). This equation states that from the marginal wage offer that a worker is willing to accept the gain $W_{d+}(W) - W$ of a higher wage in the current job is equal to the expected value gain of getting a higher wage in the new job when the economy transits to the good state, $V_{u}^{o}[W_{ud}(W)] - V_{u}^{o}[W_{d+}(W)]$, times the probability θ that this will happen during the next infinitesimal time interval. Since $W_{d+}(W) > W$ and since $V_{u}^{o}(W)$ is an increasing function of W, this implies

$$W_{ud}(W) > W_{d+}(W) > W. \tag{7}$$

The wage $W_{d+}(W)$ that makes a worker indifferent between accepting the new job that pays W and staying in the current job must be in between the wage in the new job W and the wage $W_{ud}(W)$ that she will earn in this new job in case the economy transits back to the upturn.

The second missing link is the asset value $\Pi_d^o[F, W_{d+}(W)]$ for a firm employing a worker with mismatch F during downturn of which the wage has been set during the upturn. This asset value can be solved using the solution to the first missing link, see the third equation in (5). The right hand side is the difference in asset values during the downturn for a new job that offers a wage W and a new job that offers a wage $W_{d+}(W)$, which has been set during the upturn. By construction, a worker is equally likely to quit from both jobs since they yield the same asset value for the worker. The relevant quite rate is $\lambda_d F_d(W)$. Hence, the Bellman equations for the firm's asset value for these two jobs imply that difference in asset values times the transition rate $\chi + \lambda_d F_d(W)$ is equal to the wage differential $W_{d+}(W) - W$. The final equation in (5) characterizes quit rate during the downturn from a job with mismatch F where the wage has been set during the upturn. Since this wage is higher than the current market wage for a new job with similar mismatch, the quit rate from these jobs is lower. The worker holding such a job gets paid a wage $W_u(F)$. By the definition of W_{d+}, the wage that matches the asset value of that job is $W_{d+}^{-1}(W_u(F))$. The probability that an incoming offer pays that wage is $F_d\left[W_{d+}^{-1}(W_u)\right]$. By equation (7) $W_{ud}(W_d(F)) = W_u(F) > W_{d+}(W_d)$; hence, since $W'_{d+}(W) > 0$, $W_{d+}^{-1}(W_u(F)) > W_d(F)$ and $F_d\left[W_{d+}^{-1}(W_u(F))\right] < F_d(W_d(F)) = F$: for equal F, the quit rate during the downturn from jobs that started during the upturn is lower than the quit rate from similar jobs that started during the downturn.

We can now ask ourselves whether the four observations made in the previous subsection regarding the equilibrium with flexible wages still apply in the equilibrium with downwardly rigid wages. The answer is negative in all four cases:

1. Are all job transitions efficient? No: in the downturn, job seekers whose current wage is set when the economy was still in the upturn turn down wage offers for jobs that are slightly more productive than their current job since these jobs pay lower wages during the downturn;

2. Does the worker accepts only job offers which offer a wage higher than their current job? No: in the downturn, a worker holding a job that started in the upturn might accept a job that pays slightly less because she expects the wage in this job to be raised when the economy tran-

sits back to the upturn, while the wage in her current job will remain unchanged;

3. Can the wage offer distribution be calculated without reference to the workers' asset value equation? No, during the downturn the asset value of the worker determines how much lower a wage a worker holding a job of which the wage is set in the upturn is willing to accept;

4. Can the wage offer determined separately for each state of aggregate labour market conditions? No, when making its wage offer in the upturn, the firm has to take into account that the economy might transit to the downturn in the future and that it will be unable in that situation to adjust its wage offer.

Together, these four observations explain why the equilibrium with downwardly rigid wages is so much more difficult to characterize.

2.4 Characterization of the equilibrium

Though the equations in Proposition 2 characterize the equilibrium completely, their interpretation is difficult. Three ideas help us to overcome this obstacle.

Our first idea is to focus on small differences $\Delta = (\lambda_u - \lambda_d)/\lambda$ between the up- and the downturn, so that we can applying Taylor expansions. We shall show that (leaving out the argument F of all functions)

$$\begin{aligned}
(\chi - \theta + \lambda F)\,\overline{\Pi} &= \overline{X} - \overline{W} + O(\Delta), \\
-\overline{W}' &= \lambda\overline{\Pi} + O(\Delta) \\
(\chi + \theta + \lambda F)\,\Pi_\Delta &= x\overline{X} - \lambda F\overline{\Pi} - W_\Delta + \frac{\theta}{\Delta}(\Pi_{d+} - \Pi_d) + O(\Delta), \\
-W'_\Delta &= \lambda\Pi_\Delta + \lambda\overline{\Pi} - \frac{\theta}{\Delta}\Pi_{d+W}\overline{W}' + O(\Delta),
\end{aligned} \tag{8}$$

where

$$\begin{aligned}
\overline{\Pi} &\equiv \tfrac{1}{2}(\Pi_u + \Pi_d), & \overline{X} &\equiv \tfrac{1}{2}(X_u + X_d), \\
\overline{W} &\equiv \tfrac{1}{2}(W_u + W_d) \quad \Rightarrow & \overline{W}' &= \tfrac{1}{2}(W'_u + W'_d), \\
W_\Delta &\equiv \Delta^{-1}(W_u - W_d) \quad \Rightarrow & W'_\Delta &\equiv \Delta^{-1}(W'_u - W'_d).
\end{aligned}$$

The first pair of equations in (8) characterize the "average" behavior of profits and wages throughout the business cycle. These equations are identical to the equilibrium without aggregate fluctuations, $\theta = 0$. The second pair of equations characterize the cyclical fluctuations in profits and wages divided by Δ. The terms $\lambda F\Pi$ and $\lambda\Pi$ in the third and fourth equation capture the direct effect of the change in λ_t between the up- and the downturn: $\frac{\lambda_u - \lambda_d}{\Delta} F\Pi = \lambda F\overline{\Pi}$ and likewise for $\frac{\lambda_u - \lambda_d}{\Delta}\Pi = \lambda\overline{\Pi}$.

Our second idea is to approximate, the terms Π_{d+} and Π_{d+W} by Taylor expansions, starting from the point Π_d, since in this point the partial derivative with respect to W is zero due to the envelope theorem. Using this idea, we obtain

$$\Pi_d^o(F, W) = \Pi_d + \frac{1}{2}\Pi_{d+WW}(W - W_d)^2 + O\left(\Delta^3\right),$$

where Π_{d+WW} is defined analogous to Π_{d+W}. This equation can be used to substitute for $\Pi_d^o(F, W)$ in equation (5), which can then be solved for Π_{d+}.

Our third idea is to focus on a particular functional form for the average wage distribution $\overline{W}(F)$ that fits the data well. In particular, we assume that the wage offers are Pareto distributed, so that the log wages are exponentially distributed with scale parameter σ, see Gottfries and Teulings (2016). The wages and productivities are given by

$$\begin{aligned} \overline{W}(F) &= F^{-\sigma}, \\ \overline{\Pi}(F) &= \frac{\sigma}{\lambda}\frac{\overline{W}(F)}{F}. \end{aligned} \tag{9}$$

where the second line follow from the second equation in (8). Using these ideas, we show that log wages and the quit rate for jobs with not too large a degree of mismatch satisfy two simple log linear relations, which are presented in Proposition 3.

