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LIMITED COGNITIVE ABILITY AND SELECTIVE INFORMATION PROCESSING

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This paper studies the information processing behavior of a decision maker (DM) who can only process a subset of all the information he receives: before taking an action, the DM receives sequentially a number of signals and decides whether to process or ignore each of them as it is received. The model generates an information processing behavior consistent with that documented in the psychological literature: first, the DM chooses to process signals that are strong; second, his processing strategy exhibits confirmation bias if he has a strong prior belief; third, he tends to process signals that suggest favorable outcomes (wishful thinking). As an application I analyze how the Internet and the induced change in information availability affects the processing behavior of the DM. I show that providing more/better information to the DM could strengthen his confirming bias.

Limited Cognitive Ability and Selective Information Processing[1]

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Abstract

This paper studies the information processing behavior of a decision maker (DM) who can only process a subset of all the information he receives: before taking an action, the DM receives sequentially a number of signals and decides whether to process or ignore each of them as it is received. The model generates an information processing behavior consistent with that documented in the psychological literature: first, the DM chooses to process signals that are strong; second, his processing strategy exhibits confirmation bias if he has a strong prior belief; third, he tends to process signals that suggest favorable outcomes (wishful thinking). As an application I analyze how the Internet and the induced change in information availability affects the processing behavior of the DM. I show that providing more/better information to the DM could strengthen his confirming bias.

Keywords: limited ability, information overload, information avoidance, confirmation bias, wishful thinking, polarization

JEL codes: D83, D90

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1 Introduction

There is abundant evidence that people selectively process information in a systematically biased way. For instance, investors avoid looking at their financial portfolios when the market is down; individuals tend to ignore information that challenges their existing beliefs; people tends to attend to information that support desired outcomes but ignore contradictory evidences (see Golman et al. (2017) for a review of the literature of information avoidance). These behaviors of selective information avoidance lead to biased decision making and could significantly worsen our well-being as we miss out useful information.

As bad as information avoidance sounds, we are not able to process all available information, especially in this information era. Unavoidably, everyday we have to make many decisions on whether to process or ignore pieces of information. Processing a piece of information allows us to understand better its content and update our beliefs but consumes cognitive resources like time and attention which are limited. This limitation imposes a constraint on our processing capacity[3], which gives rise to the possibility of information avoidance: it could be optimal for individuals to ignore a piece of information in order to save their cognitive resources for another piece of information.

It is increasingly important nowadays to study individuals' selective processing behavior as information overload becomes a prominent issue. There is no doubt that the advance in technology provides us with more information on different issues. However, the amount of available information clearly exceeds our processing capacity and we have to strategically use our scarce cognitive resources. Indeed, there is evidence that the Internet are associated with biased processing behavior (Flaxman et al. (2016)). In order to understand the impacts of the Internet, information policies, or in general changes in informational environment, it is important to understand how people process information when they have limited processing ability, and how their processing behavior changes with the policies.

To answer these questions, this paper proposes a simple model of sequential information processing. Consider a decision maker (DM) who wants to match his action to an unknown state of the world, *e.g.,* vote for a candidate if he/she is the best option, invest in a project if it is profitable, *etc.* Before taking the action, the DM receives sequentially T imperfect signals about the state of the world. However, he is endowed with limited cognitive resources such that he can only "process" $\bar{T}$ ($< T$) signals. In each

[3]For instance, during an election, there are millions of articles on the Internet that could guide our voting decision. However, we are endowed with limited time and attention such that we could only read a very small subset of all the available articles.

period $t = 1, \cdots, T$, the DM observes some preliminary (imperfect) information about the realization of a signal, *e.g.,* the title of an article, and decides whether to "process" it. If he processes, he learns perfectly its realization and update his belief; otherwise, he "ignores" the signal without updating his belief. In other words, in each period, he decides whether to update his belief with the signal based on his (imperfect) knowledge about its realization[4]. Once he has consumed all his cognitive resources or has received all T pieces of information, he takes an action.

The model resembles a game of sequential search of information with a constraint on processing capacity. Given the capacity constraint, there is an inter-temporal trade off between spending the unit of capacity on the current signal or some future signal. In particular, there is a loss of processing as DM could save the unit of capacity for some future signal; while there is a gain of processing as otherwise the DM will not take into account the current signal in decision making and may take suboptimal action. The loss and gain of processing varies with the realization of the current signal, which drives selective processing decisions. I show that in the equilibrium, the DM selectively ignores some of the signals he receives. Moreover, the equilibrium processing strategy is "asymmetric", *i.e.,* the processing decisions are in general different for belief-confirming and belief-challenging signals.

In the continuation game where the capacity is equal to 1, *i.e.,* $\bar{T} = 1$, the game becomes a simple stopping time problem. The equilibrium processing strategy of the DM exhibits behavioral phenomena that are well-documented in the empirical and experimental literature. First, it exhibits a preference for strong signals (Itti and Baldi (2006)), *i.e.,* the DM tends to process strong signals and ignore weak signals. Second, the optimal processing strategy of confident individuals exhibits confirmation bias (Kahan et al. (2012)), *i.e.,* if the DM a priori strongly believes that one state is more probable than the other, he tends to process information which confirms his existing belief and ignores belief-challenging information. Third, the optimal processing strategy exhibits wishful thinking (Krizan and Windschitl (2007)), *i.e.,* if one state is much more desirable than the other as it is associated with a much higher maximum payoff, the DM tends to process information which supports the more desirable state. The results suggest that these "biases" could be understood as optimal strategies, driven by the limitation in processing ability.

Lastly, I analyze how changes in information structures, for example in-

[4]This is in particular different from models of information acquisition and rational inattention, where the DM decides whether to incur a fix cost to acquire a piece of information before knowing its realization. By definition, the DM's acquisition decisions cannot be different for belief-confirming and belief-challenging information.

duced by the Internet or information policies, affect the processing behavior of individuals. I show that providing more or in average better information to the decision maker could strengthen his confirmation bias, *i.e.,* he has more incentive to process belief-confirming information and ignore belief-challenging information. Even if two individuals are exposed to the same sequence of signals, they have more incentive to "cherry pick" the information that confirms their existing belief which could lead to polarization. Moreover, media has more incentive to selectively publish biased news stories as the demand increases. These results explain a number of empirical phenomena documented in the literature of ideological polarization.

The rest of the paper is organized as follows. In the next section, I present a review of the related literature. Section 3 shows a simple version of the model to illustrate the assumption of bounded rationality and intuitions of the results. In section 4, I present the model setting. Section 5 shows the results where $\bar{T} > 1$ while section 6 turns to the continuation game where $\bar{T} = 1$. Section 7 presents a variation of the model while section 8 shows two simple applications. Lastly, I conclude.

2 Related Literature

This paper is related to a wide range of literature, spanning economics, political science and psychology. First of all, the core assumption of this paper, *i.e.,* the DM has to use his limited cognitive resources in order to update his belief, is built on psychological theories and evidence. More specifically, it could arise from different channels as suggested by the literature, including the effort required to understand the information and to memorize the information[5].

On one hand, cognitive psychological theories proposed by Langdon and Coltheart (2000), Coltheart et al. (2011) and Connors and Halligan (2015) suggest that beliefs are formed as explanations to information. It requires efforts to understand the information in order to integrate it with the individual's existing system of beliefs. On the other hand, there is evidence from psychological studies showing that memory of information plays a big role in belief formation. One example is the seminal study of availability heuristic

[5] Memorizing a piece of information involves an encoding process, which can be strengthen by different factors, including time (Goldstein et al. (2011)), attention (Shallice et al. (1994), Craik, Govoni, et al. (1996), Benjamin and Bjork (2000) and Uncapher and Rugg (2005)), and how "deep" the individual processes the information (Craik and Lockhart (1972), Craik and Tulving (1975), Wagner et al. (1998) and Brewer et al. (1998)). Strengthening the encoding process increases the probability of recalling the information, but consumes scarce cognitive resources.

by Tversky and Kahneman (1973). They find that individuals evaluate the probability of an event by how much and how easily supportive evidence can be retrieved from memory. Memory-based belief formation is also modeled in the theoretical economics literature to explain different behavioral phenomena, e.g., Bénabou and Tirole (2002) and Baliga and Ely (2011). Both mechanisms, *i.e.,* to understand information or to memorize information, imply that individuals have to spend cognitive resources to "process" the information in order to update their belief.

The idea of limited cognitive ability has been introduced in the fast-growing literature of rational inattention, e.g., Sims (2003), Matejka and McKay (2014) and many others. They study a static[6] problem of information acquisition where the cost is proportional to the reduction in Shannon entropy. Different from this paper, under the setup of information acquisition, they cannot account for the phenomena that individuals selectively update their beliefs based on the realization of the signals, and that different individuals with different prior beliefs may update differently when they receive the same signals. Moreover, as they model directly individuals' choices of the distribution of their posterior beliefs, there is limited, if not no, role of information structure on individuals' beliefs. As a result, the literature of rational inattention cannot shed light on how changes in informational environment affects the belief formation of individuals.

In terms of results, this paper contributes to the literature of information avoidance (see Golman et al. (2017) for an extensive review of the literature). For instance, Eil and Rao (2011) show that individuals update less when they receive negative information about their appearance or intelligence than when they receive positive information. Karlsson et al. (2009) and Sicherman et al. (2016) find that investors avoid checking their financial portfolios when the market is down. The papers mentioned above, alongside with many others, document different systematic biases in the processing behavior of individuals, including the well-known confirmation bias.

There are many economic theories that explain the confirmation bias in processing behavior. For instance, Akerlof and Dickens (1982), Kőszegi (2003) and Brunnermeier and Parker (2005) show that anticipatory utility or belief-dependent utility leads to the confirmation bias; Carrillo and Mariotti (2000) and Bénabou and Tirole (2002) show that confirmation bias can be used as a remedy for time inconsistent preferences; Crémer (1995) and Aghion

[6]The literature of rational inattention gets around the mechanism of belief formation and assume the cost of reduction in uncertainty is proportional to the reduction in entropy. As a result, receiving several weak supportive evidence is equivalent to receiving one strong supportive evidence, which overlooks the dynamics of information processing and belief formation. In contrast, this paper focus on the dynamics of information processing.

and Tirole (1997) explain it with interpersonal strategic concerns; and the list goes on.

In contrast, this paper belongs to a relatively small, but growing, set of literature which suggests limitation in cognitive ability explains a number of behavioral "biases". Compte and Postlewaite (2012) and Wilson (2014) assume that the belief of the DM is constrained to a finite set of discrete memory states. Both papers show that the belief of the DM is non-responsive to weak information, while the latter also shows that the DM tends to update his belief with belief-confirming signals but not with belief-challenging signals. Jehiel and Steiner (2018) assume that the decision maker chooses which action he takes based on only one signal and can decide whether to redraw that signal. They provide a micro-foundation for theoretical models that individuals place linear attention weights on information. In contrast, this paper derives a wider range of results, which explain how biased processing behavior changes with individual characteristics and the informational environment.

Lastly, the results in this paper shed light on different issues in the information era. It includes political polarization, which receives lots of attention in recent years (See Prior (2013) for a review.). On one hand, Gentzkow and Shapiro (2011) and Flaxman et al. (2016) show that online media expose individuals to belief-challenging information. On the other hand, there is evidence that the political ideology among US citizens is getting polarized (Bartels (2000), Flaxman et al. (2016)), especially among those who are more politically engaged and partisan (Baldassarri and Gelman (2008), Abramowitz and Saunders (2008), Hetherington (2009).). This paper shows that the limitation in processing ability, or information overload, could explain the wide range of phenomena documented in the literature.

3 An illustrative example

Consider a voter who must decide to vote for either a left wing or a right wing candidate. Only one of the candidates is "good". Voting for the "good" candidate yields one util while voting for the "bad" candidate yields zero util.

Before he receives any information, the voter believes that there is a 70% probability that the left wing candidate is the good candidate. Before he must vote, he knows that he will receive two tweets from two journalists whom he trusts. Each tweet provides a signal about the quality of the candidates. More precisely, the tweets could be left leaning, right leaning or neutral. When the left wing candidate is the good candidate, the tweets are more likely to be left leaning than to be neutral, and are more likely to be neutral than to be right leaning. Similarly, when the right wing candidate is the good candidate, the tweets are more likely to be right leaning than to be neutral, and more likely to be neutral than to be left leaning. (Conditional on the identity of the good candidate, the tweets are independent of each other.) The information structure is represented on table 1.

	the tweet		
	left leaning	neutral	right leaning
left wing candidate is good	0.5	0.3	0.2
right wing candidate is good	0.2	0.3	0.5

Table 1: The probabilities of the tweets depending on the identity of the good candidate.

Now, the voter knows that he can "process" and use only one tweet for his voting decision. After he sees the first tweet, he knows whether it is left leaning, neutral or right leaning, but has to read the attached long article and understand its argument if he wants to update his belief and use the tweet for his voting decision. "Processing" the tweet takes time, so if the voter processes the first tweet, he will not be able to process the second tweet and will choose whom he votes for based on the first tweet. On the other hand, if he ignores the first tweet, he processes the second tweet and decides which candidate he will vote for based on it. To summarize, the voter faces a capacity constraint on belief updating and trade-off between processing the first or the second tweet.

Beliefs and Voting decisions For simplicity, I assume the voter updates his belief with the first or the second tweet in the same way, using the following Bayesian formula:

$$\begin{array}{c}\text{Belief given the processed}\\ \text{tweet is}\left\{\begin{array}{c}\text{left leaning}\\ \text{neutral}\\ \text{right leaning}\end{array}\right\}\end{array} = \frac{\Pr\left(\begin{array}{c}\text{a tweet is}\left\{\begin{array}{c}\text{left leaning}\\ \text{neutral}\\ \text{right leaning}\end{array}\right\}\\ \text{and the left wing candidate is good}\end{array}\right)}{\Pr\left(\text{a tweet is}\left\{\begin{array}{c}\text{left leaning}\\ \text{neutral}\\ \text{right leaning}\end{array}\right\}\right)}$$

It implies that the voter updates his belief with the processed tweet as if he has only received that tweet. If the voter has ignored the first tweet, he does not infer information about whether it is left leaning, right leaning or neutral, and updates his belief as if he has only received the second tweet. Section 7 relaxes this assumption and shows that the results do not change qualitatively[7]. Given this assumption, the voter's belief given the tweet he had processed[8] is presented on table 2.

	the tweet he had processed		
	left learning	neutral	right leaning
the voter's belief that the left wing candidate is good	$^{35}/_{41}$ $\approx 85\%$	$^{7}/_{10}$ $= 70\%$	$^{14}/_{29}$ $\approx 48\%$
voting decision	left wing	left wing	right wing

Table 2: The voter's belief given the tweet he has processed.

Given the posterior beliefs, the voter will vote for the right wing candidate if he processes a right leaning tweet; in all other cases, in which he processes a neutral or left leaning tweet, he will vote for the left wing candidate.

Processing Decision Now I analyze the voter's information processing decision. First consider the case where the first tweet is left leaning. If the DM processes the tweet, he will not be able to process the second tweet and will vote for the left wing candidate. The expected utility of processing the left leaning tweet is therefore equal to the conditional probability that the left wing candidate is good given the fact that the tweet is left leaning:

$$\Pr\left(\begin{array}{c}\text{left wing}\\ \text{candidate}\\ \text{is good}\end{array}\middle|\begin{array}{c}\text{first tweet}\\ \text{is left}\\ \text{leaning}\end{array}\right) = \frac{35}{41}.$$

On the other hand, if the voter ignores the left leaning tweet, his voting decision will depend on the second tweet, which is summarized in table 2. The expected utility of ignoring the left leaning tweet is therefore equal to a

[7] In this illustrative example, the result holds as long as the voting decision of the voter given the tweet he had processed follows table 2, regardless of whether he has processed the first or the second tweet.

[8] Given a left leaning tweet, the voter's belief equals $\frac{0.7\times0.5}{0.7\times0.5+0.3\times0.2} = \frac{35}{41}$; given a neutral tweet, his belief equals $\frac{0.7\times0.3}{0.7\times0.3+0.3\times0.3} = \frac{7}{10}$; given a right leaning tweet, his belief equals $\frac{0.7\times0.2}{0.7\times0.2+0.3\times0.5} = \frac{14}{29}$.

weighted average of the expected utility of his voting decisions given different realizations of the second tweet:

$$
\begin{aligned}
&\Pr\begin{pmatrix}\text{second}\\ \text{tweet is}\\ \text{left leaning}\end{pmatrix}\times\Pr\left(\begin{matrix}\text{left wing}\\ \text{candidate}\\ \text{is good}\end{matrix}\;\middle|\;\begin{matrix}\text{first tweet}\\ \text{is left}\\ \text{leaning}\end{matrix}\;\&\;\begin{matrix}\text{second}\\ \text{tweet is}\\ \text{left leaning}\end{matrix}\right)\\
+&\Pr\begin{pmatrix}\text{second}\\ \text{tweet is}\\ \text{neutral}\end{pmatrix}\times\Pr\left(\begin{matrix}\text{left wing}\\ \text{candidate}\\ \text{is good}\end{matrix}\;\middle|\;\begin{matrix}\text{first tweet}\\ \text{is left}\\ \text{leaning}\end{matrix}\;\&\;\begin{matrix}\text{second}\\ \text{tweet is}\\ \text{neutral}\end{matrix}\right)\\
+&\Pr\begin{pmatrix}\text{second}\\ \text{tweet is}\\ \text{right leaning}\end{pmatrix}\times\Pr\left(\begin{matrix}\text{right wing}\\ \text{candidate}\\ \text{is good}\end{matrix}\;\middle|\;\begin{matrix}\text{first tweet}\\ \text{is left}\\ \text{leaning}\end{matrix}\;\&\;\begin{matrix}\text{second}\\ \text{tweet is}\\ \text{right leaning}\end{matrix}\right)
\end{aligned}
$$

Note that when the voter's first period self evaluates the voting decisions of his second period self, he takes into account his knowledge of the first tweet; while his second period self will take into account only the second tweet. Therefore, there is an inconsistency between his first and second period self in the way they evaluate the two candidates and voting decisions.

