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Abstract

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1 Introduction

Wages at the top of the distribution have been rising sharply in the United States since the early 1970s. Top 10% wage share¹ increased from 25.7% in 1970 to 34.5% in 2010 in the U.S. (Piketty (2014)). One of the reasons for the rise of top wages and wage dispersion is improved matching between firms and employees in the top positions² (Song et al. (2018)). Why did the matching improve? The conventional view attributes the improvement in matching to skill-biased technological change which raises incentives for firms and workers to be better matched. However, while this explains the rise of the upper-middle class, it cannot explain the sharp rise of top wages. Song et al. (2018) show a strong non-linearity in the sorting pattern³: a disproportional shift of high-skilled workers to high-paying firms in comparison to medium-skilled workers to medium-paying firms, and this cannot be generated by technological progress in models of labor markets.

In this paper, I show that the improved matching at the top is due to decreasing search frictions in the labor market for top positions. I develop a model where frictions are reduced by the increasing role of headhunters, or executive search firms. Headhunters started to gain market share in the U.S. in the 1970s and now assist in filling more than half of the positions in the top wage segment. They enhance matching for two reasons. First, they provide more suitable candidates for the firm because they can screen the candidates better. Second, they induce passive on-the-job search as they contact potential candidates directly, creating opportunities for new matches without active search from employed workers⁴. As a result, the headhunters restrict the pool of potential candidates to only the high-skilled workers while, at the same time, expand the pool of potential candidates to a larger number of those high-skilled workers. These two features result in better matching between high-skilled workers and firms, a higher surplus and, therefore, a higher wage in such matches. Because headhunters operate mostly on the top wage segment, such improvements in matching do not happen (or happen to a lower degree) over the rest of the distribution. Therefore, the presence of headhunters generates a strong non-linearity in sorting improvement that leads, in turn, to soaring of top wages compared to the rest of the distribution. There is a third feature of headhunters, they often play a role in salary negotiations (for example, via compensation consultancy). This may affect the wage in one direction or another, depending on the headhunter incentives.

¹The share of total wages that goes to the top 10% of all employees.

²Alternative explanations of increasing wage inequality include: i) decrease in top income taxes - Alvaredo et al. (2013); ii) direct effects of skill-biased technological change on wages - Acemoglu (2002); Autor, Katz and Kearney (2006); and many others; iii) social norms - Piketty (2014); iv) exogenous changes in random growth theories - Jones (2015); Gabaix et al. (2016); Aoki and Nirei (2017); Jones and Kim (2017); v) numerous studies on the increase of CEO pay including Gabaix and Landier (2008); Lemieux, MacLeod and Parent (2009); and Bell and Reenen (2013) among others.

³Bagger, Sørensen and Vejlin (2013) document similar findings using Danish data. They find that the correlation between worker and firm fixed effects increased from -0.07 in 1981 to 0.14 in 2001. For the top quartile of workers the correlation increased from -0.20 to 0.37, while the correlation stayed almost unchanged at around zero for the rest of the quartiles.

⁴Faberman et al. (2017) show that a significant share of unsolicited contacts and offers go to employed workers not looking for another job.

The headhunter might push the wage up to increase the fee from this assignment but might also to push the wage down to insure a long-term contract with the firm. In this paper, I will focus only on the first two features when modeling the headhunter industry. The focus is on the effects on wages of improved matching rather than on changes in wages due to bargaining or rent seeking.

To quantify the contribution of better matching induced by headhunters to the increase in top wages, I develop a labor market model along the lines of Pissarides (1985) augmented with heterogeneous workers and firms⁵. I introduce the headhunter industry by adding a new channel for matching workers and firms. Firms with an open position can either post a vacancy as in the standard model or hire through a headhunter. The difference for the firm is that it cannot screen workers coming through vacancies, while the headhunter guarantees a minimal skill level of the worker with whom the firm is matched. Consider now the worker's side. Low-skilled workers have access to the standard channel and they can search from both unemployment and employment. Every worker searching through the standard channel has to pay a per-period search cost and, therefore, search "actively". For high-skilled workers, instead, on top of the active search, there is also a possibility of "passive" search. A worker is searching passively if she agrees to consider an offer when a headhunter calls. Screening and passive search match exactly the two main features of the headhunter industry. To abstract from the third feature of headhunters in the data, potential role in compensation negotiations, I use a simple sharing rule for wage setting in the baseline model. The reason for this is that I want to isolate the effects of matching on wage distribution.

Having set up the model, I apply the following calibration strategy. First, I calibrate the model without headhunters to match moments of the wage distribution and aggregate labor market moments in the U.S. in the 1970s. The key calibrated parameters include those characterizing the exogenous distributions of workers over skills and firms over productivity. The U.S. labor market in the 1970s is well approximated by the model with no, or limited, role for headhunters. Having fixed the parameters not related to the headhunters, I then introduce the headhunter channel to the model and calibrate the related parameters to target the moments of the headhunter industry in the 2010s. At the same time, I introduce skill-biased technological change to match the increase in the 90/50 wage ratio from 1970 to 2010. I do it by increasing the degree of complementarity in the production function. Having both mechanisms in the model allows me to evaluate the relative contribution of each mechanism to the change occurring on different parts of the wage distribution.

My calibration strategy answers the question how would the distribution of wages (and, therefore, also the top wages) have changed between the 1970s and 2010s if skill-biased technological change and headhunters had been the only factors raising top wage inequality. To assess the relative contribution of the two factors, I then shut down one channel at a time: the skill-biased technological change or the headhunter channel. I

⁵Other studies including two-sided heterogeneity in the labor market include Shimer and Smith (2000); Postel-Vinay and Robin (2002); Teulings and Gautier (2004); Gautier and Teulings (2015) and many others.

also give a chance to skill-biased technological change to explain all the increase in wage inequality without the headhunters. To do that, I change the increase in the degree of complementarity to match the increase in the top 10% wage share or the 90/50 wage ratio and assess how the model fits other moments. To further study the mechanisms I then exploit the richness of the model and compare several statistics related to the wage distribution with and without the headhunters. Importantly, I perform experiments in line with Song et al. (2018) and compare results from the model-generated data to the U.S. data. This allows me to see whether the improvement in matching in the model has similar features to the observed improvement.

The main quantitative result of the paper is that the rise of headhunters accounts for 69% of the increase in the top 10% wage shares in the U.S. from the 1970s to 2010s. Skill-biased technological change contributes to another 22% of the top 10% wage share increase, and interaction between the two factors raises the top 10% share by 11%⁶. The sharp increase of top wages in the model is mainly due to improved matching after the introduction of headhunters. Comparing joint distributions of worker-firm matches in the two steady states reveals a pattern similar to empirical results of Figure 8 of Song et al. (2018)⁷, where most types of firms lose the highest-skilled workers and where the highest-paying firms gain those workers disproportionately. The headhunter channel generates the strong non-linearity in the change in assortative matching observed in the data, with a disproportionate improvement in matching for highest-skilled workers. I am not aware of other theoretical models able to generate such non-linearity. If I allow the model to match the increase in the top wages without the headhunter channel, the model overshoots the 90/50 wage ratio. This happens exactly because of the absence of a strong non-linearity of the skill-biased technological change. An increase in complementarity shifts the whole distribution of wages to the right. The non-linearity generated by the headhunters allows shifting the right tail of the distribution farther apart from the rest of the distribution without changing the shape of the distribution in the middle⁸.

The model relates to other theoretical models showing the importance of assortative matching for wage distribution. Bagger and Lentz (2017) is the closest study. They

⁶The numbers for the top 1% wage share are 35%, 11% and 22%, respectively. The remaining 47% of the increase in the top 1% wage share can be potentially attributed to the third feature of headhunters not studied in this paper.

⁷Estimating a fixed effect regression is only one way to evaluate sorting in the labor market. Eeckhout and Kircher (2011) show potential problems with identifying sorting with estimated fixed effects. Studies using non-parametric techniques, instead, find a higher degree of sorting than in the studies using fixed effects regressions. Hagedorn, Law and Manovskii (2017) find the correlation between worker and firm ranks to be 0.75 in Germany, Lise, Meghir and Robin (2016) who find significant sorting for college-graduates in the U.S., and Borovičková and Shimer (2017) who find a correlation between 0.4 and 0.6 in Austria. Schulz and Lochner (2016) show, using non-parametric techniques, that sorting increased in Germany between 1998 and 2008.