Proposition 3 *When wages are downwardly rigid and when $\overline{W}(F)$ satisfies equation (9), the wage and the quit rate satisfy*

$$\begin{array}{rcllllll} \ln W_t &=& -\sigma \ln F &+& \omega_\lambda \ln \lambda_t &+& \omega \ln \lambda_{\max a,t} & +O(\Delta^2) + O(F), \\ \ln q_t &=& \chi_F \ln F &+& (1 + \zeta\omega) \ln \lambda_t &-& \zeta\omega \ln \lambda_{\max a,t} & +O(\Delta^2) + O(F), \end{array} \tag{10}$$

a is the starting date of the current job;

$$\begin{aligned}
\lambda_{\max a,t} &\equiv \max_{\tau\in[a,t]} \lambda_\tau \\
\omega &\equiv \frac{1+\frac{\chi-\theta}{\chi+\theta}x}{1+\frac{\theta}{\chi}\left(\frac{\chi-\theta}{\chi\sigma}+1\right)}, \qquad \zeta \equiv \frac{\theta}{\chi\sigma}, \\
\omega_\lambda &= 0, \chi_F = 1.
\end{aligned}$$

The proof of Proposition 3 is in the Appendix.

For not too large a degree of mismatch (F is small) or cyclical variation in the job offer arrival rate (Δ is small), log wages and the log quit rate are linearly related to three variables:

1. log mismatch,
2. log job offer arrival rate,
3. log of the ratio of the max of this arrival rate since the date of job start to its current value.

The six coefficients of these three variables for log wages and the log quit rate ($\omega, \sigma, \omega_\lambda, \chi_\lambda, \chi_F$, and ζ) are simple analytical functions of just five parameters: the transition rate θ of the economy between the up- and the downswing, the job destruction rate δ, the discount rate ρ, the ratio x of the coefficients of variation of λ_t and X_t, and the Pareto parameter σ of the wage offer distribution. The job offer arrival rate and the job destruction rate are identified by the transition rate from unemployment to employment and vice versa. Our calculation yields an average monthly job finding rate from unemployment of 40% and a job destruction rate of about 2% per month, so that the steady state unemployment rate is 5%. The length of a business cycle identifies the transition rate for the economy, θ. Taking 6 years as a reasonable estimate of the full length of a business cycle implies that θ is about 2.8% (a complete cycle requires the arrival of two θ-shocks). The discount rate ρ is much smaller than the job destruction rate δ. Hence, little is lost for our purpose by setting it equal to zero. The same applies to the parameter x measuring the ratio of the coefficients of variation of λ_t and X_t. Only the parameter σ must be estimated from the wage regression. Hence, we have five overidentifying restrictions.

The wage equation in Proposition 3 has an appealing interpretation. Consider the second line in equation (8) for the special case $B_t = W_t(1) = 0$ for $t \in \{u, d\}$. Integrating the equation and taking logs yields

$$\ln W_t(F) = \ln \lambda_t + \ln \left(\int_F^1 \Pi_t(f)\, df \right).$$

Suppose that profits $\Pi_t(F)$ are constant over the business cycle. Then, the only reason for wages to vary over the cycle is the variation in the job offer arrival rate λ_t: firms will offer lower wages, since workers are less likely to quit. This effect has a unit elasticity: $\omega = 1$. There are two reasons for ω to be different from one, see equation (10). The first reason is the term $\frac{\chi-\theta}{\chi+\theta}x$ in the numerator of ω. This reflects the effect of cyclical variation in $X_t(F)$. When the job offer arrival rate is correlated to the monetary value of output ($x > 0$), then profits decline during the downturn and hence firms are less inclined to pay retention premiums to stop workers from quitting. This raises ω above unity. The second reason for ω to be different from one is the term $\frac{\theta}{\chi}\left(\frac{\chi-\theta}{\chi\sigma}+1\right)$ in the denominator. This term is the moderating effect of wage rigidity on the firm's wage setting policy during the upturn of the economy. Firms don't want to offer too high wages in the upturn, since they cannot reduce their offers during the downturn. This reduces ω below unity. For, $x \to 0$ there is only variation in the job offer arrival rate and not in the monetary value of output $X_t(F)$. This is the empirically relevant case as λ_t varies by a factor two or more, while the monetary value of output varies by at most a couple of percent. Hence, ω is less than one.

Our estimation results show σ to be 0.10 for low-educated workers and 0.14 for higher educated. Using these values for δ, θ, ρ and x and the value of σ for lower educated, we obtain

$$\begin{aligned} \omega &\equiv \frac{1 + \frac{\chi-\theta}{\chi+\theta}x}{1 + \frac{\theta}{\chi}\left(\frac{\chi-\theta}{\chi\sigma}+1\right)} \cong 0.25, \qquad (11) \\ \zeta &\equiv \frac{\theta}{\chi\sigma} \cong 5.8. \end{aligned}$$

The model therefore predicts that the elasticity ω of wages with respect to the job offer arrival rate is substantially below unity, while the elasticity $\zeta\omega$ of the quit rate is above unity.

3 Empirical results

3.1 Mismatch indicator

For the estimation of equation (10) we need information on the mismatch indicator F, which is not directly observed. We use a proxy for that variable using a method developed in Gottfries and Teulings (2016). We summarize the main lines of their argument which, in turn, builds on Barlevy (2008) and Hagedorn and Manovskii (2013). We follow Wolpin (1992) and refer to the time elapsing between two consecutive layoffs as an employment cycle (the first employment cycle starts at the beginning of a worker's career). Hence, a worker's current employment cycle has started either at the last layoff or -for the first employment cycle- at the start of the labour career. We normalize our measure of calendar time t such that it takes the value 0 at the start of the first job of the current employment cycle. Note that this definition of t excludes the preceding unemployment spell. We define Λ_t to be the cumulative job offer arrival rate of a worker since the start of the employment cycle:

$$\Lambda_t \equiv \int_0^t \lambda_\tau d\tau.$$

We refer to Λ_t as the labour market time elapsed since the start of the first job of the current employment cycle. The clock of labour market time runs faster relative to calendar time during the upturn, it runs slower during the downturn. We define a and b as the start and end date respectively of the current job; by construction, $0 \leq a < b$ ($a = 0$ for the first job of an employment cycle, $a > 0$ for subsequent jobs). Hence, Λ_a is the labour market time elapsed since the start of the employment cycle until the start of the current job; Λ_b is the labour market time elapsed at termination of the current job. Let n denote the number of job offers received during the current employment cycle until the end of the current job (hence: in the time interval $[0, b]$). Gottfries and Teulings (2016) show that, in a model with efficient transitions (such as the model with flexible wages in Section 2.2)

$$\begin{aligned} \mathrm{E}[n] &= \Lambda_b + 1, \\ \mathrm{E}[F] &= \Lambda_b^{-1} + O\left(\Lambda_b^{-2}\right), \end{aligned} \tag{12}$$

for jobs ending by a layoff, while for jobs that end in a quit

$$\begin{aligned} \mathrm{E}[n] &= \Lambda_b + 1 + O(\Lambda_b^{-1}), \\ \mathrm{E}[F] &= 2\Lambda_b^{-1} + O\left(\Lambda_b^{-2}\right). \end{aligned} \tag{13}$$

see Gottfries and Teulings (2016), Proposition 1. Their proposition follows from the theory of order statistics. Conditional on the value of b, jobs ending in a layoff have a lower expected mismatch than jobs ending in a quit, compare equation (12) and (13). Intuitively, the information that a job ends by a quit implies that some jobs are better and hence the job is not as a good as one would otherwise expect. This implies that wages should be higher in jobs ending by a layoff which is supported in data (Gottfries and Teulings, 2016).