With some simple algebra, the expected utility of ignoring the first leaning tweet is equal to $^{31}/_{41}$, which is smaller than the expected utility of processing. The voter processes the left leaning tweet. With similar computations, the expected utilities of processing and ignoring a neutral tweet are equal to $^{7}/_{10}$ and $^{71}/_{100}$ respectively; while the expected utilities of processing and ignoring a right leaning tweet are equal to $^{15}/_{29}$ and $^{187}/_{290}$ respectively. Therefore, the voter ignores the first tweet if it is neutral or right leaning. To summarize, *the voter processes the first tweet if and only if it is left leaning, i.e., it confirms his belief.*

The processing decisions of the voter exhibit a confirmation bias, which is well documented in the literature of information avoidance. Most of the theories in the literature explain the bias by belief-dependent utility, exogenous biases or interpersonal interaction. In contrast, the voter in this example is rational, Bayesian and cares only about maximizing the probability of voting for the good candidate. In this example, this confirmation "bias" is an optimal strategy solely driven by the limitation in processing ability.

Intuition When the voter makes his processing decisions, he trades off between the loss and gain of processing the tweet. On one hand, there is a loss of processing because there is an opportunity cost of forgoing the second tweet; on the other hand, there is a gain of processing because otherwise the voter will not take into account the first tweet and may make suboptimal voting decisions.

The loss and gain of processing the tweet varies with its realizations, which induce selective processing decision. When the voter receives a left leaning first tweet, he becomes more confident about the voting for the left wing candidate, which reduces the importance of the second tweet. In fact, in this special example, the second tweet becomes useless as it is not strong enough to alter the optimal voting decision. Even if the second tweet is right leaning, the optimal decision is still to vote the left wing candidate. On the other hand, there is a gain of processing as otherwise the voter will become under-confident about the left wing candidate and will switch sub-optimally to voting for the right wing if the second tweet is right leaning.

When the voter receives a neutral tweet, the loss and gain are different. In particular, there is no gain of processing as the voter still have the correct belief after ignoring the tweet. On the other hand, the second tweet is in average informative and processing it will (in average) improve the voting decisions of the voter.

Lastly, when the voter receives a right leaning tweet, the voter believes that the right wing candidate is marginally better. He is not confident about which action is optimal. In this case, the second tweet becomes more important and the loss of processing is big, and it outweighs the gain of processing.

In general, there is an asymmetry in the processing behavior regarding belief confirming and belief challenging information, because of the differences in loss and gain as illustrated above. However, the asymmetry does not always lead to confirmation bias. As will be shown in section 6, in the continuation game of the general model where $\bar{T} = 1$, there is a confirmation bias when the voter is a priori confident about the state, but not necessarily when the prior belief is not strong enough.

4 Model Setting

Primitives The decision maker faces a binary choice problem with two actions, $a \in \{l, r\}$. For example, l and r could represent voting for the left wing and right wing candidate respectively. There are two possible states of the world, $\omega \in \{L, R\}$. The DM wants to match his action to the state, *i.e.*, his utility function $u(a \mid \omega)$[9] follows

$$u(l \mid L) = u_L > 0;$$
$$u(r \mid R) = u_R > 0;$$
$$u(l \mid R) = u(r \mid L) = 0.$$

The DM's prior belief is (p_L, p_R), with $p_L + p_R = 1$ and $p_L, p_R > 0$. Without loss of generality, I assume $u_L p_L \geq u_R p_R$. Action l is a priori optimal.

Before the DM takes the action, he looks for information about the state of the world, for example on Google, Facebook or traditional media. The available information, for instance articles on the Google news page, is represented by a finite[10] sequence of signals, denoted $(s_t)_{t=1}^{T}$. A signal s_t is drawn from a set $\mathscr{S}_t$ with the p.d.f. in states L and R, denoted f_{tL} and f_{tR} respectively. I assume

$$f_{t\omega}(s_t) > 0 \text{ for all } s_t \in \mathscr{S}_t \text{ and both } \omega = L, R$$

which implies that no signal perfectly reveals the state. I allow for the possibility that the p.d.f. f_{tL}, f_{tR} vary with t, *i.e.,* different signals along the sequence may be drawn from different information structures. For example, articles on different positions of the Google news page could have different qualities, as they are ranked based on their features by an algorithm.

By the Bayesian formula, the information conveyed by a signal s_t is summarized by its likelihood ratio $\frac{f_{tL}(s_t)}{f_{tR}(s_t)}$. Therefore without loss of generality, I normalize signals as their likelihood ratios. For $t = 1, \cdots T$, I assume that $\mathscr{S}_t$ is the set of strictly positive real numbers $\mathscr{R}_{++}$ and

$$s_t = \frac{f_{tL}(s_t)}{f_{tR}(s_t)} \quad \text{for all } s_t \in \mathscr{R}_{++}.$$

For technical convenience, the c.d.f. F_{tL} and F_{tR} are assumed to be continuous but the results hold for more general distributions[11]. A signal $s_t > 1$ supports

[9]The model is identical up to the following transformation in the utility function:

$$u(l \mid L) = u_L + A; \qquad u(l \mid R) = B;$$
$$u(r \mid L) = A; \qquad u(r \mid R) = u_R + B,$$

for any constants $A, B \in \mathscr{R}$. What matters are the difference between the utilities of the two actions, fixing the state. The model can thus be applied to a setting where action l is risky, *i.e.,* it gives positive payoff u_L in state L and negative payoff $-u_R$ in state R, while action r is safe and gives 0 payoff in both states.

[10]The results in this paper hold for arbitrarily big T.

[11]For examples, the results hold for discrete distributions, and for distributions with mass points.

state L and confirms the DM's prior belief that action l is optimal; while a signal $s_t < 1$ supports state R and is belief-challenging[12]. Therefore, I define the set of belief-confirming information and belief-challenging information, denoted $\mathscr{S}_+$ and $\mathscr{S}_-$ respectively, as follows:

$$\mathscr{S}_+ = (1, \infty);$$
$$\mathscr{S}_- = (0, 1).$$

A signal $s_t = 1$ is pure noise which provides no information about the state of the world.

Strength of Signals The strength of a signal s_t, denoted by $STR(s_t)$, is defined as follows:

$$STR(s_t) = \max\left\{s_t, s_t^{-1}\right\} \in [1, \infty).$$

The larger s_t, the more likely that the signal is drawn in state L instead of state R, and the more convincingly it supports state L; similarly, the larger s_t^{-1}, the more convincingly it supports state R. Analogously, the strength of a set of signals $E \subset (0, \infty)$ is defined as:

$$STR(s_t \in E) = \max\left\{\frac{\int_E f_{tL}(s_t)ds_t}{\int_E f_{tR}(s_t)ds_t}, \frac{\int_E f_{tR}(s_t)ds_t}{\int_E f_{tL}(s_t)ds_t}\right\}.$$

If the DM does not know the realization of s_t but only that $s_t \in E$, the larger $\int_E f_{tL}(s_t)ds_t$ compared to $\int_E f_{tR}(s_t)ds_t$, the more probable that the true state is L.

Preliminary Information of Signals Upon receiving a signal s_t and before the DM decides whether to process or ignore it, he observes some preliminary information about its realization. For example, in the context of Google news page, the preliminary information are the titles of articles, graphics, sources, dates of publication and previews[13]. These information allows the DM to get a grasp on the content of the article with negligible time and attention, and then to decide whether to carefully read the article based on his perliminary understanding of its content.

[12]Note that the definition of belief-confirming and belief-challenging information corresponds to whether the information supports the a priori optimal action. Therefore, it depends not only on the belief of the DM, but also on the utility matrix. In particular, the set of belief-confirming and belief-challenging information may differ for two individuals who have same the beliefs but different utility functions.

[13]In general, the preliminary information could be interpreted as a brief understanding of the content of the article that consume negligible amount of cognitive resources.

Formally, the preliminary information is represented by a signal of the signal s_t, which is denoted by $\tilde{s}_t$. I assume that $\tilde{s}_t$ is drawn from a set $\mathscr{E}$ and with a p.d.f. $h(\cdot \mid s_t) : (0, \infty) \to \triangle_{\mathscr{E}}$. For example, the DM could observe whether the signal supports state L or state R but not its strength. In that case, $\mathscr{E} = \{-, \pm, +\}$ with $\tilde{s}_t = +$ if $s_t > 1$, $\pm$ if $s_t = 1$, $-$ if $s_t < 1$.

I denote by $g(\cdot \mid \tilde{s}_t)$ the conditional probability distribution of s_t given the preliminary information $\tilde{s}_t$. In the rest of the paper, without loss of generality, I often use g instead of $\tilde{s}_t$ to denote the preliminary information, as it summarizes the knowledge of the DM about the realization of s_t. Note that in the illustrative example, g is degenerate, *i.e.,* the DM observes perfectly the realization of s_t before taking the processing decision.

Processing Constraint and Timeline The DM is subject to a capacity constraint, in which he can only "process" up to $\bar{T}$ signals. If the DM processes a signal, he learns perfectly[14] the realization of s_t and updates his belief: he reads the articles, understands better the content and incroporates the information into his belief. Otherwise, he ignores the signal without updating his belief.

This mechanism of "processing" implies that the DM updates his belief only with the processed signals. Mathematically, let the set of processed signals at the beginning of period t as $\mathscr{M}_t$, which is also referred as the DM's memory[15]. By construction, $\mathscr{M}_1 = \emptyset$. If the DM processes s_t, $\mathscr{M}_{t+1} = \mathscr{M}_t \cup \{s_t\}$; otherwise, $\mathscr{M}_{t+1} = \mathscr{M}_t$. With some abuse of notations, I write $t' \in \mathscr{M}_t$ for some $t' < t$ if and only if the DM has processed the signal in period t', while $t' \notin \mathscr{M}_t$ is defined analogously. The belief of the DM at period t, denoted $(p_L^t(\mathscr{M}_t), p_R^t(\mathscr{M}_t))$, is a function of his memory $\mathscr{M}_t$ and his prior, which will be defined formally later in the section.

The timeline of the game is shown in figure 1. In period $t = 1, \cdots, T$, the DM observe the preliminary information $\tilde{s}_t$ and decides whether to process or ignore the signal. Once the DM has exhausted his processing capacity, the game jumps to period $T + 1$ and he takes one of the two actions. For simplicity, I assume that he cannot return to a piece of information he has ignored[16].

Apart from being subject to the processing constraint, the DM is rational

[14] This effect is absent in the illustrative example, where g is degenerate.

[15] This corresponds to one of the interpretation of processing, which is to memorize the information in order to update the one's belief.

[16] This assumption rules out the possibility that the DM keeps in mind the preliminary information of several pieces of news at the same time and decides which one(s) to process. It emphasizes the limitation of the DM's cognitive ability and the fact that the capacity of our working memory is small.

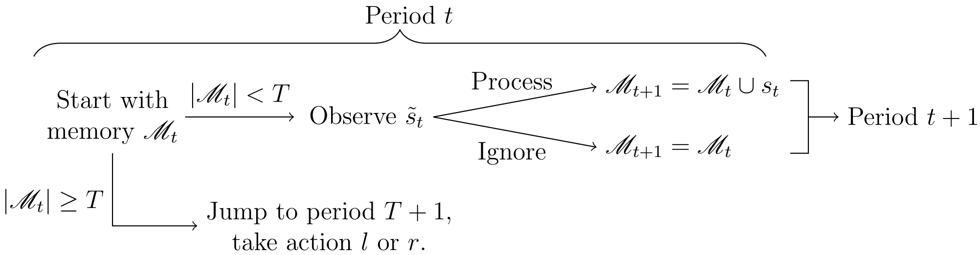


Figure 1: Timeline of the game at any period $t \leq T$.

and able to evaluate the expected utility of processing and ignoring a signal, using his memory $\mathscr{M}_t$ and the preliminary information g. The processing strategy of the DM at period t is denoted by $\sigma_t(g, \mathscr{M}_t) \in \{P, I\}$, where P represents the decision to process the signal and I represents the decision to ignore it. The processing strategy from period 1 to period T is denoted by $\sigma = (\sigma_t)_{t=1}^{T}$. I assume that if the DM is indifferent between processing or ignoring a signal, he processes it when the signal is belief-confirming and ignores it otherwise.

Posterior Belief and Equilibrium Concept I consider an "aware" and an "unaware" case on how the DM forms belief with his memory. The two cases correspond to two different levels of cognitive sophistication of the DM and are defined as follows. More specifically, the "unaware" case corresponds to the simplifying assumption made in the illustrative example, while the "aware" case relaxes the assumption.

Unaware DM In the unaware case, when the DM forms his belief, he does not infer information from the fact that he had chosen to ignore some previous signals. As a result, he updates his belief as if he has only received signals in his memory. Formally, the posterior belief of the unaware DM at period t satisfies:

$$\begin{aligned} p_L^t(\mathscr{M}_t) &= \frac{p_L \prod_{t' \in \mathscr{M}_t} f_{t'L}(s_{t'})}{p_L \prod_{t' \in \mathscr{M}_t} f_{t'L}(s_{t'}) + p_R \prod_{t' \in \mathscr{M}_t} f_{t'R}(s_{t'})} \\ &= \left(1 + \frac{p_R}{p_L \prod_{t' \in \mathscr{M}_t} s_{t'}}\right)^{-1}; \qquad (1) \\ p_R^t(\mathscr{M}_t) &= 1 - \Pr_L(\mathscr{M}_t). \end{aligned}$$

This "unawareness" in belief formation has been documented in the literature, as shown in an experimental study conducted by Enke and Zimmermann (forthcoming). They show that a significant amount of their subjects are not aware of, and do not take into account the correlation between different pieces of information when they form beliefs. Analogously, the "unaware" DM is not aware of, and does not take into account his previous processing decisions when he forms belief. This assumption is also equivalent to the equilibrium concept introduced by Jehiel (2005) and Eyster and Rabin (2005), in which they argue that individuals may not take into account how other people's action depend on their own information[17].

The game is a multi-selfs dynamic game. With the "unawareness" assumption, the DM acts as if there is no incomplete information[18]. The solution concept used is subgame perfect Nash equilibrium. At each period $t = 1, \cdots, T$, he anticipates how his processing decision affects the decisions of all his future selfs, using his belief of the state $p_L^t(\mathscr{M}_t)$ and the preliminary information g of the signal s_t. On the other hand, in period $T+1$, the DM takes actions l if $p_L^{T+1}(\mathscr{M}_{T+1}) > 1/2$ and r if $p_L^{T+1}(\mathscr{M}_{T+1}) < 1/2$. In case of indifference, I assume that the DM takes action l.

In most parts (except section 7) of the main text, I study the DM's processing behavior in only the unaware case. However, to help the reader to understand the differences between the two cases, I now briefly introduce the definition of the aware case (the formal definition is presented in section 7). The results in the two cases are similar.

Aware DM In the aware case, the DM is more sophisticated. When he updates his belief with his memory, he rationally infers information about the signals in periods $\{t' \mid t' \notin \mathscr{M}_t\}$ from the fact that he had chosen to ignore them. The game is a multi-selfs dynamic game with incomplete information and the corresponding solution concept is perfect Bayesian Nash equilibrium.

When the DM updates his belief with $\mathscr{M}_t$, he does not know his processing strategy in periods $t' < t$, but forms a conjecture denoted by $(\tilde{\sigma}_{t'})_{t'=1}^{t-1}$. In equilibrium, for all $t = 1, \cdots, T+1$, the conjecture $(\tilde{\sigma}_{t'})_{t'=1}^{t-1}$ of the DM's period t self has to be consistent with the equilibrium processing strategies of his previous selfs. On the other hand, when the DM decides whether to process or ignore a signal, he is fully aware of the fact that his future self will make rational inference. He evaluates the expected utility of processing and ignoring a signal and takes the optimal processing decision, using his

[17] In the current setting, when the unaware DM forms his belief, he does not take into account the fact that his previous selfs' processing decisions depend on their knowledge of the realization of the signals. As a result, he forms his belief as if the processing decisions of his previous selfs did not correlate with the signals' realizations.

[18] The DM acts as if he had only received the signals in his memory. In other words, it is as if he "mistakenly thinks" that he is in a singleton information set.

knowledge of the state and the corresponding signal realization.

4.1 Discussion

Before I present the analysis, I discuss the assumptions of the model. The two core building blocks of the model are the DM's inability to process all available signals and his ability to make optimal processing decisions based on his understanding of the signals realizations. While the first models the limitation in cognitive ability, the second imposes a high cognitive demand on the DM as he has to evaluate the probability of the states and anticipate how processing or ignoring the signals affects the decisions of his future selfs. Thus, this paper tries to take a minimal departure from the literature (the first building block) while keeping the rational benchmark (the second building block) to highlight how the capacity constraint can drive selective/biased processing behavior. In the following, I provide several justification/interpretations of the setting.