⁸The rise of headhunters can be also viewed as a reason for the shift in the mean income growth rates for high-skilled workers in the model of Gabaix et al. (2016). Gabaix et al. (2016) introduce an exogenous increase in mean income growth rate for some workers in 1980s, motivated by globalization and technological change. Headhunters, who allow high-skilled workers to work for the top firms, generate the increase in income growth rate for high-skilled workers due to better matches.

show that on-the-job search is a crucial mechanism to generate assortative matching in a Diamond-Mortensen-Pissarides model with two-sided heterogeneity. Bagger and Lentz (2017) consider only active on-the-job search. Uren and Virag (2011), instead, show that skill requirements are important to generate an increase in between-group inequality (increased differences between wages of workers with different skill level). Skill requirements play a similar role as does the screening by headhunters. Uren and Virag (2011) study the overall shape of the wage distribution, while this paper focuses on headhunters and the top part of the distribution.

This paper presents two blocks of independent empirical evidence supporting the mechanism. First, it uses cross-country differences in the use of headhunters in Europe in 1997 to show that in countries where headhunters were used to a larger extent the top income shares increased by more in the following years. This evidence is in line with the prediction of the model that the more the high-productive firms use headhunters, the better is the improvement of matching at the top, and the higher are the top wages. Second, the paper uses micro-level data on CEO compensation of listed companies in the U.S. to study the effects of a change of the CEO on CEO compensation in the company. The main result is that firms pay significantly more to new CEOs comparing to the previous ones, and this difference is higher during the periods when headhunters are used more intensively and in the states where there are less legal obstacles to the activity of headhunters⁹. These results suggest that headhunters do improve matching between firms and CEOs and, therefore, increase wages at the top.

The remainder of the paper is organized as follows. Section II presents the theoretical model. Section III presents the quantitative results. Section IV discusses the available empirical evidence about headhunters and headhunter industry. Section V provides the empirical evidence of cross-country differences in the patterns of top income shares over the last thirty years and their relation to the headhunter industry. Section VI presents empirical results of the effects of changing a CEO on the compensation paid by the firm. Section VII concludes.

2 Model

2.1 Environment

The economy is populated by a continuum of workers and firms. Workers differ in their skill level, e , and supply one unit of labor if employed. When a worker is unemployed she receives an unemployment benefit, $b(e)$. Firms differ in their productivity level, p . Each firm can hire one worker. Both workers and firms discount their future utility with a discounting rate β .

There are two channels for matching workers and firms. First, there is the standard, or vacancy, channel where every worker and firm can participate by paying a per-period cost.

⁹The legal obstacles are proxied by the enforceability of non-compete agreements as proposed by Garmaise (2011).

Second, there is the headhunter channel where every firm and only high-skilled workers, those with skill above a threshold, \hat{e} , can participate. To participate in the headhunter channel firms have to pay a per-period cost, while workers pay the search cost only if they are matched. Workers and firms participating in each channel are randomly matched by a standard CRS matching technology.

All workers, unemployed and employed, can search for a job. Each period workers decide whether to search for a job through vacancies (search actively) and/or to be available for a headhunter company (search passively) if her skill is higher than a threshold \hat{e} . Firms, instead, can choose only one of the channels to search for a worker. In the baseline model, the wage in a match is determined period by period as a fraction of resulting production of the match. The production of a match depends on the firm's productivity level and the worker's skill level via a supermodular production function. Separation of matches depends on two factors: idiosyncratic exogenous separation shock, s ; and endogenous worker's quit rate, $s_Q(\cdot)$. There is no aggregate uncertainty.

2.2 Timing

The time is discrete. Inside every period, first, exogenous separations happen. Then, workers and firms decide in which markets to participate, and new firms decide whether to enter the market. After that, workers searching for a job and firms searching for a worker are matched. Finally, existing matches produce and wages and unemployment benefits are paid.

2.3 Matching

The two channels are labeled as the vacancy, V , and the headhunter, H , channels. In channel $i \in \{V, H\}$ workers and firms meet via a standard CRS matching technology: $m_i = m_i(u_i + a_i, v_i)$, where m_i is the number of matches, u_i and a_i are the numbers of unemployed and employed workers participating in the channel, respectively, and v_i is the number of firms participating in the channel. Define the market tightness of channel i , θ_i , as $\theta_i = \frac{v_i}{u_i + a_i}$. The job finding rate for a worker using channel i is $f_i(\theta_i) = \frac{m_i(u_i + a_i, v_i)}{u_i + a_i} = m_i(1, \theta_i)$ and the job filling rate for a firm is $q_i(\theta_i) = \frac{m_i(u_i + a_i, v_i)}{v_i} = \frac{m_i(1, \theta_i)}{\theta_i}$.

2.4 Wages and Production Technology

In the baseline model, the wage is proportional to the match productivity: $w(e, p) = \psi \cdot y(e, p)$ with $0 < \psi < 1$. Where $y(e, p)$ is increasing and quasi-concave in both components, that is $y'_p > 0$, $y'_e > 0$, $y''_{pp} \leq 0$ and $y''_{ee} \leq 0$. Moreover, $y(e, p)$ has a property of supermodularity having positive cross-derivatives: $y''_{ep} > 0$, $y''_{pe} > 0$. Supermodularity is necessary for complementarity between the worker's skill and the firm's productivity that creates incentives for positive assortative matching.

2.5 Worker Problem

Every worker can be either employed or unemployed. An unemployed worker with skill e consumes the unemployment benefit, $b(e)$, and searches for a job in the next period. The value of unemployment, $U(e)$, can be written as:

$$U(e) = b(e) + \beta(U(e) + S_U(e)), \quad (1)$$

where $S_U(e)$ is the value of search for an *unemployed* worker.

A worker with skill e employed in a firm with productivity p consumes the wage this period, and next period the match can be exogenously separated with probability s , in which case the worker becomes unemployed, or with probability $(1 - s)$ the match survives and the worker can continue to search on-the-job. The value of work, $W(e, p)$, is:

$$W(e, p) = w(e, p) + \beta(sU(e) + (1 - s)(W(e, p) + S_E(e, p))), \quad (2)$$

where $S_E(e, p)$ is the value of search for an *employed* worker.

The value of search is different for workers with different skill level as only the high-skilled workers have a chance to be contacted by a headhunter. Consider first the problem of a low-skilled unemployed worker. The *low-skilled unemployed worker* is excluded from the headhunter channel so the only choice that she has is between searching through the vacancy channel and not searching. Low-skilled worker's value of search can be written as:

$$S_U(e) = \max\{S_{UV}(e), 0\}, \text{ if } e < \hat{e}, \quad (3)$$

where $S_{UV}(e)$ is the value of search through the vacancy channel for an unemployed worker.

For the *high-skilled unemployed worker*, the problem is the same but she chooses among four options: search through vacancies, wait for a headhunter call, do both, or be inactive. The value of the search of a high-skilled unemployed worker can be written as:

$$S_U(e) = \max\{S_{UV}(e), S_{UH}(e), S_{UVH}(e), 0\}, \text{ if } e \geq \hat{e}, \quad (4)$$

where $S_{UH}(e)$ is the value of search through the headhunter channel and $S_{UVH}(e)$ is the value of search through both channels for an unemployed worker.

When an unemployed worker is searching through the vacancy channel, with probability $f_V(\theta_V)$ she will receive an offer from a firm with productivity p that will be drawn from a distribution of firms posting vacancies. The worker then decides whether to accept the offer and receive the difference between the value of employment in this firm, $W(e, p)$, and unemployment, $U(e)$, or to stay unemployed and have no gain. To participate in the channel, the worker has to pay the search cost, $c_{wV}(e)$, every period of active search. Therefore, the value of search through the vacancy channel for an unemployed worker is the following:

$$S_{UV}(e) \equiv f_V(\theta_V) E_{p|V}[\max\{W(e, p), U(e)\} - U(e)] - c_{wV}(e).$$

The value of search through the headhunter channel, or passive search, differs in four respects: offer arrival probability, $f_H(\theta_H)$; search cost, $c_{wH}(e)$; the search cost is paid only if the offer arrives; and the offer is drawn from a different distribution (distribution of firms using headhunters). It is assumed that every eligible worker decides whether to agree to consider an offer in case of a headhunter's call in the beginning of the period before the offer is materialized. The worker will have to pay the search cost (spend time on the interviews or risk being penalized by current employer) only if she receives the call that period. The value of search through the headhunter channel is the following:

$$S_{UH}(e) \equiv f_H(\theta_H) (E_{p|H} [\max \{W(e, p), U(e)\} - U(e)] - c_{wH}(e)).$$

The value of search through both channels is just a combination of the two, with an implicit assumption that better firms are using the headhunter channel¹⁰. The value function is the following:

$$S_{UVH}(e) \equiv f_H(\theta_H) (E_{p|H} [\max \{W(e, p), U(e)\} - U(e)] - c_{wH}(e)) \\ + f_V(\theta_V) (1 - f_H(\theta_H)) E_{p|V} [\max \{W(e, p), U(e)\} - U(e)] \\ - c_{wV}(e).$$

Consider an *employed worker*. She also decides whether to participate in the channels but has a different outside option. Because the worker can always stay in the current firm, the value of search now depends also on the productivity of the current employer, p . Similarly to a low-skilled unemployed worker, a *low-skilled employed worker* may choose between searching through the vacancy channel and not searching at all, with the value of search being:

$$S_E(e, p) = \max \{S_{EV}(e, p), 0\}, \text{ if } e < \hat{e}, \quad (5)$$

where $S_{EV}(e, p)$ is the value of search through the vacancy channel for an employed worker.

