The paper analyzes the dynamics of demand for three options when agents differ in their preferences for conformity. Each agent seeks to imitate others who are more individualistic and to distinguish herself from others who are more conformist, relative to herself. In each period, every agent chooses her utility-maximizing option given each agent’s demand in the previous period. It is shown that for a large class of initial demand distributions, demand dynamics resemble fashion cycles: Total demand for each option over time is wave-like, and, when positively demanded, an option trickles through the entire population, from individualistic towards conformist agents.
Demand Cycles and Heterogeneous Conformity Preferences

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Abstract

The paper analyzes the dynamics of demand for three options when agents differ in their preferences for conformity. Each agent seeks to imitate others who are more individualistic and to distinguish herself from others who are more conformist, relative to herself. In each period, every agent chooses her utility-maximizing option given each agent’s demand in the previous period. It is shown that for a large class of initial demand distributions, demand dynamics resemble fashion cycles: Total demand for each option over time is wave-like, and, when positively demanded, an option trickles through the entire population, from individualistic towards conformist agents.

Keywords: fashion cycle, demand cycle, conformity, individuality, dynamics, distribution of demand

JEL Codes: C73, D11, D91, E21, E32, E71, Z13
1 Introduction

Many choice categories exhibit trends and fashions which come and go. Old fashions are replaced by new ones in an everlasting repetition. Once a new fashion enters the market, initial demand for it is low. Then demand grows until it reaches a peak, and finally declines until the fashion entirely disappears. These demand dynamics are the life cycle of a fashion. When one fashion disappears, the life cycle of the next fashion takes off (Kotler, 1997, p. 533). In this sense, we can speak of continuous fashion cycles. Clothing, food and nutrition, sports, neighborhoods, holiday destinations, topics in academia are only some categories among many which feature fashion cycles. This paper proposes a model to explain fashion cycles through heterogeneous conformity and individuality preferences of consumers.

Anecdotal evidence for heterogeneous preferences for individuality and conformity is manifold. Statements like “you cannot wear these glasses anymore, everybody is wearing them now” or “I do not like this band anymore, they have become so popular that they even charge for their concerts” express the wish to distinguish oneself from the majority. On the other hand, mottos like “do as your neighbors do” or “all my friends have x, so I also want x” express the wish to conform. Intuitively, this distinctive behavior of some parts of the population together with the imitative behavior of other parts of the population leads to an ever-going change in fashions.

Heterogeneous conformity and counterconformity preferences have been confirmed by sociological and psychological research. In an early seminal experiment Asch (1951) found different tendencies among subjects to conform to the majority. Some subjects conformed to the majority in order not to differ, others “withdrew” from the majority because of the desire of “being an individual”. Snyder and Fromkin (1980) introduced the uniqueness theory according to which an individual’s need for uniqueness determines how similar or dissimilar she wants to be to others. Lynn and Harris (1997), Ariely and Levav (2000) and Timmor and Katz-Navon (2008) have provided empirical evidence that an individual’s distinctive behavior in various environments is driven by her need for uniqueness. Berger and Heath (2008)
adduce an individual’s desire to distinguish herself from disliked others as a further argument for counter-conformity.

Traditionally, fashion cycles have been a topic of sociological research. Veblen (1912), Simmel (1957) and Bourdieu (1984, 1993) proposed the trickle-down theory as an explanation of fashion cycles. According to the trickle-down theory, fashion goods serve upper classes to distinguish themselves from lower classes who in turn emulate the upper classes. Thus fashions move from the upper classes down to the lower classes. In criticism of the trickle-down theory, Blumer (1969), Sproles (1981), McCracken (1988), King (1965), Field (1970), Vejlgaard (2008) separate fashion leadership from the upper class and argue that fashion innovators and leaders can come from any class, also lower classes and subcultures. They do not see the primary purpose of fashion in class distinction. Groups in the fashion diffusion process are roughly divided into “innovators, leaders, followers, and participants” (Blumer, 1969) whereby fashions trickle from innovators, over leaders and followers to participants. The underlying, not explicitly stated driving forces of these fashion cycle theories are heterogeneous conformity and counter-conformity preferences. Park (1998), Cholachatpinyo et al. (2002) and Workman and Kidd (2000) explicitly link the fashion process to heterogeneous (counter-) conformity preferences. They provide empirical evidence that fashion innovators are driven by a need to distinguish themselves and to be unique, whereas followers have stronger preferences for being similar to others and to conform.

This paper presents a formal economic model of fashion cycles which is in line with the above sociological theories. The model explains the typical fashion life cycle which is the bell-shaped demand for the fashion item over time, and the “trickling” process of the fashion item from innovators over leaders and followers to participants through heterogeneous (counter-) conformity preferences. Specifically, we propose a discrete time model in which the population is infinitely heterogeneous with respect to conformity preferences. Each agent’s conformity preference is defined by her location in the unit interval. In each period, every agent decides which one of three option to consume on the basis of the previous period distribution of demand across the population. We show that for a large class of initial distributions the
dynamics of demand converge to an infinitely repeated unique sequence of demand distributions spanning eight time periods. Given these dynamics, the demand for each option over time is given by consecutive, bell-shaped life cycles. Having completed one life cycle, an option stays out of the market for one time period, and then enters into another life cycle. Within each life cycle, the option trickles deterministically through society. The model replicates fashion cycles and predicts the revival of fashions given a limited set of choice options.