1. Short-term-long-term-memory conversion

The first interpretation of the model is a memory-based belief formation mechanism[19]. The DM forms belief only with his (long-term) memory of information. Upon receiving a piece of information, it first enters as a short-term memory. The DM then evaluates (consciously or unconsciously) the benefits and costs of processing, and decides whether or not to consume his cognitive resources to convert it into long-term memory.

2. Title (or a brief look) and the main text of articles

The second interpretation is related to the way individuals process news articles. The DM first reads the title of the article and roughly gets an impression of its main message. He then decides whether or not to read the main text based on this rough impression, which allows him to update his belief accordingly. If he chooses not to read the main text, his rough impression of the article has negligible, if not no, effect on his belief.

3. Optimal selective processing

Taking a step back, there is abundant evidence that individuals update differently with belief-confirming and belief-challenging information. This model studies the characteristics of the optimal selective processing strategies given the DM's limited processing ability, and analyzes how it resembles different empirical phenomena and how it changes with the informational environment.

4. Delegation on information acquisition

Although the aim of this paper is to analyze individuals behavior, the model

[19] See section 2 for related literature.

could be interpreted as a game between a decision maker and a delegate who acquires and presents information. Imagine a delegate who shares the same preference as the decision maker. The delegate engages in a sequential search of information and has to decide which pieces of information he present to the decision maker, who will take an action based on the presented information. The model depicts the situation where there is a capacity constraint on how many signals the delegate could present to the decision maker, which could be imposed by cost/time concerns or organization protocols.

5 Processing strategy when $\bar{T} > 1$

In this section, I characterize the processing strategy of the DM in period t, given preliminary information g. I first analyze the case where g is degenerate, *i.e.,* the preliminary information perfectly reveals the signal realization, and then proceed to discuss the differences with non-degenerate g. A degenerate distribution g is simply denoted by s_t. For simplicity of notations, I use (p^t_L, p^t_R) to denote the belief of the DM at the beginning of period t, where I assume $p^t_L u_L \geq p^t_R u_R$ without loss of generality.

To evaluate the expected utility of processing and ignoring a signal s_t, the DM has to anticipate how his processing decision will affect the action chosen by his future self. To simplify expressions, denote by $\Pr^{t+1}_{\mathscr{M}_{t+1}}(a \mid \omega)$ the probability that the DM takes action a in state ω, evaluated at the beginning of period $t+1$ given his memory $\mathscr{M}_{t+1}$. The DM's expected utility of processing s_t, denoted by $U_P(s_t, \mathscr{M}_t)$ is equal to the probability of matching his action with the state, weighted by the payoff u_L, u_R:

$$U_P(s_t, \mathscr{M}_t) = u_L \frac{p^t_L s_t}{p^t_L s_t + p^t_R} \Pr^{t+1}_{\mathscr{M}_t \cup \{s_t\}}(l \mid L) + u_R \frac{p^t_R}{p^t_L s_t + p^t_R} \Pr^{t+1}_{\mathscr{M}_t \cup \{s_t\}}(r \mid R). \quad (2)$$

In the first term, $\frac{p^t_L s_t}{p^t_L s_t + p^t_R}$ is the conditional probability of state L given the signal s_t while $\Pr^{t+1}_{\mathscr{M}_t \cup \{s_t\}}(l \mid L)$ is the probability that his future self succesfully matches his action with state L given that $\mathscr{M}_{t+1} = \mathscr{M}_t \cup \{s_t\}$. Similarly, his expected utility after ignoring the signal, denoted by $U_I(s_t, \mathscr{M}_t)$, follows:

$$U_I(s_t, \mathscr{M}_t) = u_L \frac{p^t_L s_t}{p^t_L s_t + p^t_R} \Pr^{t+1}_{\mathscr{M}_t}(l \mid L) + u_R \frac{p^t_R}{p^t_R + p^t_L s_t} \Pr^{t+1}_{\mathscr{M}_t}(r \mid R). \quad (3)$$

By comparing equation (2) and (3), we could see that the processing decision affects the utility as it induces a different distribution on the action chosen by his future self. Denote by $\triangle^t_{s_t,\mathscr{M}_t}(a \mid \omega)$ the change in probability of choosing action a in state ω after processing s_t, *i.e.*,

$$\triangle^t_{s_t,\mathscr{M}_t}(a \mid \omega) = \mathrm{Pr}^{t+1}_{\mathscr{M}_t \cup \{s_t\}}(a \mid \omega) - \mathrm{Pr}^{t+1}_{\mathscr{M}_t}(a \mid \omega),$$

we have $U_P(s_t, \mathscr{M}_t) \geq U_I(s_t, \mathscr{M}_t)$ if and only if:

$$\begin{aligned} & u_L \frac{p^t_L s_t}{p^t_L s_t + p^t_R} \triangle^t_{s_t,\mathscr{M}_t}(l \mid L) \geq -u_R \frac{p^t_R}{p^t_R + p^t_L s_t} \triangle^t_{s_t,\mathscr{M}_t}(r \mid R) \\ \iff \quad & u_L p^t_L s_t \triangle^t_{s_t,\mathscr{M}_t}(l \mid L) \geq -u_R p^t_R \triangle^t_{s_t,\mathscr{M}_t}(r \mid R). \end{aligned} \tag{4}$$

Equation (4) shows how the DM's processing decision depends on the change in the quality of decision making in the two states. For instance, if both $\triangle^t_{s_t,\mathscr{M}_t}(l \mid L)$ and $\triangle^t_{s_t,\mathscr{M}_t}(r \mid R)$ are positive, processing the signal improves the quality of decision making in both states, *i.e.*, there is a higher probability that the DM chooses the optimal action in both states. Unsurprisingly, by equation (4), the DM processes the signal. The opposite holds true when both $\triangle^t_{s_t,\mathscr{M}_t}(l \mid L)$ and $\triangle^t_{s_t,\mathscr{M}_t}(r \mid R)$ are negative[20]. The trade-off kicks in when the two functions have opposite signs, *i.e.*, processing the signal improves the quality of decision making in one state but worsens it in another state. The following lemma characterizes the DM's processing strategy given $\triangle^t_{s_t,\mathscr{M}_t}(l \mid L)$ and $\triangle^t_{s_t,\mathscr{M}_t}(r \mid R)$ are of opposite signs.

Lemma 1. *Assume $\triangle^t_{s_t,\mathscr{M}_t}(l \mid L)$ and $\triangle^t_{s_t,\mathscr{M}_t}(r \mid R)$ are of opposite signs with neither equal to 0, and suppose the DM is not indifferent between processing and ignoring. At period t, the DM's processing strategy is as follows:*

1. *if $\triangle^t_{s_t,\mathscr{M}_t}(l \mid L) > 0$, the DM chooses to process it if and only if*

$$s_t > \frac{u_R\, p^t_R}{u_L\, p^t_L} \times \frac{\triangle^t_{s_t,\mathscr{M}_t}(r \mid R)}{\triangle^t_{s_t,\mathscr{M}_t}(r \mid L)}; \tag{5}$$

2. *if $\triangle^t_{s_t,\mathscr{M}_t}(r \mid R) > 0$, the DM chooses to process it if and only if*

$$s_t^{-1} > \frac{u_L\, p^t_L}{u_R\, p^t_R} \times \frac{\triangle^t_{s_t,\mathscr{M}_t}(l \mid L)}{\triangle^t_{s_t,\mathscr{M}_t}(l \mid R)}. \tag{6}$$

[20]Note that with limited attention, processing a signal does not necessarily increase the probability of choosing the optimal action in either state. It is because processing lowers the number of signals the DM can process in the future. Take an extreme example where $s_t = 1$, processing the signal does not help the DM to know better about the state of the world, but consumes one unit of processing capacity that could be used on an informative future signal.

To understand the lemma, let us focus on case 1 where $\triangle^t_{s_t,\mathscr{M}_t}(l \mid L) > 0$. Note that the numerator of the term on the right-hand-side of equation (5) measures the loss in utility in state R while the denominator measures the gain in utility in state L, with both weighted by the prior beliefs. Thus the R.H.S. measures the loss-to-gain ratio of processing the signal s_t, adjusted with the prior belief. If s_t is big enough, the signal strongly suggests that state L is true and thus the probability of incuring the loss is small, which gives the DM incentive to process. Similar intuition applies for case 2 where $\triangle^t_{s_t,\mathscr{M}_t}(r \mid R) > 0$.

Using lemma 1 and to shed light on the role of the capacity constraint, the following proposition compares the equilibrium processing strategies of the DM without and with limited cognitive ability, *i.e,* where $\bar{T} \geq T$ and $\bar{T} < T$.

Proposition 1. *If $\bar{T} \geq T$, the DM processes almost all signals he receives in equilibrium, i.e., for all t and $\mathscr{M}_t$, $\sigma_t(s_t, \mathscr{M}_t) = P$ for almost*[21] *all $s_t \in (0, \infty)$, while that is not true if $\bar{T} < T$.*

Without limited cognitive ability, there is no trade off between current and future signals. Thus, it is always optimal to process all available signals, as otherwise the DM will have "incorrect" belief about the state and may take an suboptimal action. When $\bar{T} < T$, the trade off between current and future signals kicks in. Processing a signal does not only change the belief of the DM, but consume one unit of processing capacity which could be used for some future signals. Suppose to the contrary of proposition 1, the DM processes all signals he receives. At period t, the decision to process or ignore s_t is equivalent to the decision on whether to take action based on $(s_t, \cdots s_{t+\bar{T}-1})$ or on $(s_{t+1}, \cdots s_{t+\bar{T}})$, *i.e.,* the DM trades off between s_t and $s_{t+\bar{T}}$. When s_t is very uninformative (close to 1), the gain of processing is small. It is optimal for the DM to save his processing capacity for $s_{t+\bar{T}}$, which is in average much more informative than $s_t \approx 1$.

Apart from strategically ignoring signals, the DM's processing strategy is "asymmetric". The two inequalities (5) and (6), which characterize the processing decisions regarding to signals that increase the probability of choosing action l and r respectively, are generically different. This asymmetry drives differences between the processing behavior regarding information supporting or against the a prior optimal action. As will be explored in more details in next section, this asymmetry in processing strategy explains a number of "biases" documented in the literature, including confirmation bias for confident individuals.

[21] The DM is indifferent between processing and ignoring $s_t = 1$, which has mass equal to 0.

Non-degenerate g Now I turn to the case where g is non-degenerate, *i.e.,* the preliminary information g does not perfectly reveal the realization of the signal s_t. In this case, processing the signal has two effects: it allows the DM to learn the realization of s_t and update his belief. The expected utilities of processing and ignoring a signal with preliminary information g are:

$$U_P(g, \mathscr{M}_t) = \int U_P(s_t, \mathscr{M}_t) g(s_t \mid \tilde{s}_t)\, ds_t;$$
$$U_I(g, \mathscr{M}_t) = \int U_I(s_t, \mathscr{M}_t) g(s_t \mid \tilde{s}_t)\, ds_t,$$

which gives the following result.

Corollary 1. *Suppose when g is degenerate, the DM's processing strategy at period t is to process s_t if and only if its realization is in some set $\bar{\mathscr{S}}$, i.e., $\sigma_t(s_t, \mathscr{M}_t) = P$ if and only if $s_t \in \bar{\mathscr{S}}$.*

Now suppose the DM observes that $s_t \sim g$ for some non-degenerate g. He processes the signal if there is a high enough probability that the signal realization is in the set $\bar{\mathscr{S}}$, i.e, $\sigma_t(g, \mathscr{M}_t) = P$ if $\int_{s \in \bar{\mathscr{S}}} g(s \mid \tilde{s}_t)\, ds$ is big enough.

Importantly, corollary 1 implies that one could focus on the case where g is degenerate, without losing much qualitative insights. For instance, if the DM processes belief-confirming signals but ignores belief-challenging signals, he also processes a signal if he observes that the signal is likely to be belief-confirming. In the rest of the paper (unless when specified), I analyze only the case where g is degenerate.

6 Continuation Game where $\bar{T} = 1$

In this section, to show more detailed behavioral implications, I present the analysis of the continuation game where $\bar{T} = 1$, *i.e.,* the DM can only process one signal. Furthermore, I will only look into the case where $T = 2$, *i.e.,* there are only two available signals. In appendix B, I show that it is without loss of generality[22] to do so when $\bar{T} = 1$. By relabeling the current period as period 1, the processing decision boils down to whether the DM processes s_1 or ignores s_1 in order to save his cognitive resources for s_2. Using[23] lemma 1, the equilibrium processing strategy of the DM is characterized in the following proposition.

[22]Loosely speaking, I show that the equilibrium processing strategy of the DM in any period $t < T$ is equivalent to the equilibrium processing strategy of the DM in period 1 in a game where $T = 2$ with some information structure $\tilde{f}_{2L}, \tilde{f}_{2R}$.

[23]When $\bar{T} = 1$, $\triangle^1_{s_1, \mathscr{M}_1}(l \mid L)$ and $\triangle^1_{s_1, \mathscr{M}_1}(r \mid R)$ are always of opposite signs.

Proposition 2. *In period 1, the DM processes a signal s_1 if and only if it is strong enough, i.e.,*

- *he ignores pure noise $s_1 = 1$;*
- *he processes a belief-confirming signal $s_1 > 1$ if and only if*

$$STR\left(s_1\right) = s_1 \geq \Phi^+; \tag{7}$$

- *he processes a belief-challenging signal $s_1 < 1$ if and only if*

$$STR\left(s_1\right) = s_1^{-1} > \Phi^-, \tag{8}$$

where Φ^+ and Φ^- are defined as follows:

$$\begin{aligned} \Phi^+ &= \frac{u_R\, p_R}{u_L\, p_L} \times STR\left(s_2 \in \left(0, \frac{u_R\, p_R}{u_L\, p_L}\right)\right); \\ \Phi^- &= \frac{u_L\, p_L}{u_R\, p_R} \times STR\left(s_2 \in \left[\frac{u_R\, p_R}{u_L\, p_L}, \infty\right)\right) > \frac{u_L\, p_R}{u_R\, p_R}. \end{aligned} \tag{9}$$

The equilibrium processing strategy of the DM is a threshold-strategy, *i.e.,* he processes the signal if and only if the signal strength is larger than some threshold as illustrated in figure 2. As shown by equation (9), the two thresholds are generically different, *i.e.,* the processing strategy is asymmetric as pointed out in the previous section.

Moreover, the loss-to-gain ratio of processing s_1 is captured by the strength of a set of "contradictory" future signal s_2, which corresponds to the realizations that induce a different action compared to s_1 (see table 3). For example, consider $s_1 > 1$. The bigger $F_{2R}(\frac{u_R p_R}{u_L p_L})$ is, the higher the probability the DM will receive contradictory future signal in state R and switch (optimally) to action r; the smaller $F_{2L}(\frac{u_R p_R}{u_L p_L})$ is, the less likely the DM will receive contradictory future signal in state L and switch (sub-optimally) to action r. Therefore, the stronger the contradictory signals are, the bigger the loss-to-gain ratio of processing s_1 (or forgoing s_2) is. Loosely speaking, when an individual decides whether not to read a piece of left-wing news, he is concerned about losing time and/or attention for a right-leaning article as it provides contradictory information. Thus, the stronger the contradictory future is compared to the current signal signal, the more incentive the DM has to ignore the current signal and save his cognitive resources for the future signal. This feature has important implications on the comparative analysis, which will be discussed later in this section.

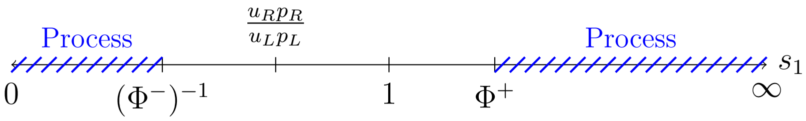


Figure 2: The DM processes s_1 if its realization is inside the shaded region.

		Final action	
s_1	s_2	process s_1	ignore s_1
> 1	$\geq \frac{u_R p_R}{u_L p_L}$	l	l
	$< \frac{u_R p_R}{u_L p_L}$	l	r
$< \frac{u_R p_R}{u_L p_L}$	$\geq \frac{u_R p_R}{u_L p_L}$	r	l
	$< \frac{u_R p_R}{u_L p_L}$	r	r

Table 3: The table shows the final action given (s_1, s_2) and the processing decision. The colored boxes highlight the configurations of s_2 that induce different actions compared to s_1, which corresponds to the subset of realizations of s_2 shown in equation (9).

6.1 Behavioral Phenomena

The equilibrium processing strategy has behavioral implications which explain some of the "biased" processing behaviors documented in the behavioral economics and psychological literature. The first implication is that there is a preference for strong signals, *i.e.,* the DM processes only strong enough signals in equilibrium. This implies that individuals react more to striking or convincing information but ignore coarse or ambiguous information. Analogously, in an experiment which studies visual attention, Itti and Baldi (2006) shows that individuals selectively allocate visual attention to details that induce a large difference between prior and posterior belief.