A *high-skilled employed worker* may choose again among four options: search through the vacancy channel, search through the headhunter channel, search through both channels, or not search at all. The value of search can be written as:

$$S_E(e, p) = \max \{S_{EV}(e, p), S_{EH}(e, p), S_{EVH}(e, p), 0\}, \text{ if } e \geq \hat{e}, \quad (6)$$

where $S_{EH}(e, p)$ is the value of search through the headhunter channel and $S_{EVH}(e, p)$ is the value of search through both channels for an employed worker.

The values of search through a particular channel differ from the ones for an unemployed worker due to a different outside option - an employed worker can always stay with the current employer if the new match is with a less productive firm. The three values of search, therefore, are:

$$S_{EV}(e, p) \equiv f_V(\theta_V) E_{p'|V} [\max \{W(e, p'), W(e, p)\} - W(e, p)] - c_{wV}(e), \\ S_{EH}(e, p) \equiv f_H(\theta_H) (E_{p'|H} [\max \{W(e, p'), W(e, p)\} - W(e, p)] - c_{wH}(e)), \\ S_{EVH}(e, p) \equiv f_H(\theta_H) (E_{p'|H} [\max \{W(e, p'), W(e, p)\} - W(e, p)] - c_{wH}(e)) \\ + f_V(\theta_V) (1 - f_H(\theta_H)) E_{p'|V} [\max \{W(e, p'), W(e, p)\} - W(e, p)] \\ - c_{wV}(e).$$

¹⁰This assumption will be satisfied in the equilibrium.

2.6 Firm Problem

Firms with vacant positions also need to choose a channel through which to find a worker. Unlike workers, all firms solve the same problem (regardless of their productivity level) and they can choose only one channel. The value of a vacant job is defined as:

$$V(p) = \max \{V_V(p); V_H(p)\}, \quad (7)$$

where $V_V(p)$ is the value of hiring through the vacancy channel and $V_H(p)$ is the value of hiring through the headhunter channel.

If the firm decides to post a vacancy, it pays the per-period cost $c_{fV}(p)$ and is matched with a worker with probability $q_V(\theta_V)$. The worker will be drawn from the distribution of workers searching through the vacancy channel. The worker will accept the new match with probability $P(A)$. It happens when the worker doesn't have a better offer at the same period and if she works in a firm with lower productivity¹¹ (if searching on-the-job). If the match is formed, the firm receives the difference between the value of a job with a worker with skill e , $J(p, e)$, and the value of a vacancy. The value of hiring through the vacancy channel for a firm is:

$$V_V(p) = -c_{fV}(p) + \beta (V(p) + q_V(\theta_V) E_{e|V} [P(A) (J(p, e) - V(p))]). \quad (8)$$

Similarly, if a firm decides to hire through the headhunter channel, it pays the per-period cost $c_{fH}(p)$ and is matched with a worker with probability $q_H(\theta_H)$. The worker will be drawn from the distribution of workers searching through the headhunter channel. The value of hiring through the headhunter channel is:

$$V_H(p) = -c_{fH}(p) + \beta (V(p) + q_H(\theta_H) E_{e|H} [P(A) (J(p, e) - V(p))]). \quad (9)$$

The value of a job is standard. The firm receives the product of the match, pays the wage and in the next period the match may be separated due to an exogenous shock or due to an endogenous worker's quit to another firm. The value of a job can be written as:

$$J(p, e) = y(e, p) - w(e, p) + \beta ((s + s_Q(\cdot)(1 - s)) V(p) + (1 - s_Q(\cdot))(1 - s) J(p, e)). \quad (10)$$

There is an ex-ante free entry condition. Firms do not know their level of productivity before entering the market. The firm draws its productivity from an exogenous distribution after paying the entry cost, F . The free entry condition for the firms is the following:

$$E_p [V(p)] = F. \quad (11)$$

¹¹As each firm hires only one worker, the internal promotion cannot be modeled explicitly. One could interpret the hires through the vacancy channel as internal promotions, in this case, the worker accepts the new position if the position is of a higher rank than her current position. Even though the firms in the model have different identities they can be a part of a large corporation that owns these firms/positions.

2.7 Steady-State Separating Equilibrium

In this section I discuss a particular structure of the equilibrium. Given supermodularity of the production function, the most reasonable equilibrium is the one where high-productive firms would hire through the headhunter channel, while low-productive firms would use the vacancy channel. Such equilibrium requires some assumptions on the cost functions, production function, and initial distributions of workers and firms. I discuss these assumptions in the Appendix. It is assumed throughout the section that such assumptions hold. In the numerical exercises I verify that such assumptions do hold under the baseline calibration.

2.7.1 Distributions

First, I need to specify distributions that will be used in expectations. Let $F(p)$ be the initial distribution of firm productivity and $G(p)$ the measure of firms with an open vacancy, both have support $[\underline{p}, \bar{p}]$. Denote as \hat{p} the cutoff level of firm productivity such that firms with productivity above \hat{p} hire through the headhunter channel and firms below \hat{p} hire through the vacancy channel. Also, let $H(e)$ be the initial distribution of workers over skill, $L_V(e)$ be the measure of employed workers searching for a job through the vacancy channel, $L_H(e)$ the measure of employed workers searching for a job through the headhunter channel, $L_{VH}(e)$ the measure of employed workers searching for a job through both channels, and $U(e)$ the measure of unemployed workers over the skill level (all with support $[\underline{e}, \bar{e}]$). Finally, let $\Phi(e, p)$ be the joint measure of active matches and $\Lambda_i(e, p)$ be the measure of active matches in which a worker is searching for a new job through channel $i \in \{V, H, VH\}$.

2.7.2 Workers

Given the structure of the equilibrium under consideration and the distributions defined above, we can now specify the expectations.

Under our assumptions, low-skilled workers are excluded from the headhunter channel so they can search only through the vacancy channel. The value of search is:

$$S_U(e) = S_{UV}(e) \equiv f_V(\theta_V) \int_{\underline{p}}^{\hat{p}} (W(e, p) - U(e)) dG(p) - c_{wV}(e). \quad (12)$$

For high-skilled unemployed workers it is optimal to search through both channels. Their value of search is then:

$$S_U(e) = S_{UVH}(e), \quad (13)$$

and the exact expression for $S_{UVH}(e)$ is defined in the Appendix. Under our assumption, better firms use the headhunter channel. If an unemployed high-skilled worker receives an offer through the headhunter channel she will accept it regardless of receiving an offer through the vacancy channel or not. Instead, this worker will accept an offer from the vacancy channel only if she doesn't receive an offer through the headhunter channel.

For employed workers the value of search is the same as the value of search for unemployed, except from the outside option. The value function of search will be, as before:

$$S_E(e, p) = \max \{S_{EV}(e, p); 0\}.$$

We can define the value of search through the vacancy channel for a low-skilled employed worker as:

$$S_{EV}(e, p) \equiv f_V(\theta_V) \int_{\underline{p}}^{\hat{p}} \max \{W(e, p') - W(e, p); 0\} dG(p') - c_{wV}(e). \quad (14)$$

Because the worker will accept offers only from more productive firms, the value can be rewritten as:

$$S_{EV}(e, p) \equiv f_V(\theta_V) \int_p^{\hat{p}} (W(e, p') - W(e, p)) dG(p') - c_{wV}(e).$$

To search from employment, the value of search for a worker with skill e and working in a firm with productivity p must be positive:

$$S_{EV}(e, p) \geq 0.$$

This equation (when satisfied with equality) implicitly determines the level of the firm productivity such that a worker with a skill level e does not search for a new job: $\tilde{p}_V(e)$ (for $e < \hat{e}$). If a worker with skill e works in a firm with productivity below $\tilde{p}_V(e)$, she searches for another job and doesn't search otherwise.