Another driver for fashion cycles, different from heterogeneous conformity preferences, has been explored by Pesendorfer (1995). He shows that when consumption choice is a signaling device for an unobserved “quality” of the consumer, then dynamic demand for different designs which are subsequently developed and sold by a monopolist resembles fashion cycles. In his model, a new design serves as a signaling device for “high” types because of a high initial price which prevents “low” types from buying the design. Over time, the monopolist decreases the price such that more “low” types buy the design, and the value as signal for high types is lost. At this point, the monopolist introduces a new design at a high price which serves as a new signaling device, and so on. This model cannot explain fashion cycles in categories in which options have the same (or no) price, for example, given first names (Yoganarasimhan, 2017). Moreover, it does not account for the fact that trends are often started by individuals with a low willingness to pay or with a low budget at low prices (Field, 1970; Sproles, 1981; Vejlgaard, 2008). For example, certain clothing and music styles were initiated in the youth and street culture, or lower income classes. Heterogeneous conformity preferences can explain both fashion cycles in the demand for goods with no price, as well as fashions being started by individuals with a low willingness to pay.

The following works have started to investigate heterogeneous conformity preferences as a driver for fashion cycles with formal models. Karni and Schmeidler (1990) consider a discrete time model with two social classes $\alpha$ and $\beta$. In each time period, $\frac{1}{3}$ of each class has to choose between three different consumption options whereby $\alpha$ members want to imitate other $\alpha$ members and distinguish themselves from $\beta$ members, and $\beta$ members want
to conform to both $\alpha$ and other $\beta$ members. Consumers are forward looking over their lifetime which is 9 periods. They show that the demand over time for each color resembles consecutive life cycles. Matsuyama (1991) analyzes a random matching game with non-conformists and conformists. Individuals choose between two consumption options before they are matched. Conformists gain a high payoff if their match chose the same option, and non-conformists gain a high payoff from miscoordination. If intergroup matching is sufficiently likely and inertia exist in the society, then the demand for the two options resembles fashion cycles with non-conformists acting as fashion leader and conformists as followers. Corneo and Jeanne (1999) again distinguish between two groups – “natives” and “tourists” – who decide whether to go to location 1 or 2. Tourists like to go where natives go, but natives do not want to go where tourists go. Location 2 is only known to some agents and information about it spreads via a random matching process between agents. If location 2 is better known among natives initially, then fashion cycles arise in which the natives act as fashion leader and the tourists as followers. Berger et al. (2011) introduce a model in which an agent chooses from a number of goods while being concerned about her social image. This induces individuals to choose what others who she likes consume, and not to choose what disliked others consume. The authors provide some qualitative results and intuition how their model might give rise to fashion cycles, but they are missing a rigorous analysis of the dynamics of demand. Granovetter and Soong (1983) model heterogeneous (counter-) conformity preferences by introducing individual-specific lower and upper thresholds on the fraction of population above and below which an individual participates in a movement. If the lower threshold has not been reached, too few people participate in the movement for the individual to join, whereas if the threshold has been surpassed, too many people participate for the individual to join. The authors find that for certain distributions of lower and upper thresholds across the population, participation rates oscillate between different levels.

This paper proposes a new simple model with an infinitely heterogeneous population regarding its conformity preferences in a three-good economy. This allows us to produce demand dynamics which show the properties of
fashion cycles as described by traditional fashion theories. Demand for each option follows consecutive bell-shaped life cycles over time, one fashion substitutes another, and fashions trickle in a deterministic way through society.

Section 2 outlines the model and section 3 presents the analysis and results.

2 Model

A unit mass population is continuously and uniformly distributed on the $[0,1]$-interval. Every agent is indexed by $i \in [0,1]$ with $i$ being equal to her location on the $[0,1]$-interval. An agent’s location is fixed and exogeneously given.

We propose two interpretations of the $[0,1]$-interval in line with our hypothesis and the sociological fashion theories. First, the $[0,1]$-interval can be interpreted as a scale of social status with $i = 0$ being the highest status (maybe richest) agent and $i = 1$ being the lowest status (maybe poorest) agent; second, it can be interpreted as a scale of conformity preference with $i = 0$ being the most individualistic agent and $i = 1$ being the most conforming agent.

As to agents’ preferences, we assume that every agent $i$ wants to be similar to all other agents $j < i$ and wants to distinguish herself from all other agents $j > i$. In the context of our two interpretations this means that every agent wants to emulate agents of higher status and distinguish herself from agents of lower status or that every agent wants to emulate more individualistic agents and distinguish herself from more conforming agents. The preferences will be formalized in the utility function given below.

We assume that the set of all available consumption options $S$ consists of three styles, $S = \{A, B, C\}$. These three styles could be three differently colored sweatshirts, three cars from different brands, or three different neighborhoods in one city. We assume an equal price for all styles. This allows us to focus on the effects of heterogeneous conformity preferences. Without loss of generality we set the price equal to zero.

The economy evolves in discrete time. Agents are assumed to be myopic.
In each time period \( t \), each agent chooses to consume a style \( s \in S \) which maximizes her utility given the distribution of demand in the previous period \( t - 1 \). The distribution of demand in the previous period is common knowledge.

In line with the conformity preferences described above, we assume that agent \( i \)'s utility in period \( t \) from style \( s \) is given by

\[
 u_i^t(s) = M_{t-1}^{j<i}(s) - M_{t-1}^{i<j}(s)
\]

where \( M_{t-1}^{j<i}(s) \) is the \( t - 1 \) market share of \( s \) (mass of agents who consume \( s \)) among all agents \( j < i \) and \( M_{t-1}^{i<j}(s) \) is the \( t - 1 \) market share of \( s \) among all agents \( j > i \). By \( M_{t-1}(s) \) we denote the total market share of \( s \) in \( t - 1 \).