Another implication relates to the confirmation bias, which is defined as follows:

Definition 1 (Confirmation Bias). *The equilibrium processing strategy exhibits confirmation bias if $\Phi^- > \Phi^+$. That is, the DM processes a larger set of belief-confirming information than belief-challenging information.*

When $\Phi^- > \Phi^+$, the DM processes a belief-confirming signal s but ignores an equally strong belief-challenging signal s^{-1} for all $s \in [\Phi^+, \Phi^-]$. On the other hand, there does not exists any $s \in (1, \infty)$ such that the DM ignores a belief-confirming signal s but processes an equally strong belief-challenging signal s^{-1}.

The following proposition presents two behavioral implications of the equilibrium processing strategy.

Proposition 3. *The equilibrium processing strategy of the DM explains the following behavioral phenomena:*

1. ***(preference for strong signals)***
   The DM processes a signal if and only if it is strong enough, as shown in equation (7) and (8).

2. ***(confirmation bias for confident individuals)***
   If the DM is a priori confident enough that state L is true, his processing strategy exhibits confirmation bias, i.e., $\Phi^- > \Phi^+$ *if* p_L *is big enough.*

The intuition of proposition 3 can be understood as a trade-off between current and future information. On one hand, processing the current signal induces a loss as the DM forgoes informative future signal; on the other hand, it induces a gain as otherwise he will not take the current signal into account when he takes action l or r. First, the stronger (more informative) the current signal is, the more important it is to take it into account in the decision making problem so the gain is bigger. Moreover, a stronger current signal implies a more extreme posterior and the future information becomes less important. Both drives the preference for strong signals.

Second, when the DM is a priori confident enough that state L is true, even if he receives a weak belief-confirming signal, he becomes almost sure that action l is the optimal action. Future information becomes less important as there is a small probability of receiving a very strong contradictory signal $s_2 < \frac{u_R p_R}{u_L p_L s_1}$. The gain of processing s_1, to prevent himself from switching sub-optimally when $s_2 \in [\frac{u_R p_R}{u_L p_L s_1}, \frac{u_R p_R}{u_L p_L})$, dominates the loss when p_L is big enough. In contrast, even when the DM has processed a strong belief-challenging signal, he is not sure about the optimality of action r. Therefore, the future information becomes important, which induces the DM to ignore the current signal and save the capacity for the future signal. This asymmetry[24] in the incentive to look for future information drives the confirmation bias.

The result of confirmation bias also holds when g is non-degenerate. To illustrate that, consider two pieces of preliminary information, g^+ and g^-,

[24]Note that the gain of processing always dominates the loss when the DM are almost sure about which action is optimal, *i.e.,* when $\frac{u_L p_L s_1}{u_R p_R} \to 0$ or $\frac{u_L p_L s_1}{u_R p_R} \to \infty$; the reverse is always true when the DM is indecisive about the two action, *i.e.,* when $\frac{u_L p_L s_1}{u_R p_R} \approx 1$.

where

$$g^+(s) = 0 \text{ for all } s \leq 1;$$
$$g^-(s) = 0 \text{ for all } s \geq 1.$$

That is, the DM observes that the signal s_1 is belief-confirming if $s_1 \sim g^+$ and is belief-challenging if $s_1 \sim g^-$. Moreover, in both cases, he is not sure about the signal strength.

Corollary 2. *Suppose a priori the DM believes strongly that state L is true, i.e., p_L is big enough. In period 1, he processes the signal if he observes that $s_1 \sim g^+$ but ignores the signal if he observes that $s_1 \sim g^-$.*

Both proposition 3 and corollary 2 explain the phenomenon of information avoidance, in particular the avoidance of belief-challenging information (Kahan et al. (2012)). Although this behavior of information avoidance looks like a systematic bias against belief-challenging information, the results aforementioned suggest that when one takes into account the limitation in cognitive ability, "confirmation bias" actually arises as an optimal strategy under a setting with rational and Bayesian individuals. This is in particular different from the literature which explains the processing behavior with belief-dependent utility, exogenous biases such as time-inconsistent preference or interpersonal interaction[25]. Moreover, the results also help us to understand how the "confirmation bias" differs across different subjects in experiments.

It is interesting to note the limitation in processing ability drives not only a confirmation bias, but also a "bias" in action. More specifically, if the DM is a priori confident enough, he chooses action l with a higher probability in the current setting, than in a setting where there is no limitation in processing ability.

Corollary 3 (Bias in action). *Suppose the DM a priori believes strongly that state L is true, i.e., p_L is big enough. He chooses action l with a higher probability when he can only process one of the two signals s_1 and s_2, than when he can process both.*

Now I present another behavioral implication, which relates to wishful thinking. It refers to individuals' tendency to form optimistic belief about some desirable outcomes. In the psychological theory proposed by Krizan and Windschitl (2007), one of the mechanisms driving wishful thinking is that individuals tend to process and encode information that suggests some desirable outcomes. In the current setting, u_L and u_R measures the how

[25]See section 2 for the references.

desirable the two states are. When u_L is larger than u_R, state L is more desirable than state R as it is associated with a higher achievable payoff. By analogy to confirmation bias, wishful thinking is defined as follows:

Definition 2 (Wishful Thinking). *Suppose state L is more desirable than state R, i.e., $u_L \geq u_R$. The equilibrium processing strategy of the DM exhibits wishful thinking if $\Phi^- > \Phi^+$. That is, he processes a larger set of information that supports state L, compared to that supports state R.*

It is clear from equation (7) and (8) that an increase in u_L/u_R has the same effect as an increase in p_L/p_R. Therefore, the second point of proposition 3 implies the following result.

Corollary 4 (Wishful thinking). *When state L is much more desirable than state R, the equilibrium processing strategy of the DM exhibits wishful thinking, i.e., $\Phi^- > \Phi^+$ when u_L/u_R is big enough*[26].

6.2 Comparative Analysis

I now analyze how a change in the information structures (f_{2L}, f_{2R}), which could be induced by the Internet or information policies, affects the information processing behavior of the DM. In particular, I study whether the change strengthens the confirmation bias. Consider two environments A and B, which are associated with two different equilibrium processing strategies characterized by two sets of thresholds (Φ_i^+, Φ_i^-), $i = A, B$. The confirmation biases under the two environments are compared as follows:

Definition 3 (Comparison of confirmation bias). *The confirmation bias is stronger under environment A than under environment B if $\Phi_A^+ \leq \Phi_B^+$ and $\Phi_A^- \geq \Phi_B^-$, where at least one of the two inequalities is strict. That is, under environment A, the DM processes only a subset of belief-challenging information that he would process under environment B; while the reverse is true for belief-confirming information.*

In the sequel, I analyze two specific types of change in the information structure, in order to illustrate how the Internet could strengthen the confirmation bias of the DM. The first type of change captures the idea that the Internet facilitates a better access to information, thanks to the decrease

[26]Note that the model also explains a reverse phenomenon of wishful thinking, namely that individuals tend to form pessimistic belief about some undesirable outcomes when the associated loss increases (Dunning and Balcetis (2013)). The formal result is presented as corollary 9 in the appendix.

in information transmission cost and the advance of search technology. Formally, I assume that under environment j, where $j = A, B$, the signal s_2 is drawn from the following distribution[27]:

$$s_2 = \begin{cases} 1 & \text{with probability } 1 - \lambda_j; \\ s & \text{otherwise, where } s \sim (f_{2L}, f_{2R}). \end{cases}$$

The bigger λ_j is, the better the access to information is. When $\lambda_A > \lambda_B$, there is a higher probability to receive informative signals $s_2 \neq 1$ under environment A than that under environment B.

Proposition 4. *If environment A facilitates a better access to information compared to environment B, then the confirmation bias is stronger under environment A than under environment B, i.e., if $\lambda_A > \lambda_B$, we have $\Phi_A^+ = \Phi_B^-$ and $\Phi_A^- > \Phi_B^+$.*

Before I present the intuition, let me first present the analysis on the second type of change. It corresponds to the idea that the Internet lowers the barrier to information production. As a result, it facilitates the development of new media, and therefore provides individuals with more information sources. The quality of the new information sources could be different from the old ones. Intuitively, the skewness of the information structure of the new sources have a great impact on the processing strategy of the DM. For instance, if the new information sources produce strong belief-challenging information but weak belief-confirming information, the DM has more incentive to process the former and ignore the latter. To avoid assuming whether the new information sources are biased towards one state or the other, I analyze the case where the information structure of the new sources are in aggregate symmetric.

Formally, I assume that under environment B, the signal s_2 is drawn from the following distribution:

$$s_2 = \begin{cases} 1 & \text{with probability } 1 - \lambda \\ s_B & \text{otherwise, where } s_B \sim (f_{2L}^B, f_{2R}^B), \end{cases} \tag{10}$$

while under environment A, the signal s_2 is drawn from the following distribution:

$$s_2 = \begin{cases} 1 & \text{with probability } 1 - \lambda - \delta \\ s_B & \text{with probability } \lambda, \text{ where } s_B \sim (f_{2L}^B, f_{2R}^B); \\ s_A & \text{with probability } \delta, \text{ where } s_A \sim (f_{2L}^A, f_{2R}^A). \end{cases} \tag{11}$$

[27]Note that unlike in previous sections, the c.d.f.s of s_2 defined here are not continuous. Nonetheless, the results in previous sections still hold.

for some symmetric distribution (f_{2L}^A, f_{2R}^A), *i.e.*, $f_{2L}^A(s) = f_{2R}^A(s^{-1})$. Put differently, environment A transfers some mass δ of pure noise to the new information sources, which is characterized by a symmetric distribution (f_{2L}^A, f_{2R}^A).

Proposition 5. *Consider two environments, A and B, defined in equation (11) and (10). The confirmation bias is stronger under environment A compared to environment B if and only if the strong belief-challenging information drawn from the new sources is weaker than the same information drawn from the old information sources, i.e.,*

$$STR\left(s_A \in \left(-\infty, \frac{u_R\, p_R}{u_L\, p_L}\right)\right) \leq STR\left(s_B \in \left(-\infty, \frac{u_R\, p_R}{u_L\, p_L}\right)\right).$$

Note that both types of change are defined in a way such that the information structure under environment A is in average more informative[28] than that under environment B. When the future information becomes more informative, intuition suggests that the DM should have more incentive to ignore the current information and process the future information. However, in this model, the processing decision depends not on the average informativeness of all realizations of s_2, but only on the set of contradictory realizations of s_2. For instance, when the DM decides whether to process an left leaning article, he does not worry about losing attention for another left leaning article but worry about losing attention for a right leaning article. While environment A increases the average strength of s_2 compared to environment B, it does not necessarily increase the average strength of the strong belief-challenging information $s_2 < \frac{u_R p_R}{u_L p_L}$. As a result, providing in average "better" information to the DM could strength his confirmation bias.

The Internet undoubtedly facilitates better access to information and lowers the cost of information production. As the Internet lowers the entry cost of information production, the new information sources are in general of lower quality. Proposition 4 and 5 contribute theoretical explanations and insights to the literature of political polarization, as it suggests that the Internet promotes biased processing behaviors or so called "cherry-picking" on information, in the presence of information overload.

7 Aware DM

In this section, I show how the results in previous sections hold in the "aware" case. Throughout the section, I assume that the preliminary information

[28]The average strength of signal s_2 in environment A is bigger than that in environment B, *i.e.*, $E_A(STR(s_2)) > E_B(STR(s_2))$.

perfectly reveals the realization of the signal *i.e.,* I focus on the case where g is degenerate.

First, let me formally define the "aware" case. When the DM forms his belief with his memory $\mathscr{M}_t$, he rationally infers information about the signals in periods $\{t' \mid t' \notin \mathscr{M}_t\}$ from the fact that he had chosen to ignore them. More specifically, he forms a conjecture $\tilde{\sigma}_{t'}$ about his processing strategies in those periods t' and his posterior is given by the following Bayesian formulas:

$$p_L^t(\mathscr{M}_t, (\tilde{\sigma}_{t'})_{t'=1}^{t-1}) = \left(1 + \frac{p_R}{p_L \prod_{t' \in \mathscr{M}_t} s_{t'} \prod_{t' \notin \mathscr{M}_t} \mathscr{R}_{t'}(\tilde{\sigma}_{t'})}\right)^{-1};$$
$$p_R^t(\mathscr{M}_t, (\tilde{\sigma}_{t'})_{t'=1}^{t-1}) = 1 - \Pr_L(\mathscr{M}_t, (\tilde{\sigma}_{t'})_{t'=1}^{t-1}),$$

where $\mathscr{R}_{t'}(\tilde{\sigma}_{t'})$ is the ratio of the conjecture probability of ignoring a period t' signal in state L over state R:

$$\mathscr{R}_{t'}(\tilde{\sigma}_{t'}) = \frac{\int (\mathbb{1}_{\tilde{\sigma}_{t'}(s_{t'}, \mathscr{M}_{t'})=I}) f_{t'L}(s_{t'})\, ds_{t'}}{\int (\mathbb{1}_{\tilde{\sigma}_{t'}(s_{t'}, \mathscr{M}_{t'})=I}) f_{t'R}(s_{t'})\, ds_{t'}}.$$

On the other hand, when he decides whether to process or ignore a signal, he knows that if he ignores it, his future selfs will rationally infer information about the signal. The game is a multi-self dynamic game with incomplete information. The solution concept used is therefore perfect Bayesian Nash equilibrium, which requires the optimality of the processing strategies as well as the consistency of the DM's conjecture, *i.e,* $\tilde{\sigma}_{t'}$ has to coincide with the equilibrium processing strategy for all $t' = 1, \cdots, T$.

First, I analyze the case where $\bar{T} > 1$. As in section 5, I analyze the processing strategy at period t, taking $\Pr^t_{\mathscr{M}_t}(a \mid \omega)$ and $\triangle^t_{s_t, \mathscr{M}_t}(a \mid \omega)$ as given. Note that two functions do not specify how the DM forms his posterior belief, and therefore encompass both the aware and unaware case. Therefore, lemma 1 holds and the processing strategy of the DM is "asymmetric". Moreover, as the definitions of "awareness" and "unawareness" make no differences when the DM processes all signals, proposition 1 also holds, *i.e.,* the DM processes almost all signals in equilibrium when $\bar{T} \geq T$ which is not true when $\bar{T} < T$.

Now I turn to the case where $\bar{T} = 1$. As in the unaware case, it is without loss of generality to further simply the model to $T = 2$. The equivalent result is presented in appendix B. Different from the unaware case, there is no guarantee that there exists a unique equilibrium in the aware case. The following proposition proves the existence of a perfect Bayesian Nash equilibrium.

Proposition 6. *There exists a perfect Bayesian Nash equilibrium in the aware case.*

The proof of the proposition is shown in the appendix. It follows classical fixed point arguments. Note that there may exist multiple equilibria because of the self-fulfilling nature of the incomplete information game. If the future selfs of the DM conjecture that his period t self ignores only belief-challenging information, his period t self will have more incentive to ignore belief-challenging information as he knows that his future selfs will rationally infer information from it. On the other hand, he also has more incentive to process belief-confirming information because otherwise, his future selfs will mistakenly conjecture that he has received a belief-challenging information in period t.

In the following, I show that the behavioral implications presented in section 6 hold qualitatively for all equilibria in the aware case. First, I characterize the equilibrium processing strategy, as in proposition 2.

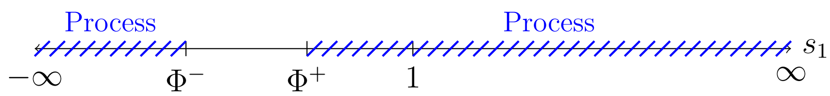
Proposition 7. *The equilibrium processing strategy of the aware DM is as follows:*

- *he processes a belief confirming signal $s_1 > 1$ if and only if*
$$STR(s_1) = s_1 \geq \Phi^+;$$
- *he processes a pure noise or a weak belief-challenging signal $s_1 \in [\frac{u_R p_R}{u_L p_L}, 1]$ if and only if*
$$s_1 > \Phi^+;$$
- *he processes a strong belief-challenging signal $s_1 < \frac{u_R p_R}{u_L p_L}$ if and only if*
$$STR(s_1) = s_1^{-1} > \Phi^-,$$

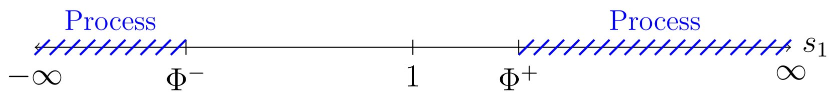
where Φ^+ and Φ^- are the fixed point of the following system of equations:

$$\begin{aligned}
\Phi^+ &= \frac{u_R\, p_R}{u_L\, p_L} STR\left(s_2 \in \left(0, \frac{u_R\, p_R}{u_L\, p_L \dfrac{F_{1L}(\Phi^+) - F_{1L}(1/\Phi^-)}{F_{1R}(\Phi^+) - F_{1R}(1/\Phi^-)}}\right)\right) \\
\Phi^- &= \frac{u_L\, p_L}{u_R\, p_R} STR\left(s_2 \in \left[\frac{u_R\, p_R}{u_L\, p_L \dfrac{F_{1L}(\Phi^+) - F_{1L}(1/\Phi^-)}{F_{1R}(\Phi^+) - F_{1R}(1/\Phi^-)}}, \infty\right)\right)
\end{aligned} \tag{12}$$

The proof is very similar to that of proposition 2 and is therefore skipped. The result is illustrate in figure 3. Note that different from the unaware



(a) $\Phi^+ < 1$.