In models with efficient job-to-job transitions, like the model with flexible wages discussed in Section 2.2, the current job is simply the job offer with the lowest mismatch that a worker has received since the start of the employment cycle. For this class of models, equation (12) and (13) hold exactly. In a model with inefficient transitions, like the model with downwardly rigid wages discussed in Section 2.3, a worker might reject an incoming offer with a lower mismatch because the wage in the current job is bound by downward rigidity. The equations will not hold exactly in that case. However, while this has a first order effect $O(\Delta)$ on the quit rate, it has a second order effect $O(\Delta^2)$ on the expected mismatch $\mathrm{E}[F|\Lambda_t]$: the worker will only reject job offers in the interval $F^{\text{new offer}} \in \left(F^{\text{current job}} - O(\Delta), F^{\text{current job}}\right)$ where the reduction in the mismatch by these "missed" job-to-job transitions is of the same order $F^{\text{current job}} - F^{\text{new offer}} = O(\Delta)$; hence, the effect on expected mismatch is of a second order $O(\Delta^2)$. Moreover, this effect is transitory because as soon as the worker accepts a new job (either during the current downswing or after transition to the upswing), that new offer must by definition have a lower mismatch than the current job, and hence the mismatch indicator is again the best job offer that a worker has received since the start of the employment cycle. Since the relations in Proposition 3 hold only up to a term of order $O(\Delta^2)$, we can ignore this effect.

If wage offers follow a Pareto distribution, see equation (9), then the expected mismatch described in equation (12) and (13) can be easily translated into an effect on the expected log wage. For layoffs, we obtain

$$\mathrm{E}[\ln W_t | t \in [a,b)] \cong \sigma \ln(\Lambda_b + 1), \tag{14}$$

while for quits

$$\mathrm{E}\left[\ln W_t | t \in [a,b)\right] \cong \sigma\left[\ln\left(\Lambda_b + 1\right) - 1\right]. \tag{15}$$

3.2 Regression specification

By equation (10) and (14), worker i's log wage $\ln W_{it}$ at time t satisfies

$$\ln W_{it} = \beta_i + \beta' X_{it} + \sigma\left[\ln\left(\Lambda_{ib} + 1\right) - D_{\text{quit}}\right] + \omega_\lambda \ln \lambda_t + \omega \ln \lambda_{i\max at} + \nu_{ia} + \epsilon_{it}. \tag{16}$$

where β_i is an individual fixed effect, measuring unobserved general human capital, where X_{it} is a vector measuring observed general human capital obtained by either education or work experience that vary over time (constant human capital variables are picked up by the worker fixed effect), where D_{quit} is dummy for those jobs that end by a quit, where $\lambda_{i\max at} = \max_{\tau \in [a_i, t]} \lambda_\tau$, where v_{ia} is random job effect, and ε_{it} is a random transitory shock. Note that while λ_t has the same value for all i at a particular time t, $\lambda_{i\max at}$ may vary between i because their current jobs started at a different points in time. This variation helps identifying the effect empirically.

We can now contrast our regression framework to that in Kudlyak (2011), Hagedorn and Manovskii (2013) and Galindo da Fonseca et al. (2016). The critical issue is how to account for the selectivity in the mismatch F. Surviving jobs are more likely to have a lower mismatch (if not, the worker is likely to have quit to a better match). Our specification is different for a number of reasons. First, the foremost mentioned papers correct for mismatch be entering both $\ln(\Theta_a)$ and $\ln(\Theta_b - \Theta_a)$ in their specification, where Θ_a refers to the integral of labour market tightness from 0 to a. Gottfries and Teulings (2016) show that the correct specification is to enter $\ln(\Lambda_b + 1)$. A matching function would imply that market tightness should be raised to some power in order to calculate the job finding rate. This is why we instead measure labour market time using job finding rates. Furthermore, while $\Lambda_b = (\Lambda_b - \Lambda_a) + \Lambda_a$, this equality does not hold for the sum of their logs. Hence, the correct specification is not nested in their specification, causing their estimates to be biased. Finally, we follow Gottfries and Teulings (2016) and include a one in order to capture the initial offer and a dummy for jobs ending via a quit, see that paper for a more detailed discussion.

For the log quit rate, we estimate a logit model for the probability that a worker quits during the current time period conditional on the observed history at the moment of observation. This yields consistent estimates of the parameter of the model for the log quit rate.

$$\ln q_{it} = \beta_q' X_{it} - \chi_F \ln(\Lambda_{it} + 1) + (1 + \zeta\omega) \ln \lambda_t - \zeta\omega \ln \lambda_{i\,\max at} + \varepsilon_{q,it} \quad (17)$$

When we estimate the above equation we include quadratic experience and years of education and a linear time trend, as well as dummies for region, marriage, and urban versus rural location in the vector X_{it}.

Note that mismatch indicator applied in both equation differs slightly. In equation (16), we apply cumulative labour market time up till the termination date b of the current job, $\ln(\Lambda_{ib} + 1)$, while in equation (17) we apply cumulative labour market time up till the moment of observation, $\ln(\Lambda_{it} + 1)$. This difference follows from econometric theory: $\ln(\Lambda_{ib} + 1)$ is the best available estimate for $-\ln F$, $\ln(\Lambda_{it} + 1)$ is the best estimate of the effect of past employment history on the current quite rate. One can show that the estimation of the logit model in equation (17) is asymptotically equivalent to maximum likelihood estimation of the job-to-job hazard rate under the model's assumption that the layoff rate is constant over time.

One can dispute whether or not both equations should include controls for tenure. As it stands, the model does not include a tenure profile in wages, see Gottfries and Teulings (2016) for a more extensive discussion, nor does the quit rate depend on tenure. However, log duration of the employment cycle $\ln(\Lambda_{it} + 1)$ is positively correlated to tenure. By not including tenure, we might therefore incorrectly attribute the effect of tenure to $\ln \Lambda_t$. Hence, we present all estimation both with and without a quadratic in tenure. All standard errors are clustered at the job level.

3.3 Data

Like Gottfries and Teulings (2016), we use the cross-sectional sample from NLSY79 over the years from 1979 to 2012. Our selections are identical to Gottfries and Teulings (2016), to which we refer to the details of the data-selection process. Since women tend to interrupt their working career for childbearing, which is not described in our model, we select only males. Similarly, since our model applies to primary jobs, the sample is restricted to the primary jobs for men over the age of 18 who are not enrolled in full-time education. Jobs for which weekly hours is less than 15 and which lasted less than four weeks or started before 1979 are excluded from the sample. When there are multiple jobs, the primary job is defined as the job with highest number of hours. Jobs with inconsistencies in their start and end date are adjusted or removed. If schooling is not reported for a given month, we assign the maximum from the previous months; if it is less than previously reported, we use the maximum previously reported.