For a high-skilled employed worker the value of search consists of four options but in this structure of equilibrium one of them (searching only through vacancies) will never be optimal¹². The value of search can be defined as:

$$\begin{aligned} S_E(e, p) &= \max \{S_{EV}(e, p); S_{EH}(e, p); S_{EVH}(e, p); 0\} \\ &= \max \{S_{EH}(e, p); S_{EVH}(e, p); 0\}. \end{aligned}$$

For a high-skilled worker with skill level e there are now two cutoff productivity levels $\tilde{p}_{VH}(e)$ and $\tilde{p}_H(e)$, with $\tilde{p}_H(e) \geq \tilde{p}_{VH}(e)$. If the worker is employed in a firm with productivity below $\tilde{p}_{VH}(e)$ she will search for another job through both channels. If she works in a firm with productivity level between $\tilde{p}_{VH}(e)$ and $\tilde{p}_H(e)$, she will search only through the headhunter channel. If she works in a firm with productivity above $\tilde{p}_H(e)$, she will not search for another job at all. Before defining the conditions that determine these cutoffs, we need to define the value functions.

The value of search through the headhunter channel for a high-skilled worker can be defined as:

$$S_{EH}(e, p) \equiv f_H(\theta_H) \left(\int_{\max\{\hat{p}; p\}}^{\bar{p}} (W(e, p') - W(e, p)) dG(p') - c_{wH}(e) \right). \quad (15)$$

¹²See Appendix.

The value of search through both channel is defined in the Appendix.

It is easy to show that, given e , $S_{EVH}(e, p)$ is higher than $S_{EH}(e, p)$ for small p , but $S_{EVH}(e, p)$ decreases faster, so they will always have just one intercept. The equality:

$$S_{EVH}(e, p) = S_{EH}(e, p)$$

defines the cutoff productivity level of searching through both channels for each worker type, $\tilde{p}_{VH}(e)$, while the equality

$$S_{EH}(e, p) = 0$$

defines the cutoff productivity level for searching only through the headhunter channel, $\tilde{p}_H(e)$.

The value functions of work and unemployment are defined as before.

2.7.3 Firms

We can also rewrite the values of hiring through the vacancy and the headhunter channels given distributions defined above. The exact expressions are presented in the Appendix.

One can show that under reasonable conditions on the values of the search costs and production function, the value of hiring through a headhunter, $V_H(p)$, is lower than the value of posting a vacancy, $V_V(p)$, for small p but $V_H(p)$ is increasing faster with p ¹³. Thus, there will be only one intercept between $V_H(p)$ and $V_V(p)$, \hat{p} , such that

$$\max \{V_V(p); V_H(p)\} = V_V(p)$$

for $p < \hat{p}$ and

$$\max \{V_V(p); V_H(p)\} = V_H(p)$$

for $p > \hat{p}$. The cutoff productivity is determined by

$$V_V(\hat{p}) = V_H(\hat{p}).$$

Finally, the value of an active match for a firm is defined as before, with the exact expression for the quit rate defined in the Appendix.

2.7.4 Equilibrium

The steady-state equilibrium, given the initial distributions of workers over skill and firms over productivity, the exogenous skill threshold, the matching functions, and the production and wage functions, is defined by the value functions, the endogenous distributions, and the decision rules. The decision rules must be consistent with the value functions. The value functions must be consistent with the endogenous distributions. Endogenous distributions must satisfy the balances given the decision rules.

The balances guarantee that the equilibrium distribution is stationary over time. In the equilibrium, the inflow of workers to every worker-firm distribution bin must be equal

¹³See Appendix.

to the outflow of workers from that bin. The equilibrium density of active matches for a pair of workers with skill e and firms with productivity p , $\phi(e, p)$, must satisfy:

$$\phi(e, p) (s + s_Q(e, p) (1 - s)) = i_E(e, p) + i_U(e, p), \quad (16)$$

where the left-hand side is the total outflow from active matches (exogenous plus endogenous separations), and the right-hand side is the total inflow into the matches from employment, $i_E(e, p)$, and unemployment, $i_U(e, p)$. The inflow rates are defined in the Appendix.

2.8 Solution Method

To find a steady state equilibrium I first guess the decision rules. Given the decision rules, I solve the system of balances equations using non-linear solution methods (trust-region or Broyden methods). The solution to the system is the stationary distribution of active matches. Then I compute the rest of endogenous distributions given the distribution of active matches and exogenous initial distributions of worker and firm types. Given distributions, I can compute the value functions for workers and firms using value function iteration. Finally, I compute new decision rules based on the value functions. I iterate these steps until convergence.

2.9 Extensions

The most important extension of the model is introduced to capture the fact that not all firms hire employees for top positions through headhunters in the data. In the baseline model, every firm above the threshold \hat{p} hires through the headhunters. I introduce an additional idiosyncratic cost for hiring through the headhunter channel. Every firm with productivity above \hat{p} and an open position draws a cost c_{fN} that is paid only if the firm wants to hire through the headhunter channel¹⁴. This cost might reflect corporate practice, an existence of a preferred candidate inside the firm, or specificity of the position. A firm with a high cost will have to post a vacancy even if it would prefer to hire through the headhunter channel absent the cost. Updated value of an open position, $\tilde{V}(p)$, can be written as:

$$\tilde{V}(p) = \max \{V_V(p); V_H(p) - c_{fN}\}.$$

This extension doesn't alter the model significantly¹⁵ but brings it closer to the data. Because the proportion of top firms using headhunters depending on productivity is unobservable, the distribution of the costs will be chosen so that the top firm (that would hire through the headhunter channel without the idiosyncratic cost) has the same chance to hire through the headhunter channel regardless of the productivity.

Other important extensions include different wage setting mechanisms (wage bargaining), and explicit modeling of headhunter's problem (choice of the threshold and the cost

¹⁴It is assumed that this cost is always large enough for firms with productivity below \hat{p} that the firm will never choose to hire through the headhunter channel.

¹⁵Equations for the value functions and the balances are presented in the appendix.

for the firm). These extensions are very important for understanding the headhunter industry and wage setting at the top, but they are beyond the focus of this paper, that is the effects of matching on the wage distribution. Therefore, these extensions are left to be discussed in the appendix.

2.10 Calibration

The calibration strategy is the following. First, the version of the model without the headhunter channel is calibrated to match the labor market in the U.S. in the 1970s. Then the parameters related to the headhunter channel are calibrated to match the moments of the headhunter industry in the U.S. in the 2010s. To take into account the skill-biased technological change from the 1970s to 2010s, I also change the degree of complementarity in the production function to match the increase in the 90/50 wage ratio.

To calibrate the model, I need to specify the exogenous distributions of workers, $H(e)$, and firms, $F(p)$, the productivity function, $y(e, p)$, the matching function, $m(u, v)$, the skill threshold, \hat{e} , the search costs, $c_{fH}(e)$, $c_{fV}(e)$, $c_{wH}(e)$, $c_{wV}(e)$, and the distribution of the idiosyncratic cost c_{fN} ¹⁶. The functional form for the initial distributions of firms and workers is chosen to be beta with the same parameters, λ_1 and λ_2 , for workers and firms, and truncated on $\bar{p} = \bar{e}$. The functional forms are presented in the Appendix.

All parameters not related to the headhunter channel are calibrated to match the wage distribution and other moments of the labor market in the 1970s¹⁷. There are seven parameters to calibrate for the steady state without the headhunters: the parameters of the distribution, λ_1 and λ_2 , the maximum type, \bar{p} or \bar{e} , the exogenous separation rate, s , the matching function efficiency, M , and the vacancy channel search costs for workers, c_{wV} , and for firms, c_{fV} . Seven targets are chosen to set these seven parameters: the top 1% and 10% wage shares, the 90/50 wage ratio, the unemployment rate, the job finding rate, the quit rate, and the estimate of vacancy cost relative to annual worker's wage. Seven parameters are jointly calibrated to match the targets.

For the headhunter channel, there are other four parameters to calibrate - the search costs for workers, c_{wH} , and firms, c_{fH} , the skill threshold, \hat{e} , and the share of top firms using headhunters, χ ¹⁸. Four targets are chosen - the estimate of the positive response rate by managers to a call by a headhunter, the average fee of headhunters, the range of positions filled by headhunters, and the share of firms that use the headhunters. The targets for the headhunter channel are taken from Cappelli and Hamori (2013). Cappelli and Hamori (2013) estimate that around 54% of managers say "yes" when a headhunter calls and asks if the manager is willing to consider an offer. They also present the evidence for the size of the average headhunter fee and the range of firms hiring through the headhunters. As the targets for the headhunter channel are not precisely estimated

¹⁶Wage share of the production, ψ , does not affect the shape of the wage distribution but only scales it. The value used in the simulations is 0.7. Matching function elasticity is fixed at 0.5 throughout the simulations.