(b) $\Phi^+ \geq 1$

Figure 3: Equilibrium Information Processing Strategy: the DM processes s_1 if and only if its realization falls on the shaded area.

case, the DM may process weak belief-challenging information (when $\Phi^+ < 1$). This difference is again driven by the self-fulfilling nature of the game: suppose the DM's period 2 self conjectures that his period 1 self ignores strong belief-challenging signals but processes weak belief-challenging signals. In the view of his period 1 self, he knows that if he ignores the weak belief-challenging signals, his period 2 self will mistakenly infer that s_1 strongly supports state R. It gives the DM's period 1 self incentive to process weak belief-challenging signals and take action l, as otherwise he will be under-confident about state L and switch to action r too easily in period 2.

Despite the slight difference, the behavior implications hold qualitatively. First, the DM processes strong enough signals, but may ignore weak signals, *i.e.,* it resembles a preference for strong signals (although it is weaker than the version in the unaware case). Moreover, by equation (12), the limits of Φ^+ and Φ^- when $u_L p_L / u_R p_R \to +\infty$ are the same as in the unaware case[29]. Hence, when $u_L p_L / u_R p_R$ is large enough, $\Phi^- > \Phi^+$. The results are summarized in the following proposition.

Proposition 8. *The equilibrium processing strategy of the DM explains the following behavioral phenomena:*

1. ***(Preference for strong signals)***
   The DM processes a signal s_1 if it is strong enough.

2. ***(Confirmation bias for confident individuals)***

[29]That is, as shown in the proof of proposition 3,

$$\lim_{u_L p_L / u_R p_R \to +\infty} \Phi^+ = 1$$
$$\lim_{u_L p_L / u_R p_R \to +\infty} \Phi^- = +\infty$$

If the DM a priori believes strongly that state L is true, his processing strategy exhibits confirmation bias, i.e., $\Phi^- > \Phi^+$ *if* p_L *is large enough.*

3. ***(Wishful thinking if one state is much more desirable than the others)***
If state L is much more desirable than state R, the DM's processing strategy exhibits confirmation bias, i.e., $\Phi^- > \Phi^+$ *if* u_L/u_R *is large enough.*

Note that the results hold in all equilibria in the aware case, *i.e.,* all equilibrium processing strategies of the DM have (qualitatively) the same behavioral implications as in the unaware case.

Lastly, because there is an issue of multiple equilibria, the result of the comparative analysis would not be as clean as in the unaware case. However, the insights still hold true. As shown in equation (12), the thresholds which characterize the equilibrium processing strategy depend only on a subset of all realizations of s_2. As in the unaware case, providing in average more informative signal to the DM does not necessarily increases the average strength of a subset of all realizations of s_2. Therefore, it could strengthen the confirmation bias of the DM.

8 Applications

In this section, I provide two simple examples, which relates to polarization and media competition in the presence of information overload. For simplicity, I only look at the case where $u_L = u_R = 1$, $T = 2$ and $\bar{T} = 1$. Moreover, as in the illustrative example, I assume s_1 and s_2 follow a symmetric information structure with three possible realizations, denoted as ${}^1\!/_q, 1, q$ where $q > 1$. The information structure is represented in the following table:

	$f_{t\omega}({}^1\!/_q)$	$f_{t\omega}(1)$	$f_{t\omega}(q)$
$\omega = L$	$\lambda(1+q)^{-1}$	$1-\lambda$	$\lambda q(1+q)^{-1}$
$\omega = R$	$\lambda q(1+q)^{-1}$	$1-\lambda$	$\lambda(1+q)^{-1}$

Table 4: The distribution of s_t, $t = 1, 2$, given the state of the world.

An increase in λ represents a better access to valuable information while q represents the strength/quality of the informative signals. In the illustrative example, $\lambda = 0.3$ and $q = 2.5$. Throughout the section, I focus on the unaware case and assume g is degenerate.

8.1 Polarization

Many empirical studies have documented the phenomenon of political polarization or stronger partisanship in the US in recent years. Bartels (1998) and Bartels (2000) show that party identification has become a better predictor of vote decisions and document a decline in volatility of election outcomes. Moreover, the polarization is stronger among citizens who are more politically engaged and partisan. (See Evans (2003), Baldassarri and Gelman (2008), Abramowitz and Saunders (2008) and Hetherington (2009)). In contrast, Gentzkow and Shapiro (2011) and Flaxman et al. (2016) show that online media exposes individuals to belief-challenging information, although somewhat counter-intuitively, Flaxman et al. (2016) also find that online media are associated to an increase in ideological distance between individuals.

In the following, I show how the model of limited cognitive ability presented in this paper sheds light on the wide range of phenomena documented in the literature of political polarization. Alice and Bob have opposite prior beliefs about the state of the world with obvious notations $p_L^a = p_R^b = p > 1/2$. I assume that the informative signals are strong enough, *i.e.*, $q > \frac{p}{1-p}$, which rules out the trivial case where the two individuals always take their a priori optimal action. Without loss of generality, I assume that L is the true state.

I analyze three indicators of polarization:

1. the probability that both individuals take the same action, which is denoted as $P_{concensus}$. It is used to asses the intuition where more/better information results in a higher probability of achieving consensus;

2. the probability that the individual takes his/her a priori optimal action, which is denoted as $P^j_{default}$, $j = A, B$. It measures how well the a priori optimal action, or analogously party identification, predicts voting decision, which is studied in the empirical literature of political science;

3. the change in the distance between the beliefs of the two individuals after receiving information. It corresponds to belief polarization and is widely analyzed in the theoretical economics literature[30].

[30]For example, see Baldassarri and Gelman (2008), Acemoglu et al. (2007).

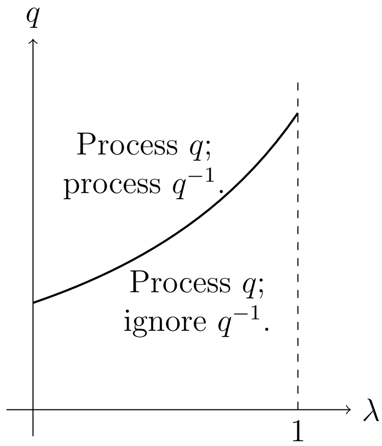


Figure 4: Alice's processing strategy in period 1 as a function of λ and q, fixing $p^a_L = p = 0.7$.

Using proposition 2, the processing strategy of the two individuals are characterized as follows:

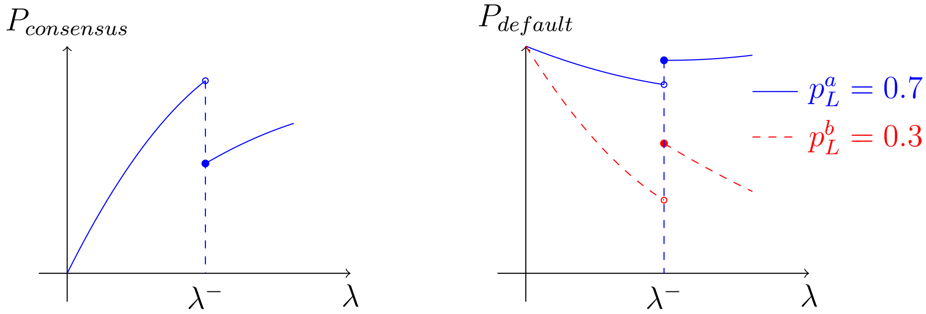
Corollary 5. *In period 1, both Alice and Bob process s_1 if it is belief-confirming and ignore it if it is pure noise. There exists some thresholds q^-, λ^- and p^- such that they process belief-challenging signal if and only if*

- *their prior is weak enough, i.e., $p < p^-$, or;*
- *the informative signals are strong enough, i.e., $q > q^-$, or;*
- *the access to information is poor enough, i.e., $\lambda < \lambda^-$.*

The proof is shown in the appendix and the result is illustrated in figure 4. The first two points correspond to the result of "preference for strong signals" and "confirmation bias for confident individuals", while the third point corresponds to proposition 4 that better access to information strengthens confirmation bias. The following proposition, illustrated in figure 5, presents how a change in the access to information λ affects the first two indicators of polarization, $P_{consensus}$ and $P^j_{default}$.

Proposition 9. *The probability that Alice and Bob take the same action, $P_{consensus}$, is non-monotonic in λ, i.e., it is increasing in the range $[0, \lambda^-)$ and $[\lambda^-, 1]$, but exhibits a downward jump at $\lambda = \lambda^-$.*

Similarly, the probability that Alice/Bob takes her/his a priori optimal action, $P^a_{default}$ or $P^b_{default}$, is also non-monotonic in λ, i.e., both are decreasing in λ in the range $[0, \lambda^-)$, but exhibit a upward jump at $\lambda = \lambda^-$.



(a) $P_{consensus}$ with $p = 0.7$ and $\Phi = 0.7$. (b) $P_{default}$ with $\Phi = 0.7$

Figure 5: The effect of λ on the first two indicators of polarization.

When there is better access to information, fixing the processing strategies of the two individuals, the probability that they takes the optimal action l increases. However, as shown in proposition 4 and corollary 5, a better access to information promotes biased processing behavior, *i.e.,* when λ is big enough, the two individuals ignore belief-challenging information. As a result, it could reduce the probability of achieving consensus and increase the probability that the individuals take their a priori optimal action, despite the availability of more valuable public information. The limitation in processing ability hinders the benefits of information technology because individuals strategically allocate their cognitive resources in the presence of information overload.

Moreover, as individuals with different prior beliefs adopt different processing strategies, their beliefs can be polarized even if they receive the same sequence of information.

Corollary 6. *When $\lambda \geq \lambda^-$, $q \leq q^-$ or $p \geq p^-$, the distance between the beliefs of Alice and Bob increases after receiving information, if the signals in the two periods support different states. More specifically, if $s_1 > 1 > s_2$ or $s_2 > 1 > s_1$, Alice becomes more confident that state L is true while Bob becomes more confident that state R is true.*

Moreover, the probability that $s_1 > 1 > s_2$ or $s_2 > 1 > s_1$ increases in λ and decreases in q.

Proof. Without loss of generality, I analyze the case where $s_1 = q$ and $s_2 = {}^1\!/_q$. When $\lambda \geq \lambda^-$, $q \leq q^-$ or $p \geq p^-$, Alice and Bob process belief-confirming information but ignore belief-challenging information in period 1. Therefore, Alice processes s_1 while Bob ignores s_1 and processes s_2. Their beliefs in period 3 are:

$$\Pr_L^a(\mathscr{M}_3^a) = \Pr_L^a(q) = \left(1 + \frac{1-p}{pq}\right)^{-1} > p_L^a \qquad ;$$

$$\Pr_L^b(\mathscr{M}_3^b) = \Pr_L^b(q^{-1}) = \left(1 + \frac{p}{(1-p)q^{-1}}\right)^{-1} < p_L^b.$$

On the other hand, the probability that $s_1 > 1 > s_2$ or $s_2 > 1 > s_1$ equals

$$2\lambda^2 q(1+q)^{-2},$$

which increases in λ and decreases in q. □

Corollary 6 shows that even when the same sequence of information is available for both individuals, the difference in their prior beliefs induces different processing strategies and could polarize their beliefs. Moreover, it happens only when there are sufficiently good access to information, or when the prior beliefs of the two individuals are strong enough. In other words, when the two individuals are partisan enough, a better access to information gives rise to the possibility of polarization even under a setting with public information. This result sheds light on the empirical evidence that polarization is much stronger among individuals who are more partisan.

On the other hand, belief polarization happens when the information available in the two periods are contradictory, *i.e.,* $s_1 > 1 > s_2$ or $s_2 > 1 > s_1$. Its probability increases when there is better access to information and when the quality of information decreases. Arguably, the Internet contributes to both. While it provides us with enormous amount of information, it also facilitates the spread of rumors, fake news and low-quality information. It is much easier to find contradictory information on the Internet, which gives more incentive for individuals to selectively attend to belief-confirming information and increases the probability of belief polarization.

8.2 Media Competition

In this second application, I study media strategy in the information era. In order to introduce the role of the media, I present a variation of the main model.

Formally, there is a continuum of media, indexed by $i \in \mathscr{I}$. In period $t = 1, 2$, each media i collects a signal s_{it} about the state of the world and publishes a piece of news m_{it}. The signal s_{it} collected by different media are independent and follow the distribution defined in the beginning of the section, *i.e.,* table 4. I assume that the media cannot post fake news and therefore either publishes the signal it receives or publishes nothing, *i.e.,* $m_{it} \in \{\emptyset, s_{it}\}$. To simplify the analysis, I assume that there are three types of non-strategic (biased) media, $\{T_L, T_R, T_N\}$, which publish according to the following fixed rules:

- media $\{i \mid i \in T_L\}$ is biased towards state L: it publishes s_{it} if it supports state L ($s_{it} = q$), but publishes $\emptyset$ if it receives $s_{it} \in \{{}^1\!/\!_q, 1\}$;
- media $\{i \mid i \in T_R\}$ has an opposite bias: it publishes a signal if it supports state R ($s_{it} = {}^1\!/\!_q$) but publishes $\emptyset$ if it receives $s_i \in \{1, q\}$;
- media $\{i \mid i \in T_N\}$ has no bias and publishes any informative signal it receives, *i.e.,* $m_{it} = s_i$ if and only if $s_i \neq 1$.

I assume that each media belongs to one and only one of the three types, and the type of each media is a public information. The time line of the game is as follows:

Period 1 The DM chooses which media he visits and processes the piece of news m_{i1} posted by the media.

Period 2 If the media visited by the DM in period 1 published nothing, he chooses again a media outlet to visit. Otherwise, he visits no media as processing the information in period 1 takes time.

Period 3 The DM forms his belief with his memory of information and takes an action l or r.

Note that this variation differs from the main model only in terms of interpretation. Here the DM control his "diet" of information by choosing which (biased or unbiased) media to visit, instead of choosing whether to process or ignore the signals he receives. For example, if the DM visits media $i \in T_L$ in period 1, it is as if he chooses to process the period 1 signal if and only if it supports state L. By corollary 5, the DM visits the biased media in period 1 if his prior belief is strong enough.

Corollary 7. *Suppose the DM has prior belief* $(p_L, 1 - p_L)$ *where* $p_L \geq 1/2$. *In period 1, there exists a threshold* $p^- \in (1/2, 1)$ *such that if* $p_L \geq p^-$, *he visits a media* i *where* $i \in T_L$; *if* $p_L \in (1/2, p^-)$, *he visits a media* i *where* $i \in T_N$.

If the media visited by the DM in period 1 published nothing, he visits a media i *where* $i \in T_N$ *in period 2.*

Now I turn to analyze the viewership of the three types of media. In the society, individuals' prior belief p_L are distributed according to $g(p)$ where $g(p) > 0$ for all $p \in (0, 1)$. Its c.d.f. is denoted by G. Define the viewership of media $\{i \mid i \in T_j\}$ as the mass of individuals which visit media $\{i \mid i \in T_j\}$ across the two periods. Denote it by V_j for $j = L, R, N$. By corollary 7,

$$V_L = 1 - G(p^-);$$
$$V_R = G(1 - p^-);$$
$$V_N = 2 - V_L - V_R - \lambda.$$

That is, biased media attracts views from individuals with strong beliefs, while unbiased media serves the others.

Corollary 8. *When there are better access to information (λ increases), the viewership of the biased media increases while the viewership of unbiased media decreases, i.e., V_L and V_R increase with λ while V_N decreases in λ.*

On the other hand, when the quality of information increases (q increases), the viewership of the biased media decreases while the viewership of unbiased media increases, i.e. V_L and V_R decrease with q while V_N increases in q.

Proof. The result follows from corollary 5. □

As shown in previous sections, a better access to information or an decrease in quality of information strengthens the confirmation bias of individuals, which in turn increases the viewership of the biased media. The increase in viewership increases the profitability of biased media and thus incentivize media to adopt a biased strategy. This result sheds light on the emergence of partisan media in recent years as the Internet promotes biased processing behavior.

9 Conclusion

In conclusion, this paper investigates the information processing behavior of a decision maker who can process only a subset of all available signals. I show that this limitation in processing ability drives a number of well-documented behavioral "biases", including preference of strong signals, confirmation bias for confident individuals and wishful thinking.

These "biases" has been attracting lots of attention in the behavioral economics literature, in which many have analyzed how these "biases" affects different market outcomes by introducing exogenous "biases" in traditional economics models. In contrast, instead of taking the "biases" as they are, this paper aims to improve our understanding by analyzing their cause. In particular, I show that these "biases" are features of optimal strategies if we take into account our limited cognitive ability as a human being.

This approach allows us to analyze the "biases" as an outcome of an optimization problem. It brings two advantages. First, not only that it explains the existence of the "biases", but using standard techniques of comparative statistics, it also explains the heterogeneity of the "biases" among individuals with different personal characteristics or abilities, and how they change in different situations faced by the individuals. Second, it allows us to study how regulatory policies could play a role in changing these "biases" and associated market outcomes.