We introduce a tie-breaking rule for the case that \( u_i^t(r) = u_i^t(s) \) with \( r \neq s \) and \( r, s \in S \). If \( u_i^t(A) = u_i^t(B) \), then \( i \) strictly prefers \( B \) over \( A \); if \( u_i^t(B) = u_i^t(C) \), then \( i \) strictly prefers \( C \) over \( B \); and if \( u_i^t(A) = u_i^t(C) \), then \( i \) strictly prefers \( C \) over \( A \). For the results, it is important that all agents agree upon which one of two styles is strictly preferred in case of utility equality.

To gain intuition for agents’ preferences consider the following example. Suppose that in \( t - 1 \) all agents \( i \in [0,0.3) \) consume \( A \), all \( i \in [0.3,0.5) \) consume \( B \), and all \( i \in [0.5,1] \) consume \( C \). Figure 1 shows agent \( i \)'s utility in \( t \) for each style given the distribution in \( t - 1 \) for all \( i \in [0,1] \).

![Figure 1: \( u_i^t(s) \) for all \( s \) and \( i \), if in \( t - 1 \) all \( i \in [0,0.3) \) consume \( A \), all \( i \in [0.3,0.5) \) consume \( B \), and all \( i \in [0.5,1] \) consume \( C \).](image-url)
The utility from style $s$ in $t$ is strictly increasing with a slope of 2 over the interval of agents who consumed $s$ in $t-1$, everywhere else it is constant. From the figure it is obvious that the consumption choices in $t$ are such that all $i \in [0, .05]$ choose $B$, all $i \in (.05, .9)$ choose $A$, and all $i \in [.9, 1]$ choose $C$.

3 Dynamics of Demand

In this section, we characterize the dynamics of demand which arise in this model. This means that we exactly characterize for each period $t$ which parts of the population consume which style, or in other words the distribution of demand across the population.

A distribution of demand in period $t$ is defined by a boundary vector $b_t$, which describes a partition of the population (unit interval) in period $t$, and a style vector $s_t$, which describes which styles the different parts of the population, as partitioned by $b_t$, consume in period $t$. This will become clear in definition 1. For tractability, we restrict initial distributions of demand in $t = 0$ to be of one of the types as defined in definition 1.

Definition 1. Distribution types in time period $t$.

(i) 1-style-distribution where $b_t = (0, 1)$ and $s_t = (x)$ with $x \in S$:

Every agent $i \in [0, 1]$ consumes the same style $x$ in $t$.

(ii) 2-styles-distribution where $b_t = (0, b^1_t, 1)$ with $0 < b^1_t < 1$ and $s_t = (x, y)$ with $x \neq y$ and $x, y \in S$:

Every agent $i \in [0, b^1_t)$ consumes style $x$, every agent $i \in (b^1_t, 1]$ consumes style $y$, and agent $i = b^1_t$ consumes $x$ or $y$ in $t$.

The results hold for all initial distributions after which the demand dynamics result into any of the distribution types as defined in definition 1 for some $t \geq 1$. 
(iii) 3-styles-distribution where
\[ b_t = (0, b_1^t, b_2^t, 1) \]
with \(0 < b_1^t < b_2^t < 1\) and
\[ s_t = (x, y, z) \]
with \(x \neq y \neq z \neq x\) and \(x, y, z \in S\):

Every agent \(i \in [0, b_1^t)\) consumes style \(x\), every agent \(i \in (b_1^t, b_2^t)\) consumes \(y\), and every agent \(i \in (b_2^t, 1]\) consumes \(z\). Agent \(i = b_1^t\) consumes \(x\) or \(y\), and agent \(i = b_2^t\) consumes \(y\) or \(z\).

In the following, it is shown that the dynamics of demand converge to the same unique sequence of distributions for any of the possible initial distributions. Denote by \(\Sigma_t\) the set of all styles \(s\) for which \(M_t(s) = 0\). Thus these are the styles which do not receive any demand in \(t\). Let \(\max\) be the function which picks the style with the highest position in the alphabet from some set of styles, with \(C\) having the highest position (3) in \(S\).

**Lemma 3.1.** If the distribution in \(t\) is a 1-style distribution, then the distribution in \(t + 1\) is a 2-styles-distribution.

**Proof.** Given a 1-style-distribution in \(t\) with \(b_t = (0, 1), s_t = (x)\) and \(\Sigma_t = \{y, z\}\), in \(t + 1\) every \(i \in [0, \frac{1}{2})\) chooses \(\max\{y, z\}\) in order to differentiate herself from more conformist agents as \(u^i_{t+1}(s) = 0 > u^i_{t+1}(x)\) for all \(s \in \Sigma_t\).

For \(i = \frac{1}{2}\), \(u^i_{t+1}(s) = 0 = u^i_{t+1}(x)\) for all \(s \in \Sigma_t\) and thus \(i = \frac{1}{2}\) chooses \(\max\{\max\{y, z\}, x\}\). The more conformist agents are still satisfied with their choice in \(t\) because everybody was consuming \(x\) in \(t\). For all \(i \in (\frac{1}{2}, 1]\), \(u^i_{t+1}(s) = 0 < u^i_{t+1}(x)\) for all \(s \in \Sigma_t\) such that every \(i \in (\frac{1}{2}, 1]\) consumes \(x\) in \(t + 1\). Figure 5 exemplifies the distributions in \(t\) and \(t + 1\).
For the next lemma, we introduce an equivalence notion for distributions. Observe that some distribution $d$ in $t$ in which $i = 0$ consumes $x$ and all $i > 0$ consume $y$ is not a 1-style-distribution. It is however mass equivalent to the 1-style-distribution with $b_t = (0, 1)$ and $s_t = (y)$ as $i = 0$ has mass zero. This means that the consumption decision of all $i$ in $t + 1$ is the same regardless of whether the distribution in $t$ is $d$ or the 1-style-distribution with $b_t = (0, 1)$ and $s_t = (y)$. We will say that two distributions of consumption $d$ and $d'$ are mass equivalent if the differences between them are mass zero.