For the construction of the variable Λ_b, we classify job terminations into either quits (belonging to the same employment cycle) or layoffs (starting a new cycle). Here, we follow Barlevy (2008). A separation is defined as a quit when the new job starts within eight weeks of the termination of the previous job and the stated reason for separation is voluntary. This definition is used to determine whether or not two consecutive jobs belong to the same employment cycle.

Having defined employment cycles, we have to decide which jobs to include in our analysis of jobs. We exclude jobs which have not ended. Jobs end if the worker reports that he no longer works at the job, if the job becomes a secondary job or if the worker at an interview during the subsequent year does not mention working for the firm during the past year. Jobs where the worker reports being self-employed or working for a family business, or where the hourly wage is below \$1 or above \$500, or where some of the covariates are missing values are dropped from the analysis. Wages are deflated using seasonally adjusted national CPI (CPIAUCSL). In order to calculate the transition rate to new jobs we create a monthly labour force record for the workers. The transition rate is calculated using the probability that the worker experiences a quit within a month.

The variable λ_t is the monthly job finding rate for unemployed workers, which is calculated from the transition rate from unemployment to employment. Since the model implies that unemployed job seekers accept any job offer, the transition rate out of unemployment is a perfect measure of λ_t.

We calculate the transition rates using the monthly CPS data. We restrict our analysis to a sample of males aged 25-54 in order to match our NLSY 79 dataset and to avoid moves involving voluntary participation decisions as opposed to job-offer arrivals.[3] To calculate the job-finding rate of the unemployed, we calculate the fraction of the workers who are unemployed less than five weeks and are employed in the next month. In our empirical application we allow for on-the-job search to be less efficient than off-the-

[3] We match the monthly CPS using variables suggested by Drew and Warren (2014). In addition, we use race and age as extra controls.

job search. We assume the job offer arrival rate for employed job seekers to be a fraction $\psi \in [0, 1]$ of the job offer arrival rate for the unemployed, see Gottfries and Teulings (2016) for details; they report ψ to be equal to 0.20.

3.4 Results on wages

Panel A of Table 1 reports the estimation results for equation (16) excluding a second order polynomial in tenure. We present the result for all data combined, and for low and highly educated workers separately. Low and high educated workers refers to those with 12 years or less and more than 12 years of education, respectively. The effect of match quality $\ln(\Lambda_{ib} + 1)$ on log wages is highly significant. The estimated coefficient σ measures the scale parameter, in the Gumbel distribution, for the wage offer distribution of log wages. In line with Gottfries and Teulings (2016), this standard deviation is higher for higher educated than for low-educated workers. The effect of the current job offer arrival rate $\ln \lambda_t$ is significant only if one does not control for the maximum of the arrival rate since the job-start, $\ln \lambda_{i\max at}$. As soon as $\ln \lambda_{i\max at}$ is entered as a regressor, $\ln \lambda_t$ becomes insignificant or weakly significant at best, which is consistent with Proposition 3: $\omega_\lambda = 0$. The coefficient for $\ln \lambda_{i\max at}$ is highly significant, which is also consistent with Proposition 3. The estimated value of 0.15 has the right order of magnitude, see equation (11), where we calculated a value of 0.25.

Panel B reports the estimation results including the polynomial on tenure. The results are virtually identical. Hence, the match quality indicator $\ln(\Lambda_{ib} + 1)$ does not serve as a proxy for tenure. Strictly speaking, the model predicts that there is no tenure profile in wages. The F-test falls substantially when $\ln \lambda_{i\max at}$ is included. This captures the fact that, $\ln \lambda_{i\max at}$, in expectation has a similar behavior to tenure, starting at zero and then rising during the time in the job. The restriction that the coefficients on tenure are zero, is therefore strongly rejected by the data. However, Gottfries and Teulings (2016) show that the return to tenure contributes less than 10% to the total increase in log wages over the career. The F-test show that part of this return to tenure is due to the effect of $\ln \lambda_{i\max at}$ (which is weakly increasing in tenure by construction): the test statistic falls substantially when $\ln \lambda_{i\max at}$ is included as an additional regressor.

Recall that our model predicts that firms pay retention premiums, but no hiring premiums. Consider the alternative case, where firms pay both hiring and retention premiums. In that case, a firm would not find it profitable to

Table 1: Wage regressions

	Without Tenure					
	All Data	All Data	Low-Edu.	Low-Edu.	High-Edu.	High-Edu.
$\ln \lambda_t$	0.064***	0.015	0.084***	0.033*	0.056**	0.013
	(0.015)	(0.015)	(0.018)	(0.019)	(0.027)	(0.027)
$\ln(\Lambda_b + 1)$	0.098***	0.088***	0.092***	0.082***	0.103***	0.095***
	(0.004)	(0.004)	(0.005)	(0.005)	(0.007)	(0.007)
$\ln \lambda_{i\,\max\,at}$		0.224***		0.213***		0.231***
		(0.023)		(0.027)		(0.043)
Observations	33386	33386	20885	20885	11722	11722

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

	With Tenure					
	All Data	All Data	Low-Edu.	Low-Edu.	High-Edu.	High-Edu.
$\ln \lambda_t$	0.068***	0.031**	0.088***	0.049***	0.060**	0.028
	(0.015)	(0.015)	(0.018)	(0.019)	(0.027)	(0.027)
$\ln(\Lambda_b + 1)$	0.084***	0.081***	0.077***	0.074***	0.092***	0.089***
	(0.004)	(0.004)	(0.005)	(0.005)	(0.007)	(0.007)
$\ln \lambda_{i\,\max\,at}$		0.163***		0.160***		0.167***
		(0.025)		(0.029)		(0.047)
Observations	33386	33386	20885	20885	11722	11722
F-test	51.9	24	31.5	14.2	15.1	6.9

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2: Wage regressions

	Without Tenure								
	All Data	All Data	All Data	Low-Edu.	Low-Edu.	Low-Edu.	High-Edu.	High-Edu.	High-Edu.
$\ln \lambda_t$	0.059***	0.015	0.013	0.077***	0.034*	0.031	0.054**	0.012	0.008
	(0.015)	(0.015)	(0.015)	(0.018)	(0.019)	(0.019)	(0.027)	(0.027)	(0.027)
$\ln \lambda_{ia}$	0.068***	-0.004	-0.096**	0.075***	0.005	0.012	0.031	-0.042	-0.191**
	(0.019)	(0.021)	(0.046)	(0.023)	(0.025)	(0.173)	(0.037)	(0.039)	(0.087)
$\ln(\Lambda_b + 1)$	0.098***	0.088***	0.088***	0.091***	0.082***	0.082***	0.103***	0.094***	0.093***
	(0.004)	(0.004)	(0.004)	(0.005)	(0.005)	(0.005)	(0.007)	(0.007)	(0.007)
$\ln \lambda_{i\,\max at}$		0.226***	0.320***		0.210***	0.218		0.251***	0.403***
		(0.024)	(0.048)		(0.029)	(0.149)		(0.045)	(0.090)
$(\ln \lambda_{i\,\max at} - \ln \lambda_a)^2$			-0.189**			-0.108			-0.304*
			(0.085)			(0.131)			(0.156)
Observations	33386	33386	33386	20885	20885	20885	11722	11722	11722