Thus, looking forward and building from the insights of this paper, there are two different ways to further develop the literature. First, in policy analysis with behavioral settings, modeling the "biases" as optimal strategies allows us to take into account the indirect effects of policy interventions on behaviors of individuals. It contributes to a more complete analysis than if we take the "biases" as they are. For example (loosely speaking), if providing more information to consumers strengthens their confirming bias, it might back fire as it could weaken competition and increase prices. Second, more experimental or empirical work has to be done to understand how "biases" are formed and vary across different individuals or settings. It will give us a better understanding on whether the "biases" do indeed relate to the limitation in ability. And as an (early) answer to that question, in an experimental study conducted by myself and co-authors, Goette et al. (2018), we find evidences that a larger cognitive load strengthens confirmation bias in belief formation.

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A Optimality of the processing strategy

Before I present the omitted proofs in the main text, I now discuss the optimality of the equilibrium processing strategy. In the main text, the DM decides whether to process or ignore the signal given the preliminary information about its realization. Therefore, the equilibrium processing strategy $\sigma_t(g, \mathscr{M}_t)$ is optimal all any preliminary information g. Moreover, as it maximizes the DM's expected utility for all possible preliminary information in each period, it also maximizes his expected ex-ante utility in every period t, which gives the following result.

Proposition A.1. *The processing strategy characterized by lemma 1 maximizes the expected utility of the DM's period t self.*

In the main text, I show that the optimal processing strategy explains some well-documented behavioral phenomena in the presence of limitation in processing ability. The optimality is achieved as the DM evaluates the expected utility of processing and ignoring a signal given his knowledge of the signal realization. In contrast, proposition A.1 suggests that the results also hold if we consider strategies that maximize the DM's expected utility at the beginning of each period. This result is also useful for the proof for the equivalence result, as shown in the next section.

B Equivalence Results

In this section, I present two equivalence results which allow me to simplify the model with $T > 2$ and $\bar{T} = 1$ to a model with $T = 2$ and $\bar{T} = 1$. The idea is to show that the equilibrium processing strategy at any period $t < T$ in a model with $T > 2$ is equivalent to the equilibrium processing strategy at period 1 in the simplified model with $T = 2$. I first present the result in the unaware case.

B.1 Unaware Case

Proposition B.1. *There exists some p.d.f. $(\tilde{f}_{2L}, \tilde{f}_{2R})$ such that the following two equilibrium strategies are equivalent:*

1. *the equilibrium processing strategy of the DM with belief (p_L^t, p_R^t) at period $t < T$, under a setting with $T > 2$, $\bar{T} = 1$ and information structure $\{(f_{t'L}, f_{t'R})\}_{t'=t+1}^{T}$;*

2. *the equilibrium processing strategy of the DM with prior belief* $(p_L, p_R) = (p^t_L, p^t_R)$ *at period* 1*, under a setting with* $T = 2$*,* $\bar{T} = 1$ *and information structure* $(\tilde{f}_{2L}, \tilde{f}_{2R})$.

Proof. I prove the proposition under the assumption the preliminary information perfectly reveals the signal realization. The proof is similar in the general setting where g could be degenerate[31]. Note that with $\bar{T} = 1$, the memory of the DM is always $\emptyset$ before processing a signal. Thus, I simply use $\sigma_t(s_t)$ to denote the processing strategy of the DM at period t. First consider the first point where $T > 2$ and without loss of generality assume $u_L\, p^t_L \geq u_R\, p^t_R$. By lemma 1, the decision maker processes signal $s_t \geq \frac{u_R\, p^t_R}{u_L\, p^t_L}$ if and only if

$$s_t \geq \frac{u_R\, p^t_R}{u_L\, p^t_L} \frac{\triangle^t_{s_t,\mathscr{M}_t}(r \mid R)}{\triangle^t_{s_t,\mathscr{M}_t}(r \mid L)}$$
$$= \frac{u_R\, p^t_R}{u_L\, p^t_L} \frac{\sum_{k=t+1}^{T} \left[\mathbb{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hR}(s')ds' + \mathbb{1}_{k=t+1}\right] \int_0^{\frac{u_R\, p^t_R}{u_L\, p^t_L}} \mathbb{1}_{\sigma_k(s)=P} f_{kR}(s)\, ds}{\sum_{k=t+1}^{T} \left[\mathbb{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hL}(s')ds' + \mathbb{1}_{k=t+1}\right] \int_0^{\frac{u_R\, p^t_R}{u_L\, p^t_L}} \mathbb{1}_{\sigma_k(s)=P} f_{kL}(s)\, ds}. \tag{B.1}$$

On the other hand, he processes signal $s_t < \frac{u_R\, p^t_R}{u_L\, p^t_L}$ if and only if

$$s_t^{-1} > \frac{u_L\, p^t_L}{u_R\, p^t_R} \frac{\triangle^t_{s_t,\mathscr{M}_t}(l \mid L)}{\triangle^t_{s_t,\mathscr{M}_t}(l \mid R)}$$
$$= \frac{u_L\, p^t_L}{u_R\, p^t_R} \frac{1 - \sum_{k=t+1}^{T} \left[\mathbb{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma^*_h(s')=I}) f_{hL}(s')ds' + \mathbb{1}_{k=t+1}\right] \int_0^{\frac{u_R\, p^t_R}{u_L\, p^t_L}} \mathbb{1}_{\sigma^*_k(s)=P} f_{kL}(s)\, ds}{1 - \sum_{k=t+1}^{T} \left[\mathbb{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma^*_h(s')=I}) f_{hR}(s')ds' + \mathbb{1}_{k=t+1}\right] \int_0^{\frac{u_R\, p^t_R}{u_L\, p^t_L}} \mathbb{1}_{\sigma^*_k(s)=P} f_{kR}(s)\, ds}. \tag{B.2}$$

Now consider point 2 where $T = 2$ with information structure $(\tilde{f}_{2L}, \tilde{f}_{2R})$ and prior belief $p_L = p^t_L$. By lemma 1, the decision maker processes signal $s_1 \geq \frac{u_R\, p_R}{u_L\, p_L}$ if

$$s_t \geq \frac{u_R\, p_R}{u_L\, p_L} \frac{\tilde{F}_{2R}\left(\dfrac{u_R\, p_R}{u_L\, p_L}\right)}{\tilde{F}_{2L}\left(\dfrac{u_R\, p_R}{u_L\, p_L}\right)}, \tag{B.3}$$

[31] The proof is identical if I replace $\mathbb{1}_{\sigma_h(s)=P}$ by $\int \mathbb{1}_{\sigma_h(s)=P} g(s)\, dg$ and $\mathbb{1}_{\sigma_h(s)=I}$ by $\int \mathbb{1}_{\sigma_h(s)=I} g(s)\, dg$.

and he processes signal $s_1 < \frac{u_R\, p_R}{u_L\, p_L}$ if and only if

$$s_t^{-1} > \frac{u_L\, p_L}{u_R\, p_R} \frac{1 - \tilde{F}_{2L}\left(\frac{u_R\, p_R}{u_L\, p_L}\right)}{1 - \tilde{F}_{2R}\left(\frac{u_R\, p_R}{u_L\, p_L}\right)}. \tag{B.4}$$

By comparing equation (B.1) to equation (B.3), and equation (B.2) to equation (B.4), the equilibrium processing strategy in the two cases are equivalent if and only if

$$\begin{aligned}
\tilde{F}_{2L}\left(\frac{u_R\, p_R}{u_L\, p_L}\right) &= \sum_{k=t+1}^{T} \left[\mathbb{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^{\infty} (\mathbb{1}_{\sigma_h^*(s')=I}) f_{hL} ds' + \mathbb{1}_{k=t+1}\right] \int_0^{\frac{u_R\, p_R^t}{u_L\, p_L^t}} \mathbb{1}_{\sigma_k^*(s)=P} f_{kL}(s) ds; \\
\tilde{F}_{2R}\left(\frac{u_R\, p_R}{u_L\, p_L}\right) &= \sum_{k=t+1}^{T} \left[\mathbb{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^{\infty} (\mathbb{1}_{\sigma_h^*(s')=I}) f_{hR} ds' + \mathbb{1}_{k=t+1}\right] \int_0^{\frac{u_R\, p_R^t}{u_L\, p_L^t}} \mathbb{1}_{\sigma_k^*(s)=P} f_{kR}(s) ds.
\end{aligned} \tag{B.5}$$

To prove that there exists p.d.f. $\tilde{f}_{2L}$ and $\tilde{f}_{2R}$ that generates the c.d.f.s evaluated at $(\frac{u_R\, p_R}{u_L\, p_L})$ with values defined in equation (B.5), it remains to prove that

$$\frac{\tilde{F}_{2R}\left(\frac{u_R\, p_R}{u_L\, p_L}\right)}{\tilde{F}_{2L}\left(\frac{u_R\, p_R}{u_L\, p_L}\right)} > \frac{u_L\, p_L}{u_R\, p_R}.$$

Note that by definition, equation (B.5) implies that

$$\frac{\tilde{F}_{2R}\left(\frac{u_R\, p_R}{u_L\, p_L}\right)}{\tilde{F}_{2L}\left(\frac{u_R\, p_R}{u_L\, p_L}\right)} = \frac{\Pr^{t+1}_{\mathscr{M}_{t+1}}(r \mid R)}{\Pr^{t+1}_{\mathscr{M}_{t+1}}(r \mid L)}.$$

On the other hand, by proposition A.1, the expected utility of the DM at the beginning of period $t+1$ is weakly greater than that if he chooses to process all s_t:

$$\begin{aligned}
&u_L\, p_L(1 - \Pr^{t+1}_{\mathscr{M}_{t+1}}(r \mid L)) + u_R\, p_R \Pr^{t+1}_{\mathscr{M}_{t+1}}(r \mid R) \\
\geq &u_L\, p_L\left(1 - F_{(t+1)L}\left(\frac{u_R p_R}{u_L p_L}\right)\right) + u_R\, p_R F_{(t+1)R}\left(\frac{u_R p_R}{u_L p_L}\right) \\
> &u_L\, p_L,
\end{aligned}$$

where the last inequality is implied by the fact receiving one signal always improves expected utility, in comparison to receiving no signal. Rearranging gives:

$$\begin{aligned} u_R\, p_R \mathrm{Pr}^{t+1}_{\mathscr{M}_{t+1}}(r \mid R) &> u_L\, p_L \mathrm{Pr}^{t+1}_{\mathscr{M}_{t+1}}(r \mid L); \\ \frac{\mathrm{Pr}^{t+1}_{\mathscr{M}_{t+1}}(r \mid R)}{\mathrm{Pr}^{t+1}_{\mathscr{M}_{t+1}}(r \mid L)} &> \frac{u_L\, p_L}{u_R\, p_R}. \end{aligned}$$

The results follow. □

B.2 Aware Case

Proposition B.2. *There exists some p.d.f.* $(\tilde{f}_{1L}, \tilde{f}_{1R})$, $(\tilde{f}_{2L}, \tilde{f}_{2R})$ *such that the following two sets of equilibrium strategies are equivalent:*

1. *the set of equilibrium processing strategies of the DM with belief* (p^t_L, p^t_R) *at period* $t < T$, *under a setting with* $T > 2$, $\bar{T} = 1$ *and information structure* $\{(f_{t'L}, f_{t'R})\}^T_{t'=t}$ *(assuming that it exists);*
2. *the set of equilibrium processing strategies of the DM with prior belief* $(p_L, p_R) = (p^t_L, p^t_R)$ *at period* 1, *under a setting with* $T = 2$, $\bar{T} = 1$ *and information structure* $(\tilde{f}_{1L}, \tilde{f}_{1R})$,$(\tilde{f}_{2L}, \tilde{f}_{2R})$.

Proof. First consider point 1 where $T > 2$ and without loss of generality assume $u_L\, p^t_L \geq u_R\, p^t_R$. For simplicity and with a bit of abuse in notations, define $\mathscr{R}^*_{t'} = \prod_{t=1}^{t'-1} \mathscr{R}_t(\tilde{\sigma}_t)$, which is the conjectured ratio of ignoring all the previous signals in state L over than in state R. Note that at the beginning of period $t' > t$, the expected utility of action l over that of action r equals $\frac{u_L\, p^t_L \mathscr{R}^*_{t'}}{u_R\, p^t_R}$. By lemma 1, the decision maker processes signal $s_t \geq \frac{u_R\, p^t_R}{u_L\, p^t_L}$ if and only if

$$\begin{aligned} s_t &\geq \frac{u_R\, p^t_R}{u_L\, p^t_L} \frac{\triangle^t_{s_t, \mathscr{M}_t}(r \mid R)}{\triangle^t_{s_t, \mathscr{M}_t}(r \mid L)} \\ &= \frac{u_R\, p^t_R}{u_L\, p^t_L} \frac{\sum_{k=t+1}^T \left[\mathbb{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hR}(s') ds' + \mathbb{1}_{k=t+1} \right] \int_0^{\frac{u_R\, p^t_R}{u_L\, p^t_L \mathscr{R}^*_k}} \mathbb{1}_{\sigma_k(s)=P} f_{kR}(s)\, ds}{\sum_{k=t+1}^T \left[\mathbb{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hL}(s') ds' + \mathbb{1}_{k=t+1} \right] \int_0^{\frac{u_R\, p^t_R}{u_L\, p^t_L \mathscr{R}^*_k}} \mathbb{1}_{\sigma_k(s)=P} f_{kL}(s)\, ds}. \end{aligned} \tag{B.6}$$

On the other hand, he processes signal $s_t < \frac{u_R\, p_R^t}{u_L\, p_L^t}$ if and only if

$$
\begin{aligned}
& s_t^{-1} > \frac{u_L\, p_L^t}{u_R\, p_R^t} \frac{\triangle^t_{s_t,\mathscr{M}_t}(l \mid L)}{\triangle^t_{s_t,\mathscr{M}_t}(l \mid R)} \\
= & \frac{u_L\, p_L^t}{u_R\, p_R^t} \frac{1-\sum_{k=t+1}^{T}\left[\mathbb{1}_{k>t+1}\prod_{h=t+1}^{k-1}\int_0^\infty(\mathbb{1}_{\sigma_h(s')=I})f_{hL}(s')ds'+\mathbb{1}_{k=t+1}\right]\int_0^{\frac{u_R\, p_R^t}{u_L\, p_L^t \mathscr{R}_k^*}}\mathbb{1}_{\sigma_k(s)=P}f_{kL}(s)\,ds}{1-\sum_{k=t+1}^{T}\left[\mathbb{1}_{k>t+1}\prod_{h=t+1}^{k-1}\int_0^\infty(\mathbb{1}_{\sigma_h(s')=I})f_{hR}(s')ds'+\mathbb{1}_{k=t+1}\right]\int_0^{\frac{u_R\, p_R^t}{u_L\, p_L^t \mathscr{R}_k^*}}\mathbb{1}_{\sigma_k(s)=P}f_{kR}(s)\,ds}.
\end{aligned} \tag{B.7}
$$

The equilibrium strategy is given by the equation (B.6) and (B.7). Now define $\tilde{s} = s\frac{\mathscr{R}_k^*}{\mathscr{R}_{t+1}^*}$, note that

$$
\begin{aligned}
& \frac{\sum_{k=t+1}^{T}\left[\prod_{h=t+1}^{k-1}\int_0^\infty(\mathbb{1}_{\sigma_h(s')=I})f_{hR}(s')ds'\right]\int_0^{\frac{u_R p_R^t}{u_L p_L^t \mathscr{R}_k^*}}\mathbb{1}_{\sigma_k(s)=P}f_{kR}(s)ds}{\sum_{k=t+1}^{T}\left[\prod_{h=t+1}^{k-1}\int_0^\infty(\mathbb{1}_{\sigma_h(s')=I})f_{hL}(s')ds'\right]\int_0^{\frac{u_R p_R^t}{u_L p_L^t \mathscr{R}_k^*}}\mathbb{1}_{\sigma_k(s)=P}f_{kL}(s)ds} \\
= & \frac{\sum_{k=t+1}^{T}\left[\prod_{h=t+1}^{k-1}\int_0^\infty(\mathbb{1}_{\sigma_h(s')=I})f_{hR}(s')ds'\right]\int_0^{\frac{u_R p_R^t}{u_L p_L^t \mathscr{R}_{t+1}^*}}\mathbb{1}_{\sigma_k(\tilde{s}/\mathscr{R}_k^*)=P}f_{kR}(\tilde{s}/\mathscr{R}_k^*)d\tilde{s}}{\sum_{k=t+1}^{T}\left[\prod_{h=t+1}^{k-1}\int_0^\infty(\mathbb{1}_{\sigma_h(s')=I})f_{hL}(s')ds'\right]\int_0^{\frac{u_R p_R^t}{u_L p_L^t \mathscr{R}_{t+1}^*}}\mathbb{1}_{\sigma_k(\tilde{s}/\mathscr{R}_k^*)=P}f_{kL}(\tilde{s}/\mathscr{R}_k^*)d\tilde{s}} \\
= & \frac{\int_0^{\frac{u_R\, p_R^t}{u_L\, p_L^t \mathscr{R}_{t+1}^*}}\sum_{k=t+1}^{T}\left[\prod_{h=t+1}^{k-1}\int_0^\infty(\mathbb{1}_{\sigma_h(s')=I})f_{hR}(s')ds'\right]\mathbb{1}_{\sigma_k(\tilde{s}/\mathscr{R}_k^*)=P}f_{kR}(\tilde{s}/\mathscr{R}_k^*)d\tilde{s}}{\int_0^{\frac{u_R\, p_R^t}{u_L\, p_L^t \mathscr{R}_{t+1}^*}}\sum_{k=t+1}^{T}\left[\prod_{h=t+1}^{k-1}\int_0^\infty(\mathbb{1}_{\sigma_h(s')=I})f_{hL}(s')ds'\right]\mathbb{1}_{\sigma_k(\tilde{s}/\mathscr{R}_k^*)=P}f_{kL}(\tilde{s}/\mathscr{R}_k^*)d\tilde{s}}.
\end{aligned}
$$