¹⁷For data on targets see Appendix.

¹⁸It is equivalent to calibrating a distribution function for the idiosyncratic cost c_{fN} .

in the data, I present robustness checks for the choice of the targets in the Appendix. On top of the headhunter channel, I also increase the degree of complementarity, γ , in order to match the change in the 90/50 wage ratio between the 1970s and 2010s.

The results of the calibration are presented in Table 1. The model matches well the main characteristics of the wage distribution and the labor market in the 1970s.

Table 1: Calibrated Parameters

Parameter	Value	Target	Data	Model
Wage distribution, 1970s				
Beta parameter, λ_1	1	Top 1% wage share	5.1%	4.82%
Beta parameter, λ_2	19.5	Top 10% wage share	25.7%	26.18%
Maximum types, \bar{p}, \bar{e}	20	90/50 wage ratio	1.91	2.08
Labor market, 1970s				
Separation rate, s	0.027	Unemployment rate	5%	5%
Matching function, M	0.55	Job finding rate	50%	50%
Search cost - vacancies, c_wV	2	Quit rate	2%	1.98%
Vacancy cost, c_fV	0.36	Vacancy cost estimates	8%	8%
Headhunter industry, 2010s				
Headhunter search cost, c_wH	8.3	Positive response rate	50%	51.3%
Headhunter firm cost, c_fH	11.7	Headhunter average fee	30%	30%
Screening threshold, \hat{e}	2.8	Range of positions	top 5%	5.25%
Share of top firms using HH, χ	0.54	Share of firms	~54%	54%
Skill-biased technological change, 1970s-2010s				
Degree of complementarity, γ	1.09	Δ 90/50 wage ratio	0.39	0.39

3 Results

3.1 Inequality

The headhunter channel changes the wage distribution. Without the headhunter channel, the distribution has a peak close to the minimal possible wage and then decreases, having a form close to Pareto (Figure 1a). When the headhunter channel is present in the model, the distribution still has a similar form, but it has a fatter right tail (Figure 1b), similar to what is observed in the data. The headhunter channel generates the fat tail of the wage distribution in this model. The reason for this is the following. Without the headhunter channel the probability of matching a high-skilled worker with a high-productive firm is lower than matching a high-skilled worker with a low-productive firm (due to the fact that there are relatively few high-productive firms), so there will be large shares of high-skilled workers working in low-productive firms and low-skilled workers in high-productive firms. Wages of low-skilled workers are lower than wages of high-skilled workers in the same type of firm. And because only some high-productive firms will be matched with high-skilled

workers without the headhunter channel, there will be a small mass of workers getting very high wages. When, instead, there is a possibility to hire only high-skilled workers through the headhunter channel, high-productive firms will be matched only with high-skilled workers and all of them will receive relatively high wages; this corresponds to the fat tail of the distribution.

There are two effects changing the wage distribution in this case - headhunters and skill-biased technological change. Skill-biased technological change increases wages of all workers and therefore moves the whole distribution to the right. To see the effect of only the headhunter channel on the wage distribution, Figure 1c plots the difference between the distributions without the effects of the skill-biased technological change. An interesting observation about the effect of headhunters on wage distribution can be done - the headhunter channel generates an effect similar to job polarization, namely, decrease in the number of medium-paying jobs and increase in the number of high- and low-paying jobs. This effect comes from the fact that low-skilled workers move from high-productive to low-productive firms (from the center to the left), and high-skilled workers move from low-productive firms to the high-productive firms (from the center to the right). The difference between the distributions also clearly indicates the appearance of a fatter right tail with the headhunter channel.

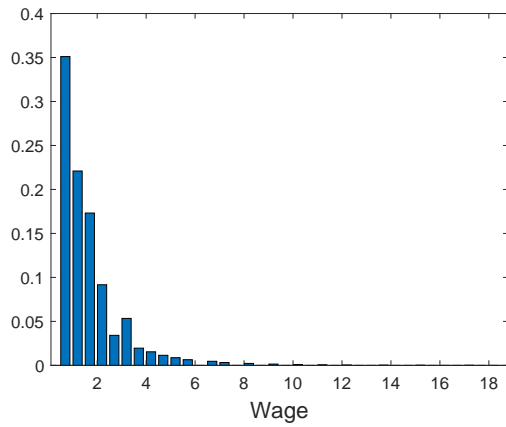
As it was stated before, the increase in wage inequality was mainly driven by the sharp increase in top wages. The top 1% wage share increased from 5.1% in 1970 to 10.9% in 2010 in the U.S., and the top 10% share increased from 25.7% to 34.5%. The shares in 1970 were targeted in the calibration but the shares in 2010 were not. The results of this experiment show how much of the overall increase in top wages can be explained by the additional channel in the labor market and an increase in the degree of complementarity in production. The results are presented in Table 2. In the model, the top 1% share increases by 3.09%, from 4.82% to 7.91%, while in the data it increases by 5.8%. The model is able to explain 53% of the actual increase in the top 1% wage share. For the top 10% wage share, the model predicts a 9.03% increase, while the actual increase is 8.8%. The model accounts for 103% of the actual increase in the top 10% wage share.

Table 2: Top Wage Shares in the Model and Data

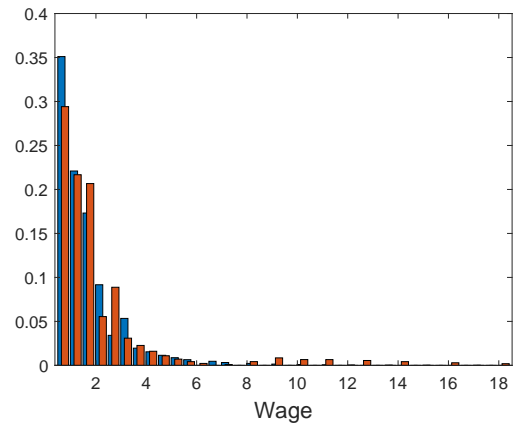
Model	Top 1%	Top 10%	Data	Top 1%	Top 10%
Without HH	4.82%	26.18%	1970	5.1%	25.7%
With HH	7.91%	35.21%	2010	10.9%	34.5%

3.2 Skill-Biased Technological Change

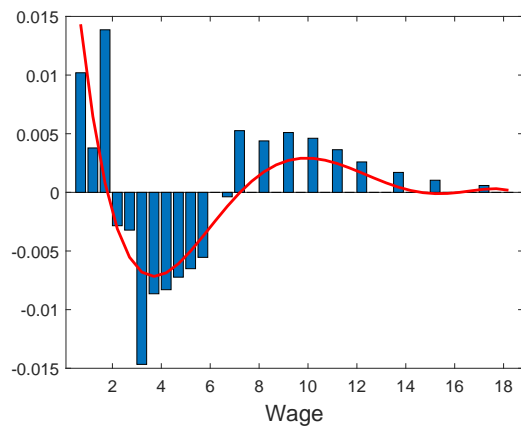
The large effect in Table 2 comes from the skill-biased technological change and the headhunter channel acting together. To assess the relative contributions of the headhunter channel and the skill-biased technological change to the increase of the top wages, I change separately only the matching technology (by adding the headhunter channel) or the degree of complementarity (SBTC). I present the results in the Table 3. First, I fix the degree



(a) Distribution of Wages without the Headhunter Channel



(b) Distribution of Wages with the Headhunter Channel



(c) Difference between the Distributions without SBTC

Figure 1: Distributions of Wages

of complementarity on the level of 1970 but add the headhunter channel (bottom-left panel). In this case, the top 1% wage share is 6.63%, instead of 7.91% in the baseline calibration (upper-left), and the top 10% wage share is 32.22% (instead of 35.21%). The headhunter channel alone contributes to 35% of the increase in the top 1% wage share and 69% of the increase in the top 10% wage share in the data. The headhunter channel also contributes to half of the rise of the upper-middle class, the 90/50 wage ratio rises by 0.21, while in the baseline calibration the rise is calibrated to be 0.39.