**Lemma 3.2.** If the distribution in $t$ is a 3-styles-distribution, then the distribution in $t + k$ for some $k \geq 1$ is mass equivalent to a 2-styles-distribution.

**Proof.** In this proof, we first derive the distribution of consumption in $t + 1$ for any 3-styles-distribution in $t$. Let $b_t = (0, b_1^t, b_2^t, 1)$ and $s_t = (x, y, z)$ with $x \neq y \neq z \neq x$ and $x, y, z \in S$.

The analysis is split into cases each of which represents a different ordering of the population masses in the consumption intervals in $t$. For convenience, the length of the most left consumption interval in $t$ will be denoted by $1^t$, the length of the central consumption interval by $2^t$, and the length of the most right consumption interval by $3^t$. Hence, $1^t := b_1^t$, $2^t := b_2^t - b_1^t$, and $3^t := 1 - b_2^t$.

We describe the case of $1^t = 2^t = 3^t$ here in detail to provide intuition for how the $t + 1$ distribution arises. All other cases are relegated to the appendix. If $1^t = 2^t = 3^t$, then $b_t = (0, \frac{1}{3}, \frac{2}{3}, 1)$. In $t + 1$, every style $s \in S$ yields $u_{i+1}^t(s) = -\frac{1}{3}$ to agent 0 and thus $i = 0$ chooses $\max S$. All $i \in (0, \frac{2}{3})$ imitate the period $t$ choice of the most individualist agents and choose $x$. All $i \in [\frac{2}{3}, 1)$ get the same positive utility from $x$ and $y$ and thus choose
max \{x, y\}$. Agent 1 obtains $u^1_{t+1}(s) = \frac{1}{3}$ for any $s \in S$ and thus chooses $\max S$.

Figure 6 depicts the distribution in $t$ and the distribution in $t+1$. Mass zero events – in this case, the choices of agent 0 and agent 1 – are not shown in this or any other figure in this paper.

$\begin{array}{cccc}
\ hline 
& x & y & z \\
\ hline 
\ t & 0 & \frac{1}{3} & \frac{2}{3} & 1 \\
\ hline 
\ t+1 & 0 & \frac{2}{3} & 1 & \max \{x, y\} \\
\ hline 
\end{array}$

**Figure 6:** Proof of Lemma 3.2, case $\mathbf{1}_t = \mathbf{2}_t = \mathbf{3}_t$.

The $t + 1$ distribution is mass equivalent to the 1-style-distribution with $b_{t+1} = (0, 1)$ and $s_{t+1} = (x)$, if $x = \max \{x, y\}$, or to the 2-styles-distribution with $b_{t+1} = (0, \frac{2}{3}, 1)$ and $s_{t+1} = (x, y)$, if $y = \max \{x, y\}$.

We summarize the results up to here, including those derived in the appendix, in figure 7. The figure provides an overview of the distributions in $t+1$ resulting from any 3-styles-distribution in $t$. The notation “$\mathbf{1} < \mathbf{2} < \mathbf{3} \rightarrow \mathbf{1} = \mathbf{2} > \mathbf{3}$” means that a 3-styles-distribution with property $\mathbf{1}_t < \mathbf{2}_t < \mathbf{3}_t$ in period $t$ results into a (distribution mass equivalent to a) 3-styles-distribution with property $\mathbf{1}_{t+1} = \mathbf{2}_{t+1} > \mathbf{3}_{t+1}$ in period $t + 1$, and analogously for all other cases. With figure 7 it is easy to verify that any 3-styles-distribution in $t$ results into a distribution mass equivalent to a 2-styles-distribution in $t + k$ for some $k \in [1, 3]$.

$\square$
Thus, eventually any 1-style- and 3-styles-distribution results into (a distribution mass equivalent to) a 2-styles-distribution. Next, we show that the dynamics following any 2-styles-distribution converge to a unique sequence of distributions of consumption.

**Lemma 3.3.** The dynamics following any 2-styles-distribution converge to a unique cycle $C$ of distributions. This unique cycle is equal to the infinite repetition of the sequence of distributions from $t$ to $t+8$, given in Figure 8, where $x \neq y \neq z \neq x$ and $x, y, z \in S$. Thus, in $C$ the distribution of consumption in any period $t$ is equal to the distribution of consumption in period $t+9$. 
Figure 8: The sequence of distributions from $t$ to $t+8$ is infinitely repeated in cycle $C$.  

<table>
<thead>
<tr>
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<th>$x$</th>
<th>$y$</th>
<th>$\Sigma_{t} = {z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$t+1$</td>
<td>$z$</td>
<td>$x$</td>
<td>$\Sigma_{t+1} = {y}$</td>
</tr>
<tr>
<td>0</td>
<td>$\frac{1}{4}$</td>
<td>1</td>
<td></td>
</tr>
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<td>$t+2$</td>
<td>$y$</td>
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<td>$\Sigma_{t+2} = {}$</td>
</tr>
<tr>
<td>0</td>
<td>$\frac{1}{8}$</td>
<td>$\frac{3}{4}$</td>
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<tr>
<td>$t+3$</td>
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<td>$\Sigma_{t+3} = {x}$</td>
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<td>0</td>
<td>$\frac{1}{2}$</td>
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<tr>
<td>$t+4$</td>
<td>$x$</td>
<td>$y$</td>
<td>$\Sigma_{t+4} = {z}$</td>
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<td>$t+7$</td>
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<td>$z$</td>
<td>$\Sigma_{t+7} = {x}$</td>
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<td>$\frac{1}{4}$</td>
<td>1</td>
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<td>$x$</td>
<td>$y$</td>
<td>$\Sigma_{t+8} = {}$</td>
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<tr>
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<td>$z$</td>
<td>$y$</td>
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<td>0</td>
<td>$\frac{1}{2}$</td>
<td>1</td>
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Proof. We show that the dynamics of the distribution of consumption following any 2-styles-distribution in $t$ with $b_t = (0, b_{1t}, 1)$, $s_t = (x, y)$ and $\Sigma_t = \{z\}$ where $x \neq y \neq z \neq x$ and $x, y, z \in S$ converge to $C$. For the proof, we divide all 2-styles-distributions in $t$ into eight different cases depending on their value of $b_{1t}$ and derive the resulting dynamics for each case. The case of $0 < b_{1t} \leq \frac{1}{3}$ will be discussed in detail here, whereas all other cases are relegated to the appendix.