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

	With Tenure								
	All Data	All Data	All Data	Low-Edu.	Low-Edu.	Low-Edu.	High-Edu.	High-Edu.	High-Edu.
$\ln \lambda_t$	0.062***	0.033**	0.031**	0.080***	0.051***	0.049***	0.058**	0.028	0.023
	(0.015)	(0.015)	(0.015)	(0.018)	(0.019)	(0.019)	(0.027)	(0.028)	(0.027)
$\ln \lambda_{ia}$	0.078***	0.030	-0.081*	0.083***	0.036	-0.044	0.041	-0.013	-0.178**
	(0.020)	(0.022)	(0.046)	(0.023)	(0.026)	(0.054)	(0.037)	(0.040)	(0.088)
$\ln(\Lambda_b + 1)$	0.083***	0.081***	0.080***	0.076***	0.074***	0.074***	0.092***	0.089***	0.088***
	(0.004)	(0.004)	(0.004)	(0.005)	(0.005)	(0.005)	(0.007)	(0.007)	(0.007)
$\ln \lambda_{i\,\max at}$		0.144***	0.256***		0.137***	0.217***		0.174***	0.342***
		(0.027)	(0.049)		(0.033)	(0.058)		(0.050)	(0.092)
$(\ln \lambda_{i\,\max at} - \ln \lambda_a)^2$			-0.229***			-0.168			-0.340**
			(0.086)			(0.105)			(0.158)
Observations	33386	33386	33386	20885	20885	20885	11722	11722	11722

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

increase its wage offer when the arrival rate of outside offers increases only slightly. The firm's wage offer at the date of hiring included both hiring and retention premiums. Since hiring is completed, the firm has no incentive to continue paying the premium apart from its implicit or explicit contractual obligation. When the optimal retention premium goes up due to an increase in the arrival rate of outside offers, the firm has first to deplete this buffer of by now useless hiring premiums, before it becomes profitable for the firm to raise its wage offer for ongoing jobs. This has two empirical implications. First, $\ln \lambda_{i \max at}$ only becomes relevant for the wage at time t when it is substantially above than the log arrival rate at the date of hiring, $\ln \lambda_{ia}$. We can test this by adding three variables to the regression: $\ln \lambda_{ia}$ and $(\ln \lambda_{i \max at} - \ln \lambda_{ia})^2$. If firms were to pay hiring premiums, the arrival rate at the date of hiring $\ln \lambda_{ia}$ should drive out the maximum arrival rate $\ln \lambda_{i \max at}$, while $(\ln \lambda_{i \max at} - \ln \lambda_{ia})^2$ captures the fact that $\ln \lambda_{i \max at}$ comes in only when it is substantially above $\ln \lambda_{ia}$. Hence, $(\ln \lambda_{i \max at} - \ln \lambda_{ia})^2$ and $\ln \lambda_{ia}$ should come in with a positive coefficient, while $\ln \lambda_{i \max at}$ should be insignificant. These predictions are tested in Table 2. In the regressions $\ln \lambda_{i \max at}$ crowds out $\ln \lambda_{ia}$, consistent with retention premiums only. The term $(\ln \lambda_{i \max at} - \ln \lambda_{ia})^2$ is significant is some of the regressions but has the opposite sign is expected from a model with hiring premiums. Again, comparing Panel A and B, the conclusion does not depend on whether or not controls for tenure are included in the model.

We conclude that the model with downwardly rigid wages fits the data on wages very well. The key variable $\ln \lambda_{i \max at}$ has the right sign and is highly significant for both education levels and all coefficients have the right order of magnitude. Moreover, the evidence suggests that firms do not pay hiring premiums.

3.5 Results on the quit rate

Table 4 reports the estimation results for a logit model for the probability that a worker quits from her current job. The coefficients of this logit model converge to the coefficients of the log quit rate in equation (17). In both panels, we report the estimation results for models ex- and including $\ln \lambda_{i \max at}$, measuring the inefficiency in job-to-job transitions during the downturn. In fact, $\ln \lambda_{i \max at}$ can be interpreted as an instrument for the effect of cyclical variation in wages. In that interpretation, equation (16) is the first stage regression. The estimation results, discussed in the previous section, show the instrument to be highly relevant. Then, equation (17) is the second stage regression. First, we consider Panel A without controls for tenure. The coefficient $\ln(\Lambda_{it}+1)$ varies roughly between 0.60 and 0.80, being somewhat lower for models in- rather than excluding $\ln\lambda_{i\max at}$ as a regressor. Proposition 3 predicts the coefficient on $\ln(\Lambda_{it}+1)$ to be equal to unity. The coefficient has therefore the right order of magnitude, but is lower then its expected theoretical value. Recall that our estimate for Λ_{it} is based on the observed transition rate from unemployment to employment, presuming that the job offer arrival rate for employed job seekers varies proportionally to that for unemployed. The lower coefficient could therefore be due to an imperfect correlation between both job offer arrival rates. The surprise is that the model works for wages but not for quits. The coefficient on $\ln\lambda_t$ is around 0.30 for specifications including $\ln\lambda_{i\max at}$, while it is between 0.70 and 0.80 for specifications including $\ln\lambda_{i\max at}$. The predicted value of this coefficient is greater than unity for a model including $\ln\lambda_{i\max at}$, see Proposition 3. The coefficient between 0.70 and 0.80 for the specification that do control for $\ln\lambda_{i\max at}$ therefore smaller than predicted. The key coefficient on $\ln\lambda_{i\max at}$ has the right sign and is highly significant. Proposition 3 predicts this coefficient to be $-\zeta\omega=-5.8\times 0.25=-1.45$. The estimation results are closely in line with this prediction.

The results in Panel B are less favorable. When we add controls for tenure, all coefficients drop in absolute value and the coefficient on the match quality indicator $\ln(\Lambda_{it}+1)$ becomes weakly significant at best. However, the key coefficient on $\ln\lambda_{i\max at}$ remains highly significant. By construction, Λ_{it} is highly correlated to tenure, if only because the tenure in the current job can never exceed the length of employment cycle up till the moment of observation. Moreover, the model predicts the most recent job in the employment cycle to have the longest expected duration, strengthening the positive correlation. However, the model predicts that $\ln(\Lambda_{it}+1)$ would drive out tenure. In fact, the opposite happens. We have no good explanation for this.

The model with downwardly rigid wages fits the data on the cyclical behavior of quit rates well, though the coefficients is too small. The key variable $\ln\lambda_{i\max at}$ has the right sign and magnitude and is highly significant for both education levels. As soon as controls for tenure are added to the specification, these covariates drive out $\ln(\Lambda_{it}+1)$.