Table 3: Relative Contribution of Headhunters and SBTC

		HH	no HH
SBTC	Top 1%	7.91%	5.44%
	Top 10%	35.21%	28.14%
	Δ 90/50	0.39	0.14
no SBTC	Top 1%	6.83%	4.82%
	Top 10%	32.22%	26.18%
	Δ 90/50	0.21	0

If, instead, I increase the degree of complementarity to the level of the baseline calibration without the headhunter channel (upper-right), the top 1% wage share increases just to 5.44% and the top 10% wage share increases to 28.14%. The relative contribution of the degree of complementarity is about 11% (out of 53% in the baseline) for the top 1% wage share, and 22% (out of 103%) for the top 10% wage share. SBTC also explains one-third of the increase of the 90/50 wage ratio.

The interaction between SBTC and the headhunter channel is also very important. The interaction explains around 7% of the increase in top 1% wage share (53%-35%-11%) and 10% of the increase in top 10% wage share (103%-69%-22%). With a higher degree of complementarity the importance of having a better match increases. Relative productivity of a firm with a high-skilled worker is even higher with respect to a similar firm with a low-skilled worker in case of high degree of complementarity. Better assortative matching reinforces the effects of SBTC.

To give a chance to SBTC to explain a higher proportion of the rise in top shares, I recalibrate the SBTC to match the increase in the top 10% wage share or the 90/50 wage ratio without the headhunter channel. I present the results in Table 4. We can see that the degree of complementarity must increase up to 1.23 without the headhunter channel to match the increase in 90/50 wage ratio, and up to 1.38 to match the top 10% wage share. When I match the 90/50 wage ratio, the model explains a smaller proportion of the increase in top 1% wage share (30%) and the top 10% wage share (50%). If I match the top 10% wage share, instead, the model explains a larger part of the top 10% wage share (all of it was targeted, while the headhunter channel alone explains 70%) and a similar increase in the top 1%. However, in this case, the model predicts a very large increase in the 90/50 wage ratio (0.67) that is 59% higher than the actual increase. The reason for this is that the rise of the degree of complementarity alone raises all the wages and the rise must be enormous to match the top 10% wage share. We can see it in Figure

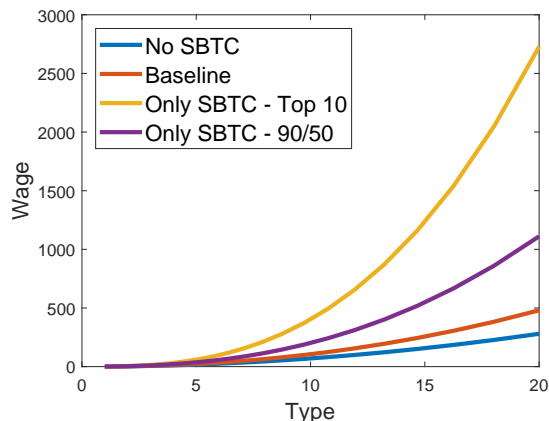


Figure 2: Wages of Workers in the Best-Fit Firms

2. With the headhunter channel all the wages rise only slightly due to a higher degree of complementarity (movement of the curve) and the top wages rise more than that due to improvements in the assortative matching (movement along the curve). With the headhunter channel the high-skilled workers move up with the curve and move along it to the right, and without the headhunter channel they can only move up with the curve.

Table 4: Alternative Calibration of SBTC

	Data	Baseline	no HH, SBTC - 90/50	no HH, SBTC top 10
	2010	$\gamma = 1.09$	$\gamma = 1.23$	$\gamma = 1.38$
Top 1%	10.9%	7.91%	6.54%	7.91%
Top 10%	34.5%	35.21%	30.61%	34.97%
Δ 90/50	0.39	0.39	0.39	0.67

These experiments show that skill-biased technological change helps to explain the rise in the 90/50 wage ratio that corresponds to the rise of the upper-middle class relative to the bottom but fails to replicate the sharp increase of the top wages. The headhunter channel, instead, has the main effect on the top wages, rather than on the upper-middle class. Skill-biased technological change stretches the whole distribution to the right, while the headhunter channel fixes the left part of the distribution and moves the right tail further away.

3.2.1 Income Growth

To further demonstrate the lack of sufficient non-linearity of SBTC I compare the distribution of income growth rates in different specifications of the model. Figure II in Piketty, Saez and Zucman (2018) shows the average annual income growth for earners in each percentile of the U.S. population from 1980 to 2014. The figure demonstrates that the

growth rates increase slightly with income for most of the distribution and then explode for top income earners. Solid line on Figure 3 plots corresponding annualized growth rates for wages in the model with baseline calibration (with headhunters and SBTC). The shape of the line at the top percentiles is very similar to the data with headhunters generating exploding top incomes. The scale is different possibly due to lack of “average” technological growth in the model and focus on only the labor part of income, both likely benefiting richer workers more. Figure 3 also shows similar distributions for alternative calibrations of the model. It is evident that SBTC alone is not able to reproduce the exploding pattern observed in the data.

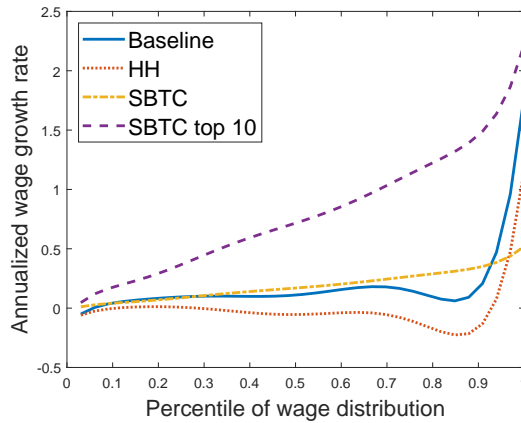
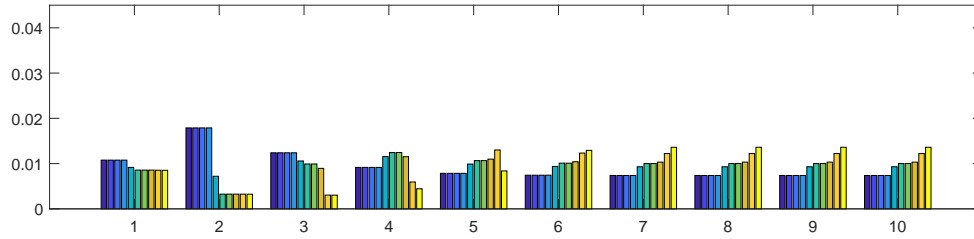


Figure 3: Wage growth by percentile of wage distribution, model-generated data, calibration comparison

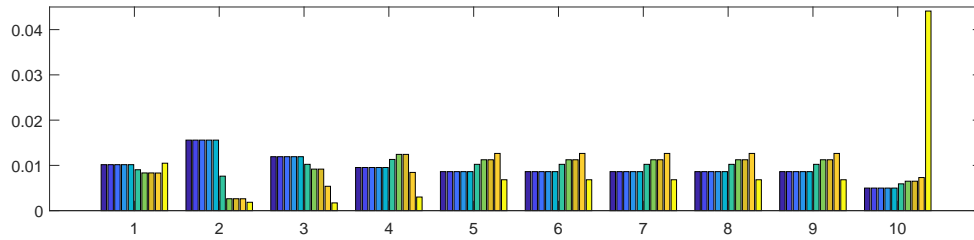
3.3 Assortative Matching

The main mechanism behind the increase in the wage inequality in the model is the increase in sorting between workers and firms, especially at the very top. With headhunters, high-skilled workers have an exclusive opportunity to be matched with high-productive firms, and high-productive firms, instead, have an exclusive opportunity to be matched with high-skilled workers. Empirically, there are two widely used ways to look at the assortative matching between workers and firms. First, one can directly compare the joint distributions of worker-firm matches over estimated types. And second, one can just look at the correlation between the types. I compute both statistics using the data simulated from the model in the baseline calibration in order to compare them to empirical estimates in the literature. The major drawback of this experiment, however, is that I can observe the real type of workers and firms directly, while in the data it is impossible.