If $0 < b_{1t} \leq \frac{1}{3}$, then the distribution in $t + 1$ is a 3-styles-distribution with $b_{t+1} = (0, \frac{1}{2}b_{1t}, \frac{1}{2}(2b_{1t} + 1), 1)$ and $s_{t+1} = (z, x, y)$. Thus, $2_{t+1} > 3_{t+1} \geq 1_{t+1}$. In $t + 2$, the distribution is mass equivalent to the 2-styles-distribution with $b_{t+2} = \frac{1}{2}(2b_{t+1} + b_{t+1}^2)$ and $s_{t+2} = (z, x)$. These dynamics are shown in figure 9.

![Figure 9: Proof of Lemma 3.3, case $0 < b_{1t} \leq \frac{1}{3}$.](image)

For the remainder of this proof it is useful to further divide this case into four sub-cases and to provide the possible values of consumption interval boundaries for each sub-case.

(1a) If $0 < b_{1t} \leq \frac{1}{12}$, then $b_{t+1} \in (0, \frac{1}{24}]$, $b_{t+1}^2 \in (\frac{1}{2}, \frac{7}{12}]$, and $b_{t+2} \in (\frac{1}{4}, \frac{1}{3}]$.

(1b) If $\frac{1}{12} < b_{1t} < \frac{1}{4}$, then $b_{t+1} \in (\frac{1}{24}, \frac{1}{8})$, $b_{t+1}^2 \in (\frac{7}{12}, \frac{3}{4})$, and $b_{t+2} \in (\frac{1}{3}, \frac{1}{2})$. 

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(1c) If $b_t^1 = \frac{1}{4}$, then $b_{t+1}^1 = \frac{1}{8}$, $b_{t+1}^2 = \frac{3}{4}$, and $b_{t+2}^1 = \frac{1}{2}$.

(1d) If $\frac{1}{4} < b_t^1 \leq \frac{1}{3}$, then $b_{t+1}^1 \in (\frac{1}{8}, \frac{1}{6}]$, $b_{t+1}^2 \in (\frac{3}{4}, \frac{5}{6}]$ and $b_{t+2}^1 \in (\frac{1}{2}, \frac{7}{12}]$.

The cases of $\frac{1}{3} < b_t^1 < \frac{1}{2}$ and $\frac{1}{2} \leq b_t^1 < 1$ (the latter one split into three sub-cases) are relegated to the appendix.

The analysis up to here reveals that any 2-styles-distribution in $t$ is followed by a 2-styles-distribution again in $t + 3$ or earlier. We summarize the results from the analysis, including the ones in the appendix, in figure 10. The notation “$0 < b_t^1 \leq \frac{1}{12} \to \frac{1}{4} < b_t^1 \leq \frac{1}{3}$” means that a 2-styles-distribution in $t$ with $0 < b_t^1 \leq \frac{1}{12}$ is followed by a 2-styles-distribution in $t + k$ with $\frac{1}{4} < b_{t+k}^1 \leq \frac{1}{3}$ for some $k \in [1, 3]$, and analogously for all other cases.

Thus, it remains to show that the two sequences of distributions implied by I. and II. where

I. implies

a 2-styles-distribution in $t$ with $b_t^1 = \frac{1}{4}$,

a 2-styles-distribution in $t + 2$ with $b_{t+2}^1 = \frac{1}{2}$,

a 2-styles-distribution in $t + 3$ with $b_{t+3}^1 = \frac{1}{4}$,

and so on, and

II. implies

a 2-styles-distribution in $t$ with $b_t^1 \in (\frac{1}{4}, \frac{1}{3}]$,

a 2-styles-distribution in $t + 2$ with $b_{t+2}^1 \in (\frac{1}{2}, \frac{2}{3}]$,
a 2-styles-distribution in $t + 3$ with $b_{t+3}^1 \in (\frac{1}{4}, \frac{1}{3}]$, and so on,

both converge to $C$.

First, observe that the sequence of distributions implied by I. is equal to $C$. Second, we show that the sequence of distributions implied by II. converges to $C$. Assume w.l.o.g. that the distribution in $t$ is a 2-styles-distribution with $b_t^1 \in (\frac{1}{3}, \frac{5}{6}]$. Hence, in $t + 1$ the distribution is a 3-styles-distribution with $b_{t+1}^1 = \frac{1}{2} b_t^1 \in (\frac{1}{8}, \frac{1}{6}]$ and $b_{t+1}^2 = \frac{1}{2} (2b_t^1 + 1) \in (\frac{3}{4}, \frac{5}{6}]$. In $t + 2$ the distribution is mass equivalent to a 2-styles-distribution with $b_{t+2}^1 = \frac{1}{2} (2b_{t+1}^1 + b_{t+1}^2) = b_t^1 + \frac{1}{4} \in (\frac{1}{2}, \frac{7}{12}]$. The distribution in $t + 3$ is a 2-styles-distribution with $b_{t+3}^1 = \frac{1}{2} b_{t+2}^1 = \frac{1}{2} b_t^1 + \frac{1}{8} \in (\frac{1}{4}, b_t^1)$ because $\frac{1}{4} < b_t^1$. Since $b_{t+3}^1 < b_t^1$, then also $b_{t+4}^1 < b_{t+1}^1$ and $b_{t+4}^2 < b_{t+1}^2$, and $b_{t+5}^1 < b_{t+2}^1$. Thus, $\lim_{k \to \infty} b_{t+3k}^1 = \frac{1}{4}$, $\lim_{k \to \infty} b_{t+3k+1}^1 = \frac{1}{8}$, $\lim_{k \to \infty} b_{t+3k+1}^2 = \frac{3}{4}$ and $\lim_{k \to \infty} b_{t+3k+2}^1 = \frac{1}{2}$ and in the limit the sequence of distributions implied by II. converges to $C$. □

With the previous lemmata, we establish our main result.