Overall, the model fits the data surprisingly well. The model yields five overidentifying restrictions on the six coefficients for the key variables,

Table 3: Logit regressions for quits

	Without Tenure					
	All Data	All Data	Low-Edu.	Low-Edu.	High-Edu.	High-Edu.
$\ln \lambda_t$	0.323**	0.805***	0.290*	0.764***	0.372*	0.803***
	(0.128)	(0.150)	(0.164)	(0.194)	(0.210)	(0.238)
$\log(\Lambda_t + 1)$	-0.821***	-0.644***	-0.866***	-0.693***	-0.736***	-0.575***
	(0.033)	(0.041)	(0.044)	(0.056)	(0.050)	(0.060)
$\ln \lambda_{i\,\max\,at}$		-1.665***		-1.549***		-1.622***
		(0.199)		(0.268)		(0.299)
Observations	340007	340007	199022	199022	131174	131174

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

	With Tenure					
	All Data	All Data	Low-Edu.	Low-Edu.	High-Edu.	High-Edu.
$\ln \lambda_t$	0.242*	0.529***	0.191	0.453**	0.306	0.582**
	(0.128)	(0.146)	(0.163)	(0.189)	(0.209)	(0.233)
$\log(\Lambda_t + 1)$	-0.160***	-0.114**	-0.095	-0.053	-0.217***	-0.170**
	(0.046)	(0.045)	(0.063)	(0.062)	(0.067)	(0.066)
$\ln \lambda_{i\,\max\,at}$		-0.951***		-0.828***		-0.995***
		(0.190)		(0.254)		(0.286)
Observations	340007	340007	199022	199022	131174	131174

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 4: Logit regressions for quits: subsequent jobs

	Without Tenure					
	All Data	All Data	Low-Edu.	Low-Edu.	High-Edu.	High-Edu.
$\ln \lambda_t$	0.573**	0.844***	0.781**	1.015***	0.348	0.630
	(0.256)	(0.273)	(0.348)	(0.375)	(0.388)	(0.406)
$\log(\Lambda_t + 1)$	-0.768***	-0.662***	-0.828***	-0.740***	-0.705***	-0.594***
	(0.071)	(0.080)	(0.098)	(0.113)	(0.109)	(0.118)
$\ln \lambda_{i\,\max at}$		-1.200***		-0.972*		-1.328***
		(0.371)		(0.559)		(0.489)
Observations	91018	91018	48078	48078	40152	40152

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

	With Tenure					
	All Data	All Data	Low-Edu.	Low-Edu.	High-Edu.	High-Edu.
$\ln \lambda_t$	0.525**	0.516**	0.737**	0.570	0.297	0.426
	(0.254)	(0.263)	(0.346)	(0.359)	(0.385)	(0.397)
$\log(\Lambda_t + 1)$	-0.294***	-0.295***	-0.250**	-0.271**	-0.361***	-0.346***
	(0.081)	(0.081)	(0.109)	(0.111)	(0.124)	(0.124)
$\ln \lambda_{i\,\max at}$		0.038		0.635		-0.569
		(0.370)		(0.548)		(0.498)
Observations	91018	91018	48078	48078	40152	40152

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

$\ln(\Lambda_{it} + 1)$, $\ln \lambda_t$ and $\ln \lambda_{i\,\max at}$ in the models for both log wages and the quit rate. The sign of all six estimated coefficients fits the prediction in Proposition 3 for all three versions of the model (high and low-educated, and both groups combined). When predicted to be different from zero, the coefficients are mostly highly significant and they have roughly the magnitude predicted in Proposition 3, except for the coefficient on $\ln \lambda_t$ in the logit model for the quit rate.

4 Conclusion

The ordeal of a job seeker is not much different from that of the mythological character Sisyphos. Sisyphos has to role a heavy boulder up on a steep hill. Each time when he has almost reached the top, the boulder slips out of his hands, rolling down the slope and crushing anything that is along its pathway. Afterwards, Sisyphos has no other choice then starting its laborious task all over again. This fate looks very much similar to that of a job seeker in an OJS model by Burdett and Mortensen (1998). Job seekers climb the hill of rents by hopping from on job to another. Their ultimate goal is the most productive and hence best paying job, on the top of the hill. Just when they almost reached the top, an unexpected job destruction shocks throws them off its slopes, back in the deep trough of unemployment. This leaves them no alternative than to restart their ascent from the lowest rungs of the hill of rents.

In our previous paper Gottfries and Teulings (2016), we have shown that the steady state version of this model has all first-order predictions on wage dynamics and job-to-job mobility right. Job seekers do gradually climb the hill of rents by selecting the ever better draws from the offer distribution. They face a layoff every now and then, after which they have to restart this selection process again, receiving draws from largely the same offer distribution as during their first attempt to climb the ladder. The randomness of this process explains some 9% of total wage dispersion.

This paper shows that this model is also useful to study the long-standing problem of the aggregate wage rigidity. Firms seek to credibly commit to a posted wage in order to convince workers not to quit. The necessity to commit provides a forceful motive for wage-rigidity. Remarkably, the literature has largely avoided this idea, either by using search models without OJS, as in Shimer (2004) and Hall (2005), or by tweaking the model such that the posted wages can respond flexibly to aggregate shocks, as in Moscarini and Postel-Vinay (2013) and Coles and Mortensen (2016). Until so far, no paper has analyzed the models with OJS, wage rigidity and inefficient job-to-job transitions, probably because these model are complicated. This paper fills the gap by using the wage posting model to analyze the implications of wage rigidity.

This model yields inefficiently low job-to-job transitions during the downturn because workers reject outside offers that are more productive than their current job. The theoretical structure in the model is derived from first principles. It yields two simple linear relations, for log wages and for quits respectively. The over-identifying restrictions implied by the theoretical structure hold surprisingly well. To our knowledge, this model is the first model with OJS and wage rigidity that derives prediction regarding the degree of wage cyclicality from a theoretical structure and that yield micro-predictions which are supported reasonably well by the data. In particular, downward rigid in wages is supported empirically looking at both wages and quit rates.

Until now, macro-economics has largely relied on search models without OJS. Our paper shows that this is the wrong benchmark. The main implications of our model contrast sharply from standard models used till so far. When a negative shock hits the (nominal) value of output, wages in ongoing jobs are bound by downward rigidity, unlike wages in new jobs. Nevertheless, a downward shock has a negative effect on vacancy creation, not so much due to the fall in the nominal value of output, but due to a commitment problem. Employed job seekers correctly expect firms not to pay high wages, because the job offer arrival rate is low. Hence, workers are reluctant to give up their high paying job. Hence, fewer vacancies are opened. The lower job offer arrival rate allows other firms that open new vacancies to offer lower wages because worker retention is less of an problem. Only in course of time, jobs of which the wages are set before the onset of the recession are faced out by gradual job destruction. Then, the economy converges gradually to its new steady state equilibrium. The economy suffers from a collective action problem. Firms cannot commit to pay high wages. In turn, this makes hiring harder which further weakens the incentive to pay higher wages. Uncovering the details of these dynamics is fertile avenue for future research.