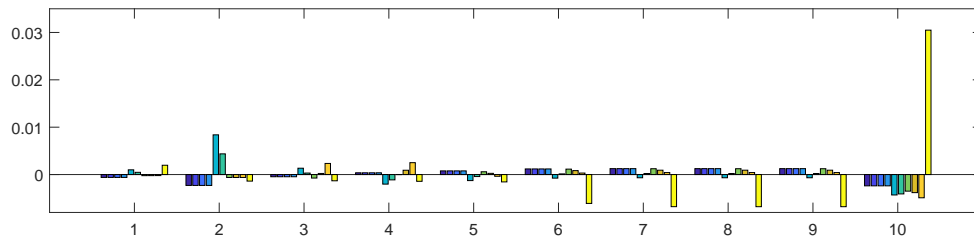
First, I study the change in the joint distribution of worker-firm matches. To do it I split workers and firms into ten categories by their skill or productivity level and plot the joint distribution before and after introducing the headhunter channel. Figure 4a shows



(a) Joint Distribution without the Headhunter Channel



(b) Joint Distribution with the Headhunter Channel



(c) Change in the Joint Distribution

Figure 4: Joint Distributions of Worker-Firm Matches

the distribution without the headhunter channel, Figure 4b shows the distribution with the headhunter channel, and Figure 4c shows the change of the distribution. Numbers 1,2,3,...,10 in the figure correspond to the firm types, with 1 being the least productive firms and 10 being the most productive firms, and the colors correspond to the types of workers, with dark blue being the least skilled and yellow being the most skilled workers.

As it can be seen from the figures, almost all high-skilled workers (within the top 10%) move to the best firms (top 10%). All other firms lose significantly in the share of top workers and gain in the share of lower-skilled workers. This pattern is strikingly similar to the findings of Song et al. (2018) who plot similar distributions for workers and firms fixed effects estimated on the U.S. data. Comparing to the data, the model exaggerates the increase in the number of top workers working in top firms. In the data, there is an error in estimating the true type of the worker that may result in smoothing the figure. Furthermore, the headhunters might make mistakes when assessing the skill of workers,

therefore smooth the resulting difference even more.

The second way to analyze sorting in the labor market is by computing correlations between the types of workers and firms. In order to do it, I draw 100000 matches from the joint worker-firm distribution in the model and decompose the variance of log wages into worker type, firm type, and covariance between the two. Table 5 presents the results of this experiment for the steady-state without headhunters, with headhunters, and the difference between the two.

Table 5: Log-Wage Variance Decomposition and Correlation of Worker and Firm Types

	Without HH	With HH	Difference	Share
$Var(\log(w))$	0.2948	0.4065	0.1117	100%
$Var(\log(e))$	0.1256	0.1482	0.0226	20%
$Var(\log(p))$	0.1235	0.1467	0.0232	21%
$2Cov(\log(e), \log(p))$	0.0456	0.1117	0.0661	59%
$Cor(\log(e), \log(p))$	0.1834	0.3789	0.2023	-

We can see that covariance and correlation of the worker and firm types increase significantly after the introduction of the headhunter channel. Indeed, the increase is not only in the top part of the distribution but over the whole distribution. This happens because the high-skilled workers that move to the best firms free the positions for low- and medium-skilled workers in the rest of the firms also improving the matching for them. Again, these results are in line with the empirical results by Song et al. (2018) who show that increased covariance between worker and firm types was one of the major drivers in the increase in the variance of wages in the U.S. between the 1980s and 2000s.

Improvement in assortative matching naturally translates to a higher aggregate production in the economy. Better matches at the top significantly improve average productivity. In the baseline calibration, the aggregate production increases by 19.5% driven by better matching and SBTC. The improvement in productivity due to matching alone is naturally smaller. Better matching leads to a 5.7% increase in production with technology at the level of 1970 and to a 7.2% increase with technology at the level of 2010. It is clear that stronger complementarity makes matching more important and, therefore, the increase in production is higher for 2010.

4 Headhunter Industry

Individual headhunters are typically focused on a specific position or industry and collect detailed databases with information on the majority of potential candidates for such position or industry. With this detailed information already collected, when asked to assist to fill a position, they can choose the best fitting candidate and improve matching. As the headhunters already possess the information on the majority of candidates, they are more efficient in screening than firms. Firms could carry out the selection without the help of a headhunter but they would have to pay the screening cost, that can be

immensely high, just to hire one candidate (say a CEO). Headhunters, using the same database to place candidates in different companies, spread the screening cost across many hires, therefore improving efficiency. After a headhunter chooses a candidate from its database, it calls the candidate directly and asks whether she wants to consider a job offer (without specifying the offer). The headhunter contacts any candidate who is perceived to be the best fit for the position without the candidate having to signal interest in changing her job. A worker who has not put effort into receiving an offer from a headhunter and who agrees to consider the offer is essentially searching passively on the job¹⁹. Passive search helps high-skilled workers not to get stuck for long in positions that do not fit them, moving them to a better fitting position and improving matching.

The main question is how big is the headhunter industry. To determine the exact market share of a very closed and private industry is a difficult task. Few headhunter companies release the number of hires in a given year. To overcome the shortage of data, one can use IACPR report (2003) that claims that 54% of the positions above \$150,000 a year were filled by headhunters in 2003. Another way to determine the market share of the headhunters is to compare the estimates of the fee revenues to the ones implied by the total wage bill. Total fee revenues in the U.S. are around \$4.6 billion as estimated by the Association of Executive Search and Leadership Consultants (AESC). It is possible to compute the total wages that go to the top 5% of the U.S. employees using the top 5% wage share. Then, using the hiring rate, one can determine total wages that go to the new hires in a given year. Headhunters receive a fee of around 30% of the first year wage paid to the new hires. It is possible to determine what would be the aggregate fee revenues for a given market share of headhunters. Given the average hiring rate of 3.5%, the share of headhunters in the labor market for positions in the top 5% must be around 15%, to be consistent with the estimates by AESC. However, the hiring rate at the top is, in general, much lower than in the lower-paying jobs, with tenures being significantly longer. With a more realistic hiring rate, the implied market share of headhunters is more than 30% but still below 54% estimated by IACPR.

How are the revenues distributed? Headhunters cover a wide range of positions: CEOs, board directors, CFOs, senior executives, general management, top professionals in finance and control, information systems, marketing, and sales. They are not focused only on CEOs, as is sometimes perceived, but cover almost all the top positions in companies. The industry composition of headhunter operations is also dispersed; they operate in all industries as illustrated by the distribution of fee revenues. Distribution of fee revenues by industry in the 4th quarter of 2015 is presented in Table 6.

Geographical distribution of fee revenues, instead, is not so homogeneous, as can be seen in Table 7. Headhunters receive most of the revenue from North America, and mainly the U.S. Europe is lagging behind and the major part of European revenues comes from the U.K. There might be several reasons for such difference. One possible reason is the difference in labor market legislation. It is more difficult to be an intermediary in a European labor market than in the U.S. Another possible reason is the cost of creating

¹⁹Cappelli and Hamori (2013) show that more than half of executives are willing to consider an offer when a headhunter calls them.

Table 6: Fee Revenues of Headhunters by Industry, 4th Quarter 2015, from AESC

Industry	Share (%)
Industrial	23.5
Financial	21.0
Consumer products	18.0
Technology	16.2
Life Sciences/Healthcare	15.2
Non-Profit	4.4
Other	1.7

Source: AESC Insights Q4 2015 State of the Executive Search Industry.

a database of potential candidates in a new country. The headhunter must know the specifics of the labor market and the companies operating in the country in order to understand the skills demanded by companies as well as the value of observable signals, such as particular diplomas and experience in particular companies. Because headhunters first appeared in the U.S. they established the databases and acquired the knowledge of the labor market and the signals there first.

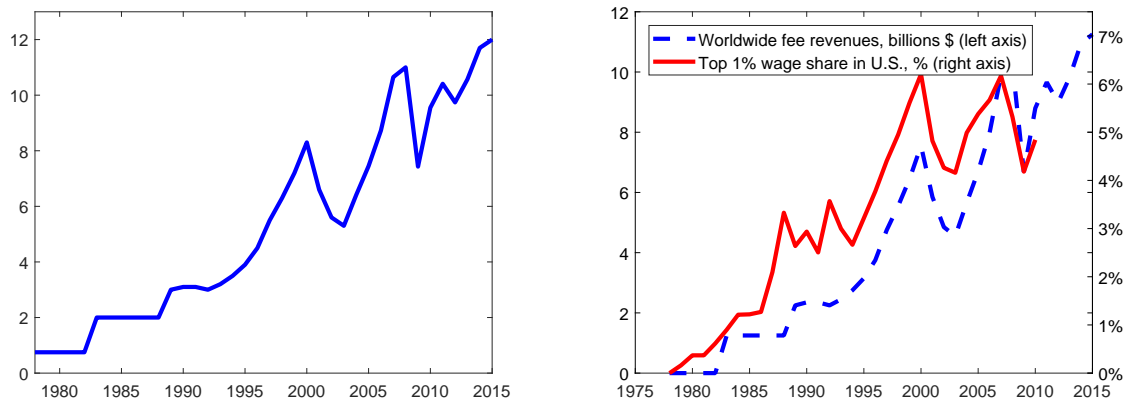
Table 7: Fee Revenues of Headhunters by Region, 4th Quarter 2015, from AESC

Region	Share (%)
North America	45.5
EMEA	33.2
Asia Pacific	16.5
Latin America	4.8

Source: AESC Insights Q4 2015 State of the Executive Search Industry.