**Proposition 3.4.** The dynamics of the distribution of consumption converge to $C$ for any initial 1-style-, 2-styles- or 3-styles-distribution.

**Proof.** By the previous lemmata, any 1-style- and 3-styles-distribution in $t$ is followed by a 2-styles-distribution in $t + k$ for some $k \geq 1$ and the dynamics of the distribution of consumption after any 2-styles-distribution converge to $C$. □

Within cycle $C$, the dynamics of overall demand possess the properties ascribed to fashion cycles. First, the demand for each style follows recurring life cycles, second, there is an ever-going change in fashions because the life cycles of different options peak at different points in time, and, third, within one life cycle of a style, the style “trickles” deterministically through society from individualist/higher status individuals to conformist/lower status individuals. This is synthesized in the following two corollaries and explained in the next paragraphs.
Corollary 3.5. In cycle $C$, the demand for every style follows a unique sequence of market shares $M$, which resembles two consecutive life cycles, in infinite repetition. The sequence of market shares is given by $M = (0, \frac{1}{8}, \frac{5}{8}, \frac{1}{2}, 0, \frac{1}{8}, \frac{1}{2}, \frac{3}{8}, \frac{1}{2})$.

In cycle $C$, each style faces a demand of zero in some period $t$ and in $t+1$ it enters the market with a demand of $\frac{1}{8}$ of the population. In $t+2$, demand for this style grows to $\frac{5}{8}$ and the style is at the peak of its life cycle. In $t+3$, demand for the style declines to $\frac{1}{2}$ before the style is abandoned by every consumer and is not consumed for one period in $t+4$. Having been out of the market for one period, the style sees a revival in $t+5$ and enters into another life cycle experiencing a positive demand for exactly four consecutive time periods. Demand starts out at $\frac{1}{8}$ in $t+5$, increases to $\frac{1}{2}$ in $t+6$ and further to $\frac{3}{4}$ in $t+7$. In $t+8$ demand for the style drops to $\frac{1}{4}$ before it leaves the market again in $t+9$ for one period. Then, the process starts over again; period $t+9$ is equal to period $t$ in the life cycle process. The reappearance of a product in the market after a period of zero demand in our model is due to the limited set of consumption options. Revivals of fashions and brands can be commonly observed in reality. Ray Ban glasses, platform shoes and flares are examples for this.

Corollary 3.5 is consistent with research on product life cycles (for a discussion see Rink and Swan, 1979) which divides the evolution of demand for a product into four stages: the initial phase of introduction with low demand, the growth phase with rising demand, the maturity phase where demand has reached its peak, and finally a phase of decline where demand decreases until the product is out of the market.

The life cycles of different styles peak at different points in time. This reproduces an ever-going change in fashions where one fashion is substituted by another. Figure 11, which plots the demand for each style in cycle $C$ over 9 periods, exemplifies this feature. Given $C$, there exists period $t$ where styles $x$ and $y$ with $x \neq y \neq z \neq x$ each receive $\frac{1}{2}$ of total demand.
Corollary 3.6. Within one life cycle, a style trickles from the “left” to the “right” of the population. In the first life cycle in \( M \), the style is initially consumed by all \( i \leq \frac{1}{4} \), subsequently by all \( i \in [\frac{1}{8}, \frac{3}{8}] \), and final demand comes from all \( i \geq \frac{1}{2} \). In the second life cycle in \( M \), the style is initially consumed by all \( i \leq \frac{1}{8} \), subsequently by all \( i \leq \frac{1}{2} \), then by all \( i \geq \frac{1}{4} \), and final demand comes from all \( i \geq \frac{3}{4} \).

Within one life cycle, each consumption option trickles deterministically through society from the left to the right of the unit interval. Initial demand for a product comes from consumers “at the left” of society, then the product moves over “centre” consumers to consumers “at the right” of society, when the “left” of society has abandoned the product already. Finally the product trickles out of the market. This is in line with the different “trickling” theories for fashion which were reviewed in the introduction.

4 Conclusion

This paper proposes a model to explain fashion cycles through heterogeneous conformity preferences only. The overall demand dynamics for three options
which result from the model resemble fashion cycles which are in line with traditional sociological fashion theories. The demand for each style over time follows consecutive life cycles in each of which demand for the style takes a bell-shaped pattern and the style trickles in a deterministic way through society. Moreover, an ever-going change of fashions arises because the life cycles for different styles peak at different times.

This followed from the result that the demand dynamics converge to a unique cycle $C$ of demand distributions after any 1-style-, 2- and 3-styles-distribution. In cycle $C$, demand over time exhibits the above discussed fashion cycle properties.

We hold a strong conjecture that the demand dynamics possess the same fashion cycle properties if the number of consumption options is generalized to $n$. To prove that the results are robust to increasing the number of consumption options would be an interesting extension. Another promising direction for future research would be to include firms into the model as strategic actors which can set prices and decide about when to introduce a new style.
Appendix

Remainder of the Proof for Lemma 3.2.