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A Proofs

Preliminary steps, conjecturing the differentiability W_Δ

- We leave out the argument F of all functions.
- We conjecture that the solution for W_u and W_d takes the following form

$$\begin{aligned} W_d &= \overline{W} \exp\left[-\frac{1}{2}\omega\Delta + O\left(\Delta^2\right) + O\left(F\right)\right], \\ W_u &= \overline{W} \exp\left[\frac{1}{2}\omega\Delta + O\left(\Delta^2\right) + O\left(F\right)\right], \end{aligned} \tag{18}$$

  where ω is a parameter that has to be determined. At the end of the proof, we show that this conjecture is correct.

- Equation (9) and (18) imply

$$\begin{aligned} \frac{W_\Delta}{\overline{W}} &= \omega + O\left(\Delta\right) + O\left(F\right), \\ \frac{FW'_\Delta}{\overline{W}} &= -\sigma\omega + O\left(\Delta\right) + O\left(F\right), \\ \frac{F\overline{X}}{\overline{W}} &= \frac{\sigma}{\lambda}\left(\chi - \theta\right) + O\left(\Delta\right) + O\left(F\right). \end{aligned} \tag{19}$$

- Equation (7) implies $W_u = W_{ud}\left(W_d\right) < W_{d+}\left(W_d\right) < W_d$. Since $W_u - W_d = W_\Delta \Delta$, this implies

$$\begin{aligned} W_{d+}\left(W_d\right) &= W_u + O\left(\Delta\right) = W_d + O\left(\Delta\right), \\ W'_{d+}\left(W_d\right) &= 1 + O\left(\Delta\right). \end{aligned} \tag{20}$$

Step 1: derivation of expressions for $V_u^{o\prime}\left(W_u\right)$ and $V_u^{o\prime\prime}\left(W_u\right)$

- Equation (2) and (6) apply identically for all W. Hence, their derivatives must apply. For the first derivatives we obtain:

$$\begin{aligned} \left[\chi + \lambda_u F_u\left(W\right)\right] V_u^{o\prime}(W) &= 1 + \theta V_{d+}^{o\prime}(W), \\ \left[\chi + \lambda_d F_d\left(W_{d+}^{-1}\left(W\right)\right)\right] V_{d+}^{o\prime}\left(W\right) &= 1 + \theta V_u^{o\prime}\left(W\right), \end{aligned}$$

  and for the second derivatives:

$$\begin{aligned} \left[\chi + \lambda_u F_u\left(W\right)\right] V_u^{o\prime\prime}(W) &= \theta V_{d+}^{o\prime\prime}(W) - \lambda_u u_u\left(W\right) V_u^{o\prime}(W), \\ \left[\chi + \lambda_d F_d\left(W_{d+}^{-1}\left(W\right)\right)\right] V_{d+}^{o\prime\prime}\left(W\right) &= \theta V_u^{o\prime\prime}\left(W\right) - \lambda_d u_d\left(W_{d+}^{-1}\left(W\right)\right) W_{d+}^{-1\prime}\left(W\right) V_{d+}^{o\prime}\left(W\right). \end{aligned}$$

- We evaluate these equations for $W = W_u$, using $F_u(W_u) = F$, $\lambda_u - \lambda_d = O(\Delta)$, $f_u(W_u) = W_u'^{-1}$ (see equation (1)) and

$$\begin{aligned} F_d\left[W_{d+}^{-1}(W_u)\right] &= F + O(\Delta), \\ f_d\left[W_{d+}^{-1}(W)\right] W_{d+}^{-1'}(W) &= f_u(W_u) + O(\Delta), \end{aligned}$$

due to equation (20). The solution of the first derivatives for $V_u^{o\prime}(W_u)$ and $V_{d+}^{o\prime}(W)$ reads:

$$V_u^{o\prime}(W_u) = V_{d+}^{o\prime}(W) = (\chi - \theta + \lambda F)^{-1} + O(\Delta), \tag{21}$$

while the equations for the second derivatives simplify to

$$\begin{aligned} \lambda(\chi - \theta + \lambda F)^{-1} + (\chi + \lambda F) V_u^{o\prime\prime}(W) W_u' &= \theta V_{d+}^{o\prime\prime}(W) W_u' + O(\Delta), \\ \lambda(\chi - \theta + \lambda F)^{-1} + (\chi + \lambda F) V_{d+}^{o\prime\prime}(W) W_u' &= \theta V_u^{o\prime\prime}(W) W_u' + O(\Delta). \end{aligned}$$

which can be solved for $V_u^{o\prime\prime}(W_u)$ and $V_{d+}^{o\prime\prime}(W)$:

$$V_u^{o\prime\prime}(W_u) W_u' = -\lambda(\chi - \theta + \lambda F)^{-2} + O(\Delta). \tag{22}$$

Step 2: use Step 1 for the derivation of $W_{d+}(W)$ and $W_{d+}'(W)$

- Equation (5) applies identically for all W and hence its first derivative with respect to W must apply, which reads:

$$W_{d+}'(W) - 1 = \theta\left(V_u^{o\prime}\left[W_{ud}(W)\right] W_{ud}'(W) - V_u^{o\prime}\left[W_{d+}(W)\right] W_{d+}'(W)\right).$$

- We evaluate equation (5) and its derivative above for $W = W_d$. A first order Taylor expansion of $V_u^o(W_u)$ and $V_u^{o\prime}(W_u)$ yields

$$\begin{aligned} W_{d+}(W_d) - W_d &= \theta V^{o\prime}{}_u(W_u)\left[W_\Delta \Delta + W_d - W_{d+}(W_d)\right] + O\left(\Delta^2\right), \\ W_{d+}'(W_d) - 1 &= \theta V_u^{o\prime}(W_u)\left[W_\Delta' \Delta / W_d' + 1 - W_{d+}'(W_d)\right] \\ &\quad - \theta V_u^{o\prime\prime}(W_u)\left[W_\Delta \Delta + W_d - W_{d+}(W_d)\right] W_{d+}'(W_d) + O\left(\Delta^2\right). \end{aligned} \tag{23}$$

We use the definitional relations $W_{ud}(W_d) \equiv W_u \equiv W_\Delta \Delta + W_d$ and hence $W_{ud}'(W_d) = W_\Delta' \Delta / W_d' + 1$.

- Substitution of equation (21) and (22) in equation (23) and using equation (20) yields

$$
\begin{aligned}
W_{d+}(W_d) - W_d &= \frac{\theta}{\chi+\lambda F} W_\Delta \Delta + O\left(\Delta^2\right), \\
W'_{d+}(W_d) - 1 &= \frac{\theta}{\chi+\lambda F}\left(W'_\Delta + \frac{\lambda}{\chi+\lambda F} W_\Delta\right)\frac{\Delta}{\overline{W}'} + O\left(\Delta^2\right).
\end{aligned} \tag{24}
$$