Now I define $(\tilde{f}_{2L}, \tilde{f}_{2R})$ as follows and verify whether it is a probability distribution function and whether there exists $(\tilde{f}_{1L}, \tilde{f}_{1R})$ such that the equilibrium processing strategy of the DM in period $t = 1$ where $T = 2$ is equivalent to that characterized in equation (B.6) and (B.7):

$$
\begin{aligned}
\tilde{f}_{2L}(\tilde{s}) &= \sum_{k=t+1}^{T}\left[\prod_{h=t+1}^{k-1}\int_0^\infty(\mathbb{1}_{\sigma_h(s')=I})f_{hL}(s')ds'\right]\mathbb{1}_{\sigma_k(\tilde{s}/\mathscr{R}_k^*)=P}f_{kL}(\tilde{s}/\mathscr{R}_k^*); \\
\tilde{f}_{2R}(\tilde{s}) &= \sum_{k=t+1}^{T}\left[\prod_{h=t+1}^{k-1}\int_0^\infty(\mathbb{1}_{\sigma_h(s')=I})f_{hR}(s')ds'\right]\mathbb{1}_{\sigma_k(\tilde{s}/\mathscr{R}_k^*)=P}f_{kR}(\tilde{s}/\mathscr{R}_k^*).
\end{aligned}
$$

First, it is clear that $\int \tilde{f}_{2L}(\tilde{s})d\tilde{s} = \int \tilde{f}_{2R}(\tilde{s})d\tilde{s} = 1$ as the DM always processes one signal in equilibrium. On the other hand,

$$\begin{aligned}
\frac{\tilde{f}_{2L}(\tilde{s})}{\tilde{f}_{2R}(\tilde{s})} &= \frac{\sum_{k=t+1}^{T}\left[\prod_{h=t+1}^{k-1}\int_0^\infty(\mathbb{1}_{\sigma_h(s')=I})f_{hL}(s')ds'\right]\mathbb{1}_{\sigma_k(\tilde{s}/\mathscr{R}_k^*)=P}f_{kL}(\tilde{s}/\mathscr{R}_k^*)}{\sum_{k=t+1}^{T}\left[\prod_{h=t+1}^{k-1}\int_0^\infty(\mathbb{1}_{\sigma_h(s')=I})f_{hR}(s')ds'\right]\mathbb{1}_{\sigma_k(\tilde{s}/\mathscr{R}_k^*)=P}f_{kR}(\tilde{s}/\mathscr{R}_k^*)} \\
&= \frac{\sum_{k=t+1}^{T}\mathscr{R}_k^*\left[\prod_{h=t+1}^{k-1}\int_0^\infty(\mathbb{1}_{\sigma_h(s')=I})f_{hR}(s')ds'\right]\mathbb{1}_{\sigma_k(\tilde{s}/\mathscr{R}_k^*)=P}f_{kL}(\tilde{s}/\mathscr{R}_k^*)}{\sum_{k=t+1}^{T}\left[\prod_{h=t+1}^{k-1}\int_0^\infty(\mathbb{1}_{\sigma_h(s')=I})f_{hR}(s')ds'\right]\mathbb{1}_{\sigma_k(\tilde{s}/\mathscr{R}_k^*)=P}f_{kR}(\tilde{s}/\mathscr{R}_k^*)} \\
&= \frac{\sum_{k=t+1}^{T}\tilde{s}\left[\prod_{h=t+1}^{k-1}\int_0^\infty(\mathbb{1}_{\sigma_h(s')=I})f_{hR}(s')ds'\right]\mathbb{1}_{\sigma_k(\tilde{s}/\mathscr{R}_k^*)=P}f_{kR}(\tilde{s}/\mathscr{R}_k^*)}{\sum_{k=t+1}^{T}\left[\prod_{h=t+1}^{k-1}\int_0^\infty(\mathbb{1}_{\sigma_h(s')=I})f_{hR}(s')ds'\right]\mathbb{1}_{\sigma_k(\tilde{s}/\mathscr{R}_k^*)=P}f_{kR}(\tilde{s}/\mathscr{R}_k^*)} \\
&= \tilde{s}.
\end{aligned}$$

Denote $(\tilde{F}_{2L}, \tilde{F}_{2R})$ as the c.d.f. associated with $(\tilde{f}_{2L}, \tilde{f}_{2R})$. In point 2 where $T = 2$, there exists $(\tilde{f}_{1L}, \tilde{f}_{1R})$ such that the DM processes signal $s_1 \geq \frac{u_R\, p_R}{u_L\, p_L}$ if and only if

$$s_1 \geq \frac{u_R\, p_R}{u_L\, p_L}\frac{\tilde{F}_{2R}\left(\frac{u_R\, p_R}{u_L\, p_L\mathscr{R}_1^*}\right)}{\tilde{F}_{2L}\left(\frac{u_R\, p_R}{u_L\, p_L\mathscr{R}_1^*}\right)},$$

and processes signal $s_t < \frac{u_R\, p_R}{u_L\, p_L}$ if and only if

$$s_1^{-1} \geq \frac{u_L\, p_L}{u_R\, p_R}\frac{1-\tilde{F}_{2L}\left(\frac{u_R\, p_R}{u_L\, p_L\mathscr{R}_1^*}\right)}{1-\tilde{F}_{2R}\left(\frac{u_R\, p_R}{u_L\, p_L\mathscr{R}_1^*}\right)}.$$

where $(\tilde{F}_{1L}, \tilde{F}_{1R})$ satisfy

$$\mathscr{R}_1^* = \frac{\tilde{F}_{1L}\left(\frac{u_R\, p_R}{u_L\, p_L}\frac{\tilde{F}_{2R}\left(\frac{u_R\, p_R}{u_L\, p_L\mathscr{R}_1^*}\right)}{\tilde{F}_{2L}\left(\frac{u_R\, p_R}{u_L\, p_L\mathscr{R}_1^*}\right)}\right) - \tilde{F}_{1L}\left(\frac{u_L\, p_L}{u_R\, p_R}\frac{1-\tilde{F}_{2L}\left(\frac{u_R\, p_R}{u_L\, p_L\mathscr{R}_1^*}\right)}{1-\tilde{F}_{2R}\left(\frac{u_R\, p_R}{u_L\, p_L\mathscr{R}_1^*}\right)}\right)}{\tilde{F}_{1R}\left(\frac{u_R\, p_R}{u_L\, p_L}\frac{\tilde{F}_{2R}\left(\frac{u_R\, p_R}{u_L\, p_L\mathscr{R}_1^*}\right)}{\tilde{F}_{2L}\left(\frac{u_R\, p_R}{u_L\, p_L\mathscr{R}_1^*}\right)}\right) - \tilde{F}_{1R}\left(\frac{u_L\, p_L}{u_R\, p_R}\frac{1-\tilde{F}_{2L}\left(\frac{u_R\, p_R}{u_L\, p_L\mathscr{R}_1^*}\right)}{1-\tilde{F}_{2R}\left(\frac{u_R\, p_R}{u_L\, p_L\mathscr{R}_1^*}\right)}\right)}.$$

The result follows. □

C Omitted Results and Proofs

C.1 Proof of Lemma 1

Proof. The lemma is implied by inequality (4) combined with

$$\triangle^t_{s_t,\mathscr{M}_t}(r \mid L) = -\triangle_{s_t,\mathscr{M}_t}(l \mid L);$$
$$\triangle^t_{s_t,\mathscr{M}_t}(l \mid R) = -\triangle_{s_t,\mathscr{M}_t}(r \mid R).$$

□

C.2 Proof of Proposition 1

Proof. I first consider the case where $\bar{T} \geq T$, and prove that the best response processing strategy is to process s_t for all $s_t \neq 1$ given that all his future selfs process all $s_{t'} \neq 1$ for $t' = t+1, \cdots T$. Denote the c.d.f. of $s_{t+1}s_{t+2}\cdots s_T$ in state $\omega = L, R$ as $F_{(t+1\to T)\omega}$, *i.e.*,

$$F_{(t+1\to T)\omega}(s) = \int_0^\infty \cdots \int_0^\infty F_{T\omega}\left(\frac{s}{s_{t+1}\cdots s_{T-1}}\right)\left(\prod_{t'=t+1}^{T-1} f_{t'\omega}(s_{t'})\right) ds_{t+1}\cdots ds_{T-1}.$$

Notice that

$$\begin{aligned} f_{(t+1\to T)L}(s) &= \int_0^\infty \cdots \int_0^\infty f_{TL}\left(\frac{s}{s_{t+1}\cdots s_{T-1}}\right)\left(\prod_{t'=t+1}^{T-1} \frac{f_{t'L}(s_{t'})}{s_{t'}}\right) ds_{t+1}\cdots ds_{T-1} \\ &= s\int_0^\infty \cdots \int_0^\infty \frac{f_{TR}\left(\frac{s}{s_{t+1}\cdots s_{T-1}}\right)}{s_{t+1}\cdots s_{T-1}}\left(\prod_{t'=t+1}^{T-1} f_{t'R}(s_{t'})\right) ds_{t+1}\cdots ds_{T-1} \\ &= sf_{(t+1\to T)R}(s). \end{aligned}$$

Thus, I can treat the information conveyed by the future signals $(s_{t'})_{t'=t+1}^T$ as one single signal with c.d.f. $F_{(t+1\to T)\omega}$ and p.d.f. $f_{(t+1\to T)\omega}$ defined above.

Now consider the case where $s_t > 1$, if the DM processes the signal, he updates his belief and will choose action r if and only if $s_{t+1}\cdots s_T < \frac{u_R p_R}{u_L p_L s_t}$; otherwise, he chooses action r if and only if $s_{t+1}\cdots s_T < \frac{u_R p_R}{u_L p_L}$. Thus,

$$\triangle^t_{s_t,\mathscr{M}_t}(l \mid L) = F_{(t+1\to T)L}\left(\frac{u_R p_R}{u_L p_L}\right) - F_{(t+1\to T)L}\left(\frac{u_R p_R}{u_L p_L s_t}\right) > 0$$
$$\triangle^t_{s_t,\mathscr{M}_t}(r \mid R) = F_{(t+1\to T)R}\left(\frac{u_R p_R}{u_L p_L s_t}\right) - F_{(t+1\to T)R}\left(\frac{u_R p_R}{u_L p_L}\right) < 0$$

By lemma 1, the DM processes $s_t > 1$ as

$$\begin{aligned}
&\frac{u_R p_R}{u_L p_L} \times \frac{F_{(t+1\to T)R}(\frac{u_R p_R}{u_L p_L s_t}) - F_{(t+1\to T)R}(\frac{u_R p_R}{u_L p_L})}{F_{(t+1\to T)L}(\frac{u_R p_R}{u_L p_L s_t}) - F_{(t+1\to T)L}(\frac{u_R p_R}{u_L p_L})} \\
=&\frac{u_R p_R}{u_L p_L} \times STR\left(s_{t+1}\cdots s_T \in [\frac{u_R p_R}{u_L p_L s_t}, \frac{u_R p_R}{u_L p_L}]\right) \\
<&\frac{u_R p_R}{u_L p_L} \times \frac{u_L p_L s_t}{u_R p_R} = s_t.
\end{aligned}$$

Similarly, the DM processes $s_t < 1$ as,

$$\begin{aligned}
&\frac{u_L p_L}{u_R p_R} \times \frac{\triangle^t_{s_t,\mathscr{M}_t}(l \mid L)}{\triangle^t_{s_t,\mathscr{M}_t}(l \mid R)} \\
=&\frac{u_L p_L}{u_R p_R} \times \frac{F_{(t+1\to T)L}(\frac{u_R p_R}{u_L p_L}) - F_{(t+1\to T)L}(\frac{u_R p_R}{u_L p_L s_t})}{F_{(t+1\to T)R}(\frac{u_R p_R}{u_L p_L}) - F_{(t+1\to T)R}(\frac{u_R p_R}{u_L p_L s_t})} \\
<&\frac{u_L p_L}{u_R p_R} \times \frac{u_R p_R}{u_L p_L s_t} = s_t^{-1}.
\end{aligned}$$

I the case where $\bar{T} < T$, proposition 2 implies that in the continuation game where $\bar{T} = 1$, the DM ignores signals in $[1-\epsilon, 1+\epsilon]$ with small enough ϵ. The result follows.

□

C.3 Proof of Proposition 2

Proof. By lemma 1, the decision maker processes $s_1 \geq \frac{u_R\, p_R}{u_L\, p_L}$ if and only if

$$s_1 \geq \frac{u_R\, p_R}{u_L\, p_L} \frac{F_{2R}\left(\frac{u_R\, p_R}{u_L\, p_L}\right)}{F_{2L}\left(\frac{u_R\, p_R}{u_L\, p_L}\right)} = \frac{u_R\, p_R}{u_L\, p_L} \times STR\left(s_2 \in \left(0, \frac{u_R\, p_R}{u_L\, p_L}\right)\right) > 1. \quad \text{(C.1)}$$

The last inequality follows from the fact that the strength of the set of signals $s_2 \in (0, \frac{u_R\, p_R}{u_L\, p_L})$ is higher than the strength of $s_2 = \frac{u_R\, p_R}{u_L\, p_L}$, *i.e.*,

$$\frac{F_{2R}\left(\frac{u_R\, p_R}{u_L\, p_L}\right)}{F_{2L}\left(\frac{u_R\, p_R}{u_L\, p_L}\right)} > \frac{f_{2R}\left(\frac{u_R\, p_R}{u_L\, p_L}\right)}{f_{2L}\left(\frac{u_R\, p_R}{u_L\, p_L}\right)} = \frac{u_L\, p_L}{u_R\, p_R}.$$

The last inequality of equation (C.1) also implies that the DM ignores all weak belief-challenging signals $s_1 \in [\frac{u_R\, p_R}{u_L\, p_L}, 1]$.

On the other hand, the DM processes $s_1 < \frac{u_R p_R}{u_L p_L}$ if and only if

$$s_1^{-1} > \frac{u_L p_L}{u_R p_R} \frac{1 - F_{2L}\left(\frac{u_R p_R}{u_L p_L}\right)}{1 - F_{2R}\left(\frac{u_R p_R}{u_L p_L}\right)} = \frac{u_L p_L}{u_R p_R} \times STR\left(s_2 \in \left[\frac{u_R p_R}{u_L p_L}, \infty\right)\right) > \frac{u_L p_L}{u_R p_R}. \tag{C.2}$$

The last inequality is implied by

$$\begin{aligned} F_{2R}\left(\frac{u_R p_R}{u_L p_L}\right) &> F_{2L}\left(\frac{u_R p_R}{u_L p_L}\right); \\ 1 - F_{2L}\left(\frac{u_R p_R}{u_L p_L}\right) &> 1 - F_{2R}\left(\frac{u_R p_R}{u_L p_L}\right); \\ STR\left(s_2 \in \left[\frac{u_R p_R}{u_L p_L}, \infty\right)\right) &> 1. \end{aligned}$$

Combining inequalities (C.1) and (C.2) proves the results. □

C.4 Proof of Proposition 3

Proof. Point 1 of the proposition is directly implied by the proposition 2. For point 2, by proposition 2, $\Phi^- > \Phi^+$ if and only if:

$$\frac{u_L^2 p_L^2}{u_R^2 p_R^2} \times \frac{\left(1 - F_{2L}\left(\frac{u_R p_R}{u_L p_L}\right)\right) F_{2L}\left(\frac{u_R p_R}{u_L p_L}\right)}{\left(1 - F_{2R}\left(\frac{u_R p_R}{u_L p_L}\right)\right) F_{2R}\left(\frac{u_R p_R}{u_L p_L}\right)} > 1.$$

First, note that

$$\begin{aligned} \lim_{u_L p_L / u_R p_R \to \infty} \frac{F_{2L}\left(\frac{u_R p_R}{u_L p_L}\right)}{F_{2R}\left(\frac{u_R p_R}{u_L p_L}\right)} &= \lim_{u_L p_L / u_R p_R \to \infty} \frac{f_{2L}\left(\frac{u_R p_R}{u_L p_L}\right)}{f_{2R}\left(\frac{u_R p_R}{u_L p_L}\right)} \\ &= \lim_{u_L p_L / u_R p_R \to \infty} \frac{u_R p_R}{u_L p_L} \end{aligned}$$

which implies

$$\lim_{u_L p_L/u_R p_R \to \infty} \frac{u_L p_L F_{2L}\left(\dfrac{u_R p_R}{u_L p_L}\right)}{u_R p_R F_{2R}\left(\dfrac{u_R p_R}{u_L p_L}\right)} = 1 > 0;$$

$$\lim_{u_L p_L/u_R p_R \to \infty} \frac{u_L^2 p_L^2}{u_R^2 p_R^2} \times \frac{\left(1 - F_{2L}\left(\frac{u_R p_R}{u_L p_L}\right)\right) F_{2L}\left(\frac{u_R p_R}{u_L p_L}\right)}{\left(1 - F_{2R}\left(\frac{u_R p_R}{u_L p_L}\right)\right) F_{2R}\left(\frac{u_R p_R}{u_L p_L}\right)} = +\infty > 0.$$