1. If $1_t > 2_t = 3_t$, then the $t+1$ distribution is a 2-styles-distribution with $b_{t+1}^1 = \frac{1}{2} (b_t^1 - M_t (\max \{y, z\}))$ and $s_{t+1} = (\max \{y, z\}, x)$.

$$
\begin{array}{c|ccc}
   t & x & y & z \\
   \hline
   0 & b_t^1 & b_t^2 & 1 \\
\end{array}
$$

$$
\begin{array}{c|c}
   t + 1 & \max \{y, z\} & x \\
   \hline
   0 & b_{t+1}^1 & 1 \\
\end{array}
$$

2. If $1_t < 2_t = 3_t$, then the $t+1$ distribution is mass equivalent to a 2-styles-distribution with $b_{t+1}^1 = \frac{1}{2} (2b_t^1 + b_t^2)$ and $s_{t+1} = (x, y)$.

$$
\begin{array}{c|ccc}
   t & x & y & z \\
   \hline
   0 & b_t^1 & b_t^2 & 1 \\
\end{array}
$$

$$
\begin{array}{c|cc}
   t + 1 & x & y \\
   \hline
   0 & b_{t+1}^1 & 1 \\
\end{array}
$$

3. If $1_t = 2_t > 3_t$, then the $t+1$ distribution is a 2-styles-distribution with $b_{t+1}^1 = \frac{1}{2} (b_t^1 - 3_t)$ and $s_{t+1} = (z, x)$ in case $x = \max \{x, y\}$. In case $y = \max \{x, y\}$, the $t + 1$ distribution is a 3-styles-distribution with $b_{t+1} = (0, \frac{1}{2} (b_t^1 - 3_t), b_t^2, 1)$ and $s_{t+1} = (z, x, y)$. Given a 3-styles-distribution in $t + 1$, the relation between population masses in consumption intervals is $1_{t+1} < 2_{t+1} > 3_{t+1}$.
4. If $1_t = 2_t < 3_t$, then the $t+1$ distribution is a 2-styles-distribution with $b_{t+1}^1 = \frac{1}{2} (b_t^2 + 1_t + 1)$, and $s_{t+1} = (x, z)$ in case $x = \max\{x, y\}$. In case $y = \max\{x, y\}$, the $t + 1$ distribution is mass equivalent to a 3-styles-distribution with $b_{t+1} = (0, b_t^2, \frac{1}{2} (b_t^2 + 2_t + 1), 1)$ and $s_{t+1} = (x, y, z)$. Given a 3-styles-distribution in $t + 1$, $2_{t+1} > 3_{t+1}$.

5. If $1_t > 2_t > 3_t$, then the $t + 1$ distribution is a 2-styles-distribution with $b_{t+1}^1 = \frac{1}{2} (b_t^1 - 3_t)$ and $s_{t+1} = (z, x)$. 

\[\begin{array}{c|c|c|c}
  t & x & y & z \\
  \hline
  0 & b_t^1 & b_t^2 & 1 \\

t + 1 & z & x & \max\{x, y\} \\
  \hline
  0 & b_{t+1}^1 & b_{t+1}^2 & 1 \\
\end{array}\]

\[\begin{array}{c|c|c|c|c|c}
  t & x & y & z \\
  \hline
  0 & b_t^1 & b_t^2 & 1 \\

t + 1 & \max\{x, y\} & x & z \\
  \hline
  0 & \frac{1}{2} (b_t^1 + 1) & b_t^2 & \frac{1}{2} (b_t^1 + 1) & 1 \\
\end{array}\]

\[\begin{array}{c|c|c|c}
  t & x & y & z \\
  \hline
  0 & b_t^1 & b_t^2 & 1 \\

t + 1 & z & x & \max\{x, y\} \\
  \hline
  0 & b_{t+1}^1 & 1 \\
\end{array}\]
6. If $1_t > 3_t > 2_t$, then the $t + 1$ distribution is a 2-styles-distribution with $b^{t+1}_1 = \frac{1}{2} (b^t_1 - 2_t)$ and $s_{t+1} = (y, x)$.

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<td>0</td>
<td>$b^{t+1}_1$</td>
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7. If $1_t = 3_t > 2_t$, then the $t + 1$ distribution is mass equivalent to a 2-styles-distribution with $b^{t+1}_1 = \frac{1}{2} (b^t_1 - 2_t)$ and $s_{t+1} = (y, x)$.

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<tbody>
<tr>
<td>0</td>
<td>$b^{t+1}_1$</td>
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8. If $3_t > 1_t > 2_t$, then the $t + 1$ distribution is a 3-styles-distribution with $b^{t+1}_1 = \frac{1}{2} (b^t_1 - 2_t)$ and $b^{t+1}_2 = \frac{1}{2} (b^t_2 + 1_t + 1)$, and $s_{t+1} = (y, x, z)$.