Step 3: derivation of $W_{d+}^{-1}(W)$ and $W_{d+}^{-1\prime}(W)$ from Step 2

- Since $W'_{d+}(W_d) = 1 + O(\Delta)$ (see equation (20)), equation (24) implies

$$
\begin{aligned}
W_{d+}^{-1}(W_u) - W_u &= W_{d+}^{-1}(W_d) - W_d + O\left(\Delta^2\right) \\
&= W_d - W_{d+}(W_d) + O\left(\Delta^2\right) \\
&= -\frac{\theta}{\chi+\lambda F} W_\Delta \Delta + O\left(\Delta^2\right), \\
W_{d+}^{-1}(W_u) - W_d &= \frac{\chi - \theta + \lambda F}{\chi+\lambda F} W_\Delta \Delta + O\left(\Delta^2\right), \\
W_{d+}^{-1\prime}(W_u) - 1 &= W_{d+}^{-1\prime}(W_d) - 1 + O\left(\Delta^2\right) = 1 - W'_{d+}(W_d) + O\left(\Delta^2\right) \\
&= -\frac{\theta}{\chi+\lambda F}\left(W'_\Delta + \frac{\lambda}{\chi+\lambda F} W_\Delta\right)\frac{\Delta}{\overline{W}'} + O\left(\Delta^2\right).
\end{aligned} \tag{25}
$$

Step 4: derivation of an expression for Π_{dWW}

- The expression for Π_d^o in equation (2) holds identically for all W. Hence, its derivatives with respect to W must hold.

$$
\begin{aligned}
\left[\chi + \lambda_d F_d(W)\right] \Pi_{dW}^o(F,W) + \lambda_d f_d(W) \Pi_d^o(F,W) &= -1, \\
\left[\chi + \lambda_d F_d(W)\right] \Pi_{dWW}^o(F,W) + 2\lambda_d f_d(W) \Pi_{dW}^o(F,W) + \lambda_d f'_d(W) \Pi_d^o(F,W) &= 0.
\end{aligned}
$$

- Evaluating the last expression for $W = W_d$, using the envelope theorem result $\Pi_{dW} = 0$ and substitution of equation (1) for $f'_d(W)$ yields

$$
\Pi_{dWW} = \frac{\lambda_d \Pi_d}{\chi + \lambda_d F} \frac{W''_d}{W'^3_d} = -\frac{1}{\chi+\lambda F} \frac{\overline{W}''}{\overline{W}'^2} + O(\Delta). \tag{26}
$$

where we use $W'_d = -\lambda_d \Pi_d$, see Proposition 2.

Step 5: substitution of Step 3 and 4 in Π_{d+} and Π_{d+W}

- Using equation (25) and (26), $\Pi^{o}_{d+}(F, W_u)$ and its first derivative with respect to W_u can be written as

$$\begin{aligned}
\Pi^{o}_{d+}(F, W_u) &= \Pi_d + \frac{1}{2}\Pi_{dWW}\left[W^{-1}_{d+}(W_u) - W_d\right]^2 \qquad (27)\\
&\quad + \left[\chi + \lambda_d F_d(W^{-1}_{d+}(W_u))\right]^{-1}\left[W^{-1}_{d+}(W_u) - W_u\right] + O\left(\Delta^3\right)\\
&= \Pi_d - \frac{\theta}{(\chi+\lambda F)^2} W_\Delta \Delta + O\left(\Delta^2\right),\\
\Pi^{o}_{d+W}(F, W_u) &= \Pi_{dWW}\left[W^{-1}_{d+}(W_u) - W_d\right] W^{-1\prime}_{d+}(W_u)\\
&\quad + \left[\chi + \lambda_d F_d(W^{-1}_{d+}(W_u))\right]^{-1}\left[W^{-1\prime}_{d+}(W_u) - 1\right]\\
&\quad + \lambda_d f_d\left[W^{-1}_{d+}(W_u)\right]\frac{W^{-1\prime}_{d+}(W_u)}{\left[\chi + \lambda_d F_d(W^{-1}_{d+}(W_u))\right]^2}\left[W^{-1}_{d+}(W_u) - W_u\right]\\
&= -\frac{\theta}{(\chi+\lambda F)^2}\left(\frac{\chi - \theta + \lambda F}{\theta}\frac{\overline{W}''}{\overline{W}'}W_\Delta + W'_\Delta - 2\frac{\lambda}{\chi+\lambda F}W_\Delta\right)\frac{\Delta}{\overline{W}'}\\
&\quad + O\left(\Delta^2\right).
\end{aligned}$$

Step 6: substitution of equation (9) in equation (27)

- Using equation (9) for $\overline{W}'$ and $\overline{W}''$ and using equation (19) for W_Δ and W'_Δ, we obtain

$$\begin{aligned}
\frac{\Pi_{d+} - \Pi_d}{\Delta \overline{W}} &= -\frac{\theta}{\chi^2}\omega + O(\Delta) + O(F), \qquad (28)\\
\frac{\Pi_{d+W}}{\Delta} &= \frac{\theta}{\sigma\chi^2}\left(\frac{\chi - \theta}{\theta}(\sigma+1) + \sigma\right)\omega + O(\Delta) + O(F).
\end{aligned}$$

Step 7: derivation of equation (8)

- The Bellman equations for the firm's profit in Proposition 2 can be written as

$$\begin{aligned}
(\chi + \lambda_u F)\,\Pi_u &= \overline{X}(1 + \frac{\Delta}{2}x) - W_u + \theta\Pi_{d+} + O(\Delta^2),\\
(\chi + \lambda_d F)\,\Pi_d &= \overline{X}(1 - \frac{\Delta}{2}x) - W_d + \theta\Pi_u + O(\Delta^2).
\end{aligned}$$

- Adding and substracting and division by Δ yields the first pair of equations in (8)

$$\begin{aligned}(\chi - \theta + \lambda F)\,\overline{\Pi} &= \overline{X} - \overline{W} + O(\Delta), \\ (\chi + \theta + \lambda F)\,\Pi_\Delta + \lambda F\overline{\Pi} &= x\overline{X} - W_\Delta + \theta\frac{\Pi_{d+} - \Pi_d}{\Delta} + O(\Delta).\end{aligned}$$

- The second pair of equations in (8) can be derived similarly

$$\begin{aligned}-\overline{W}' &= \lambda\overline{\Pi} + O(\Delta), \\ -W'_\Delta &= \lambda\Pi_\Delta + \lambda\overline{\Pi} + \theta\frac{\Pi_{d+W}}{\Delta}\overline{W}' + O(\Delta).\end{aligned}$$

- Hence, ω satisfies

$$\omega = \frac{1 + \frac{\chi-\theta}{\chi+\theta}x}{1 + \frac{\theta}{\chi}\left(\frac{\chi-\theta}{\chi\sigma} + 1\right)}.$$

- Substitution in equation (18) and taking logs yields Proposition 3. This also confirms the initial conjecture regarding the functional form of $W_t(F)$.

Step 8: derivation of the quit rate for jobs started in the upturn

-

$$\begin{aligned}q_{d+} &= \lambda_d F_d\left[W_{d+}^{-1}(W_u)\right] = \lambda_d F + \lambda_d\frac{dF}{d\overline{W}}\left[W_{d+}^{-1}(W_u) - W_u\right] + O\left(\Delta^2\right) \\ &= \lambda_d F - \lambda_d\frac{F}{\sigma\overline{W}}\frac{\theta}{\chi + \lambda F}W_\Delta\Delta + O\left(\Delta^2\right) \\ &= \lambda_d F\left[1 - \frac{\theta}{\sigma\chi}\omega\Delta\right] + O\left(\Delta^2\right) + O(F),\end{aligned}$$

where we use equation (23) in the second line and equation (19) in the third line.

- Taking logs yields Proposition 3.