The result follows given the continuity of Φ^- and Φ^+ in p_L. □

C.5 Proof of Corollary 2

Proof. I prove the corollary at the limit where $p_L \to 1$. Follow from the proof of proposition 3,

$$\lim_{u_L p_L/u_R p_R \to \infty} \Phi^+ = 1;$$
$$\lim_{u_L p_L/u_R p_R \to \infty} \Phi^- = +\infty.$$

It is then obvious that as $p_L \to 1$, for all $s > 1$,

$$U_P(s, \mathscr{M}_1) > U_I(s, \mathscr{M}_1)$$
$$U_P(s^{-1}, \mathscr{M}_1) < U_I(s^{-1}, \mathscr{M}_1).$$

The result then follows from corollary 1. □

C.6 Proof of Corollary 3

Proof. I prove the corollary at the limit where $p_L \to 1$. Follow from the proof of proposition 3,

$$\lim_{u_L p_L/u_R p_R \to \infty} \Phi^+ = 1;$$
$$\lim_{u_L p_L/u_R p_R \to \infty} \Phi^- = +\infty.$$

Now suppose $\Phi^+ \to 1$ and $\Phi^- \to +\infty$. The probability that the DM chooses action r converges to

$$p_L F_{1L}(1) F_{2L}\left(\frac{u_R p_R}{u_L p_L}\right) + p_R F_{1R}(1) F_{2R}\left(\frac{u_R p_R}{u_L p_L}\right). \tag{C.3}$$

On the other hand, if the DM can process both signals, the probability that he chooses action r equals

$$p_L \int_0^\infty F_{2L}\left(\frac{u_R\, p_R}{u_L\, p_L s}\right) f_{1L}(s)ds + p_R \int_0^\infty F_{2R}\left(\frac{u_R\, p_R}{u_L\, p_L s}\right) f_{1R}(s)ds. \quad \text{(C.4)}$$

Equation (C.3) is strictly smaller than equation (C.4) as for $\omega = L$ and R,

$$\begin{aligned}
\int_0^\infty F_{2\omega}\left(\frac{u_R\, p_R}{u_L\, p_L s}\right) f_{1\omega}(s)ds &> \int_0^1 F_{2\omega}\left(\frac{u_R\, p_R}{u_L\, p_L s}\right) f_{1\omega}(s)ds \\
&> \int_0^1 F_{2\omega}\left(\frac{u_R\, p_R}{u_L\, p_L}\right) f_{1\omega}(s)ds \\
&= F_{2\omega}\left(\frac{u_R\, p_R}{u_L\, p_L}\right) F_{1\omega}(1).
\end{aligned}$$

The result follows by the continuity of the probability of choosing action r in Φ^+ and Φ^-. □

C.7 Reverse Wishful Thinking

To illustrate reverse wishful thinking, I normalize the utility function as follows:

$$\begin{aligned}
u(r \mid L) &= -u_L < 0; \\
u(r \mid R) &= u_R > 0; \\
u(l \mid R) = u(l \mid L) &= 0.
\end{aligned}$$

State L is the undesirable outcome which yields weakly negative utility while state R yields weakly positive utility. By analogy to the definition of wishful thinking, the processing strategy of the DM exhibits reverse wishful thinking when $\Phi^+ < \Phi^+$. That is, he processes a larger set of information that supports the undesirable state, compare to that supports the desirable state. The following result hold as the

Corollary 9 (Reverse Wishful thinking). *When state L is very undesirable, the equilibrium processing strategy of the DM exhibits reverse wishful thinking, i.e., $\Phi^- > \Phi^+$ when $|-u_L/u_R|$ is big enough.*

C.8 Proof of Proposition 4

Proof. By proposition 2,

$$\begin{aligned}\Phi_A^+ &= \frac{u_R\,p_R}{u_L\,p_L} \times \frac{F_{2R}\left(\frac{u_R\,p_R}{u_L\,p_L}\right)}{F_{2L}\left(\frac{u_R\,p_R}{u_L\,p_L}\right)} \\ &= \Phi_B^+\end{aligned}$$

On the other hand,

$$\begin{aligned}\Phi_A^- &= \frac{u_L\,p_L}{u_R\,p_R} \times \frac{1-\lambda_A+\lambda_A\left(1-F_{2L}\left(\frac{u_R\,p_R}{u_L\,p_L}\right)\right)}{1-\lambda_A+\lambda_A\left(1-F_{2R}\left(\frac{u_R\,p_R}{u_L\,p_L}\right)\right)} \\ &> \frac{u_L\,p_L}{u_R\,p_R} \times \frac{1-\lambda_B+\lambda_B\left(1-F_{2L}\left(\frac{u_R\,p_R}{u_L\,p_L}\right)\right)}{1-\lambda_B+\lambda_B\left(1-F_{2R}\left(\frac{u_R\,p_R}{u_L\,p_L}\right)\right)} \\ &= \Phi_B^-\end{aligned} \tag{C.5}$$

where the second inequality of equation (C.5) follows from the fact that $1-F_{2L}\left(\frac{u_Rp_R}{u_Lp_L}\right) > 1-F_{2R}\left(\frac{u_Rp_R}{u_Lp_L}\right)$. □

C.9 Proof of Proposition 5

Proof. First, by proposition 2,

$$\begin{aligned}\Phi_A^- &= \frac{u_L\,p_L}{u_R\,p_R} \frac{1-\lambda-\delta+\lambda\left(1-F_{2L}^B\left(\frac{u_R\,p_R}{u_L\,p_L}\right)\right)+\delta\left(1-F_{2L}^A\left(\frac{u_R\,p_R}{u_L\,p_L}\right)\right)}{1-\lambda-\delta+\lambda\left(1-F_{2R}^B\left(\frac{u_R\,p_R}{u_L\,p_L}\right)\right)+\delta\left(1-F_{2R}^A\left(\frac{u_R\,p_R}{u_L\,p_L}\right)\right)} \\ &> \frac{u_L\,p_L}{u_R\,p_R} \frac{1-\lambda+\lambda\left(1-F_{2L}^B\left(\frac{u_R\,p_R}{u_L\,p_L}\right)\right)}{1-\lambda+\lambda\left(1-F_{2R}^B\left(\frac{u_R\,p_R}{u_L\,p_L}\right)\right)} \\ &= \Phi_B^-\end{aligned} \tag{C.6}$$

where the second inequality of equation (C.6) follows from the fact that $1 - F^A_{2L}\left(\frac{u_R\, p_R}{u_L\, p_L}\right) > 1 - F^A_{2R}\left(\frac{u_R\, p_R}{u_L\, p_L}\right)$.

Now I compare Φ^+_A and Φ^+_B. By proposition 2,

$$\Phi^+_A = \frac{u_R\, p_R}{u_L\, p_L} \frac{\lambda F^B_{2R}\left(\frac{u_R\, p_R}{u_L\, p_L}\right) + \delta F^A_{2R}\left(\frac{u_R\, p_R}{u_L\, p_L}\right)}{\lambda F^B_{2L}\left(\frac{u_R\, p_R}{u_L\, p_L}\right) + \delta F^A_{2L}\left(\frac{u_R\, p_R}{u_L\, p_L}\right)};$$

$$\Phi^+_B = \frac{u_R\, p_R}{u_L\, p_L} \frac{\lambda F^B_{2R}\left(\frac{u_R\, p_R}{u_L\, p_L}\right)}{\lambda F^B_{2L}\left(\frac{u_R\, p_R}{u_L\, p_L}\right)}.$$

Therefore, $\Phi^+_A \leq \Phi^+_B$ if and only if

$$\frac{F^A_{2R}\left(\frac{u_R\, p_R}{u_L\, p_L}\right)}{F^A_{2L}\left(\frac{u_R\, p_R}{u_L\, p_L}\right)} \leq \frac{F^B_{2R}\left(\frac{u_R\, p_R}{u_L\, p_L}\right)}{F^B_{2L}\left(\frac{u_R\, p_R}{u_L\, p_L}\right)}.$$

□

C.10 Proof of Proposition 6

Proof. By proposition B.2, it is sufficient to prove that there exists a perfect Bayesian Nash equilibrium for $T = 2$. First note that the equilibrium strategy is a threshold strategy, with thresholds (Φ^+, Φ^-). Denote $\mathscr{R}_1$ as

$$\mathscr{R}_1 = \frac{F_{1L}(\Phi^+) - F_{1L}(1/\Phi^-)}{F_{1R}(\Phi^+) - F_{1R}(1/\Phi^-)}.$$

As $\Phi^+ \geq \frac{u_R\, p_R}{u_L\, p_L}$ and $\Phi^- > \frac{u_L\, p_L}{u_R\, p_R}$, $\mathscr{R}_1 \in \left(\frac{F_{1L}(1)}{F_{1R}(1)}, \frac{1-F_{1L}(1)}{1-F_{1R}(1)}\right)$. Given the optimality of the processing strategy, the perfect Bayesian Nash equilibrium is a fixed point of the following equation:

$$\mathscr{R}_1 = \frac{F_{1L}\left(\frac{u_R\, p_R}{u_L\, p_L} \frac{F_{2R}\left(\frac{u_R\, p_R}{u_L\, p_L \mathscr{R}_1}\right)}{F_{2L}\left(\frac{u_R\, p_R}{u_L\, p_L \mathscr{R}_1}\right)}\right) - F_{1L}\left(\frac{u_R\, p_R}{u_L\, p_L} \frac{1 - F_{2R}\left(\frac{u_R\, p_R}{u_L\, p_L \mathscr{R}_1}\right)}{1 - F_{2L}\left(\frac{u_R\, p_R}{u_L\, p_L \mathscr{R}_1}\right)}\right)}{F_{1R}\left(\frac{u_R\, p_R}{u_L\, p_L} \frac{F_{2R}\left(\frac{u_R\, p_R}{u_L\, p_L \mathscr{R}_1}\right)}{F_{2L}\left(\frac{u_R\, p_R}{u_L\, p_L \mathscr{R}_1}\right)}\right) - F_{1R}\left(\frac{u_R\, p_R}{u_L\, p_L} \frac{1 - F_{2R}\left(\frac{u_R\, p_R}{u_L\, p_L \mathscr{R}_1}\right)}{1 - F_{2L}\left(\frac{u_R\, p_R}{u_L\, p_L \mathscr{R}_1}\right)}\right)}$$

Denote the equation as $\mathscr{R}_1 = \psi(\mathscr{R}_1)$. The mapping ψ is continuous mapping from $\left[\frac{F_{1L}(1)}{F_{1R}(1)}, \frac{1-F_{1L}(1)}{1-F_{1R}(1)}\right]$ to itself, which is a convex and compact set. By the Brouwer's fixed point theorem, there exists a fixed point of the equation which belongs to the set $\left[\frac{F_{1L}(1)}{F_{1R}(1)}, \frac{1-F_{1L}(1)}{1-F_{1R}(1)}\right]$. As the two end points $\frac{F_{1L}(1)}{F_{1R}(1)}$ and $\frac{1-F_{1L}(1)}{1-F_{1R}(1)}$ are not a fix point of the equation, there exists a fixed point $\mathscr{R}_1^*$ such that

$$\mathscr{R}_1^* \in \left(\frac{F_{1L}(1)}{F_{1R}(1)}, \frac{1-F_{1L}(1)}{1-F_{1R}(1)}\right);$$
$$\mathscr{R}_1^* = \psi(\mathscr{R}_1^*).$$

□

C.11 Proof of Corollary 5

Proof. Without loss of generality, I analyze only the processing strategy of Alice. First by corollary 2, Alice processes signal q because

$$q > \frac{1-p}{p} \times q,$$

while she processes signal q^{-1} if and only if

$$q > \frac{p}{1-p} \times \frac{1-\lambda+\lambda q(1+q)^{-1}}{1-\lambda+\lambda(1+q)^{-1}} = \frac{p}{1-p} \times \frac{1-\lambda+q}{1+q(1-\lambda)}.$$

Define $\mathscr{F}(p,q,\lambda) = q - \frac{p}{1-p} \times \frac{1-\lambda+q}{1+q(1-\lambda)}$. Its first derivatives w.r.t. p and λ are

$$\frac{\partial \mathscr{F}}{\partial p} = -\frac{1}{(1-p)^2}\frac{1-\lambda+q}{1+q(1-\lambda)} < 0;$$
$$\frac{\partial \mathscr{F}}{\partial \lambda} = -\frac{p}{1-p}\frac{q^2-1}{(1+q(1-\lambda))^2} < 0.$$

Moreover,

$$\mathscr{F}(0,q,\lambda) = q > 0 > \lim_{(p/1-p)\to q} \mathscr{F}(p,q,\lambda) = q\frac{\lambda(1-q)}{1+q(1-\lambda)};$$
$$\mathscr{F}(p,q,0) = q - \frac{p}{1-p} > 0 > \mathscr{F}(p,q,1) = q(1-\frac{p}{1-p}).$$

Therefore, there exists p^- and λ^- such that $\mathscr{F} > 0$ if and only if $p < p^-$ and if and only if $\lambda < \lambda^-$. It remains to prove point 2 of the corollary. First, $\mathscr{F}$ is strictly convex in q

$$\frac{\partial^2 \mathscr{F}}{\partial q^2} = \frac{p}{1-p} \frac{2\lambda(1-\lambda)(2-\lambda)}{(1+q(1-\lambda))^3} > 0,$$

and the two roots of $\mathscr{F}(q) = 0$ are

$$\frac{-(1-\frac{p}{1-p}) \pm \sqrt{(1-\frac{p}{1-p})^2 + 4(1-\lambda)^2 \frac{p}{1-p}}}{2(1-\lambda)}. \tag{C.7}$$

One of the two roots is negative, which contradicts the fact that $q > 1$. Hence, there exists a q^-, which is the positive root defined in equation (C.7), such that $\mathscr{F} > 0$ if and only if $q > q^-$. □

C.12 Proof of Proposition 9

Proof. When $\lambda < \lambda^-$, Alice and Bob processes both belief-confirming and belief-challenging information. They take the same action if and only if at least one of s_1 and s_2 is informative.

$$P_{concensus} = 1 - (1-\lambda)^2 = \lambda(2-\lambda). \tag{C.8}$$

On the other hand, when $\lambda \geq \lambda^-$, they ignore belief-challenging signals in period 1. Therefore, Alice and Bob take the same action if and only if the signals in both periods support the same state, or if s_1 is pure noise while s_2 is not.

$$P_{concensus} = \lambda^2(q^2(1+q)^{-2} + (1+q)^{-2}) + \lambda(1-\lambda) \tag{C.9}$$

Similarly, the probability that Alice/Bob takes her/his a priori optimal action follows:

$$\begin{aligned}
P^a_{default} &= \begin{cases} 1 - \lambda(1+q)^{-1} - \lambda(1-\lambda)(1+q)^{-1} & \text{if } \lambda < \lambda^- \\ 1 - \lambda^2(1+q)^{-2} - \lambda(1-\lambda)(1+q)^{-1} & \text{if } \lambda \geq \lambda^- \end{cases} \\
P^b_{default} &= \begin{cases} 1 - \lambda q(1+q)^{-1} - \lambda(1-\lambda)q(1+q)^{-1} & \text{if } \lambda < \lambda^- \\ 1 - \lambda^2 q^2(1+q)^{-2} - \lambda(1-\lambda)q(1+q)^{-1} & \text{if } \lambda \geq \lambda^- \end{cases}
\end{aligned} \tag{C.10}$$

First, I prove the first part of the proposition. First, notice that equation (C.8) is increasing and convex in λ. On the other hand, the first and second derivative of equation (C.9) w.r.t. λ are given by:

$$\frac{\partial P_{concensus}}{\partial \lambda} = 1 - 2\lambda(1 - (q^2(1+q)^{-2} + (1+q)^2)) > 0$$
$$\frac{\partial P^2_{concensus}}{\partial^2 \lambda} = -2(1 - (q^2(1+q)^{-2} + (1+q)^2)) < 0$$

respectively. The two inequalities are implied by the fact that $q > 1$ and $\lambda \in [0, 1]$.[32] Secondly, equation (C.9) is clearly smaller than equation (C.8) for any given λ, which implies that there is a downward jump at $\lambda = \tilde{\lambda}$.

Now I move on to the proof for $P^j_{default}$, $j = a, b$. First, part 1 is implied by the fact that both functions

$$P^a_{default} = 1 - \lambda(1+q)^{-1} - \lambda(1-\lambda)(1+q)^{-1}$$
$$P^b_{default} = 1 - \lambda q(1+q)^{-1} - \lambda(1-\lambda)q(1+q)^{-1}$$

decrease in λ as $\lambda(2-\lambda)$ increases in λ for $\lambda \in [0, 1]$. On the other hand, from equation (C.10), it is obvious that there is a upward jump at $\lambda = \tilde{\lambda}$. □

[32]First, $[q^2(1+q)^{-2} + (1+q)^{-2}] \in (1/2, 1)$ as it is increasing in q. Therefore, $1-(q^2(1+q)^{-2} + (1+q)^{-2}) \in (0, 1/2)$. Combined with the fact that $\lambda \in [0, 1]$, it implies the first inequality.