The relation between population masses is $1_{t+1} < 2_{t+1} > 3_{t+1}$.
9. If $2_t > 1_t > 3_t$, then the $t+1$ distribution is a 3-styles-distribution with $b_{t+1}^1 = \frac{1}{2}(b_t^1 - 3_t)$ and $b_{t+1}^2 = \frac{1}{2}(2b_t^1 + b_t^2)$, and $s_{t+1} = (z, x, y)$. The relation between population masses is $1_{t+1} < 2_{t+1} > 3_{t+1}$. Observe that $1_{t+1} < 1_t$ and $3_{t+1} > 3_t$ because $b_{t+1}^1 < b_t^1$ and $b_{t+1}^2 < b_t^2$. Hence, eventually every 3-styles-distribution with $2_t > 1_t > 3_t$ results into a 3-styles-distribution with $2_{t+1} > 3_{t+1} ≥ 1_{t+1}$.

\[
\begin{array}{c|ccc}
 t & x & y & z \\
 \hline
 0 & b_t^1 & & b_t^2 \\
\end{array}
\]

\[
\begin{array}{c|ccc}
 t+1 & z & x & y \\
 \hline
 0 & b_{t+1}^2 & b_{t+1}^2 & 1 \\
\end{array}
\]

10. If $2_t > 1_t = 3_t$, then the $t + 1$ distribution is mass equivalent to a 2-styles-distribution with $b_{t+1}^1 = \frac{1}{2}(2b_t^1 + b_t^2)$, and $s_{t+1} = (x, y)$.

\[
\begin{array}{c|ccc}
 t & x & y & z \\
 \hline
 0 & b_t^1 & & b_t^2 \\
\end{array}
\]

\[
\begin{array}{c|ccc}
 t+1 & x & y \\
 \hline
 0 & b_{t+1}^1 & 1 \\
\end{array}
\]

11. If $2_t > 3_t > 1_t$, then the $t + 1$ distribution is a 2-styles-distribution with $b_{t+1}^1 = \frac{1}{2}(2b_t^1 + b_t^2)$, and $s_{t+1} = (x, y)$.

\[
\begin{array}{c|ccc}
 t & x & y & z \\
 \hline
 0 & b_t^1 & & b_t^2 \\
\end{array}
\]

\[
\begin{array}{c|ccc}
 t+1 & x & y \\
 \hline
 0 & b_{t+1}^1 & 1 \\
\end{array}
\]
12. If $1_t < 2_t < 3_t$, then the $t + 1$ distribution is a 3-styles-distribution with $b_{t+1}^1 = \frac{1}{2} (2b_t^1 + b_t^2)$ and $b_{t+1}^2 = \frac{1}{2} (b_t^2 + 2_t + 1)$, and $s_{t+1} = (x, y, z)$. Thus, $2_{t+1} > 3_{t+1}$.

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<tbody>
<tr>
<td>0</td>
<td>$b_{t+1}^1$</td>
<td>$b_{t+1}^2$</td>
<td>1</td>
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**Remainder of the Proof for Lemma 3.3.**

1. If $\frac{1}{3} < b_t^1 < \frac{1}{2}$, then the distribution in $t + 1$ is a 3-styles-distribution with $b_{t+1}^1 = \frac{1}{2} b_t^1 \in (\frac{1}{6}, \frac{1}{3})$ and $b_{t+1}^2 = \frac{1}{2} (2b_t^1 + 1) \in (\frac{5}{6}, 1)$. Thus, $2_{t+1} > 1_{t+1} > 3_{t+1}$. Moreover, $s_{t+1} = (z, x, y)$.

In $t+2$ the distribution is a 3-styles-distribution with $b_{t+2}^1 = \frac{1}{2} (b_{t+1}^1 - 3_{t+1}) \in (0, \frac{1}{3})$ and $b_{t+2}^2 = \frac{1}{2} (2b_{t+1}^1 + b_{t+1}^2) \in (\frac{7}{12}, \frac{3}{4})$. Thus, $2_{t+2} > 3_{t+2} > 1_{t+2}$. Moreover, $s_{t+2} = (y, z, x)$.

In $t+3$ a 2-styles-distribution is reached again with $b_{t+3}^1 = \frac{1}{2} (2b_{t+2}^1 + b_{t+2}^2) \in (\frac{7}{24}, \frac{1}{2})$.  

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Thus, a 2-styles-distribution in $t$ with $\frac{1}{3} < b_t^1 < \frac{1}{2}$ results into a 2-styles-distribution with either $\frac{1}{3} < b_{t+3}^1 < \frac{1}{2}$ again or $\frac{1}{4} < b_{t+3}^1 \leq \frac{1}{3}$ in the short run; in the long run, however, it results into a 2-styles-distribution with $\frac{1}{4} < b_{t+3}^1 \leq \frac{1}{3}$ with certainty because $b_{t+3}^1 < b_t^1$ for the 2-styles-distribution in $t + 3$ following from any 2-styles-distribution with $\frac{1}{3} < b_t^1 < \frac{1}{2}$ in $t$.

2. If $\frac{1}{2} \leq b_t^1 < 1$,

then the distribution in $t+1$ is mass equivalent to a 2-styles-distribution with $b_{t+1}^1 = \frac{1}{2} b_t^1$. Moreover, $s_{t+1} = (z, x)$. 

\begin{center}
\begin{tabular}{c|c|c}
$t$ & $x$ & $y$ \\
0 & $b_t^1$ & 1 \\

$t+1$ & $z$ & $x$ \\
0 & $b_{t+1}^1$ & $b_{t+1}^2$ & 1 \\

$t+2$ & $y$ & $z$ \\
0 & $b_{t+2}^1$ & $b_{t+2}^2$ & 1 \\

$t+3$ & $y$ & $z$ \\
0 & $b_{t+3}^1$ & 1
\end{tabular}
\end{center}
Again, it is useful to further divide this case into sub-cases and provide the possible values of the consumption interval boundary $b_{t+1}^l$ for each sub-case.

(a) If $b_t^l = \frac{1}{2}$, then $b_{t+1}^l = \frac{1}{4}$.

(b) If $\frac{1}{2} < b_t^l \leq \frac{2}{3}$, then $b_{t+1}^l \in (\frac{1}{4}, \frac{1}{3})$.

(c) If $\frac{2}{3} < b_t^l < 1$, then $b_{t+1}^l \in (\frac{1}{3}, \frac{1}{2})$. 
References


URL: http://ssrn.com/abstract=1828848


**URL:** https://doi.org/10.1509/jmr.15.0119