The history of the rise of headhunters started in the U.S. already in the 1950s. However, the first decades were not very successful for them. Only in the late 1970s and early 1980s, the industry started to expand sharply with worldwide fee revenues rising from \$0.75 billion in 1978 to \$3.9 billion in 1990. The fee revenues kept rising up to \$12 billion in 2015 (Figure 5a). This rise was partly mechanical because top wages were also rising over the same period (Figure 5b), but the revenues increase was much larger in proportion to the increase of the wages. Another indicator of the expansion of the industry is the number of hires by headhunters. For example, the number of assignments of one of the historical leaders of the industry Korn/Ferry increased from 42 in 1969 to 8,480 in 2015 according to the financial statements.

There might be several reasons for the rise of headhunters. The most important reason is the technological progress in IT and communication that increased the quality of the services provided by headhunters. Development and growing availability of computers reduced the costs of managing and searching through databases of potential candidates. Communication technology (mobile phones and emails), instead, made it easier to contact potential candidates and allowed headhunters to expand their networks of potential



(a) Estimated Worldwide Fee Revenues of Headhunters (b) Cumulative Change in Headhunters Fee Revenues and the Top 1% Wage Share

Figure 5: Estimated Worldwide Fee Revenues of Headhunters and Top Wages, Calculated from AESC and Piketty (2014)

candidates. Companies that adopted new technologies earlier were more successful²⁰. Another effect of technology goes through the demand side: better technology made it easier to apply for jobs (especially in the late 1990s and 2000s). More applications increased the amount of information that the firms had to evaluate to hire a worker, and the higher was the position, the more information was there to evaluate. It became more efficient for the firms to delegate the screening of applicants for top positions to intermediaries - the headhunters - and the demand for headhunter service increased. One more potential reason for the rise that goes through the demand side is related to the nature of the skills required from employees in the top positions. Because of technological change, globalization, or change of company structure, it became more important for the firm to hire employees with higher general skill in comparison to 1970s. Firms started to use headhunters more because in the 1990s the skill of the CEO, for example, affected the performance of the company much more than in 1970s. Even though there is more evidence in support of the supply story, this paper doesn't exclude other reasons for the rise of headhunters. The actual reason for the rise doesn't play a big role in the results because of the nature of the experiments studied in this paper.

²⁰Jenn (1995) writes: "The drive towards a more consistent quality of service throughout the world has been greatly assisted by the application of information technology to the search business and the use of global databases. Technological advances have allowed firms to search more widely and communicate more efficiently. Virtually all executive search firms are attempting to modernise their communications and database systems on a global basis. ... This is the area where the search world is changing most dramatically. Firms have a tremendous opportunity to improve their efficiency, achieve better margins and differentiate themselves from their competitors."

5 Cross-Country Comparison

Headhunters entered labor markets of different countries in different periods and therefore were used by firms to a different extent²¹. This variation in headhunters' activity allows me to argue about the causality between the role of headhunters and the growth of top wages. This section uses the data on the major headhunter companies in European labor markets in 1997 and the data on top income shares between 1980 and 2010. Ideally, top wage shares should be used in the analysis, however, such data is not available for all countries over the whole period in question.

Data on major headhunter companies operating in Europe in 1997 is available in Jenn (1999). The data includes the distribution of fee revenues as well as the number of hires by country²². I aggregate the data by country to get total fees and a total number of hires by headhunters in a country in 1997. I normalize the data by GDP in 1997 (for fee revenues) or total employment in 1997 (for hires). The normalization allows comparing the share of headhunters between countries. The question that this analysis answers is what is the relation between the headhunter activity and the dynamics of top incomes? To answer this question, Figure 6²³ plots the relations between normalized hires by headhunters and the top 10% income share, or growth of the top 10% income share. Similar relations for the top 1% income share as well as for normalized fee revenues are presented in the Appendix. Figure 6a shows that there is a strong positive correlation between normalized hires by headhunters and the future growth of the top 10% income share. Only Norway falls from the general pattern. Norway experienced a change in capital income taxation in 2006, so most likely this drop is not related to labor incomes.

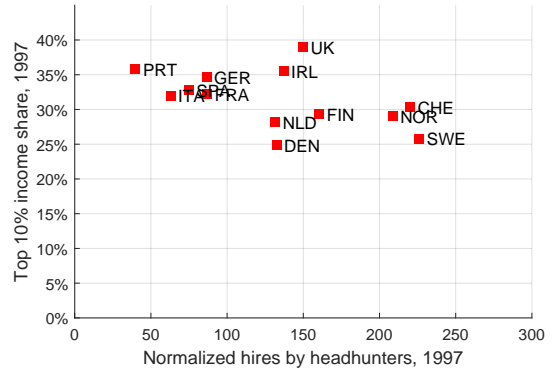
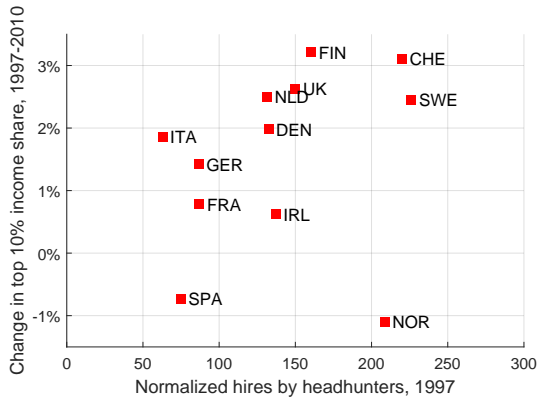
To address the concern that headhunters were more active in countries where top wages were already higher, Figure 6b plots the relation between top income shares in 1997 and normalized hires by headhunters in 1997. As it is evident from the figure, there is no correlation between headhunter activity and top income shares in 1997. It means that differences of headhunter intensity across countries are driven by other factors, exogenous to top income levels. To further strengthen this claim, Figure 6c plots the growth of top income shares before 1997 against normalized hires in 1997. Lack of positive correlation shows that headhunters intensity is not driven by the previous growth of top incomes. Headhunters didn't choose countries with fast-growing top incomes.

All the results for normalized hires hold also for normalized fee revenues. This analysis shows two important facts. First, it suggests that headhunter activity, indeed, signals the future growth of top incomes. In the model, the increase of top incomes happens because of improved matching at the top, with headhunters inducing the better matching. This evidence, however, doesn't provide any hints on the mechanism of the top incomes increase, or the degree of the quality of the matching. Second, this analysis suggests that the distribution of headhunter activity over countries is exogenous to the level of

²¹For example, due to different labor market legislation, language barriers and other institutional reasons.

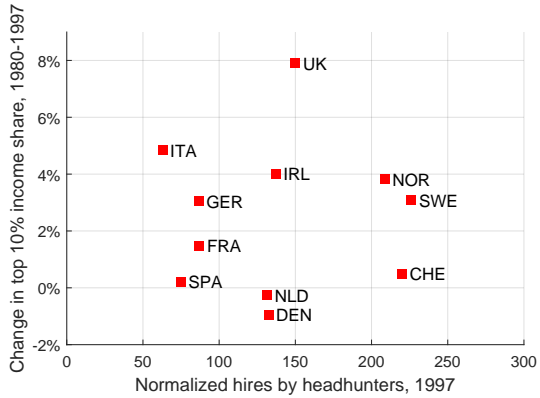
²²The number of hires is estimated and not exactly observed for some companies.

²³Sources: <http://www.wid.world/>, Jenn (1999), <https://data.oecd.org/>, and author's calculations.



(a) Top 10% Income Share Growth, 1997-2010, and Normalized Hires by Headhunters, 1997

(b) Top 10% Income Share, 1997, and Normalized Hires by Headhunters, 1997



(c) Top 10% Income Share Growth, 1980-1997, and Normalized Hires by Headhunters, 1997

Figure 6: Top 10% Income Share and Normalized Hires by Headhunters, Calculated from Jenn (1999), OECD, and WID

top incomes and the history of the growth of top incomes. There must be other factors limiting headhunter activity in some countries, for example, labor market legislation or higher costs of establishing detailed databases of potential candidates.

The importance of this empirical evidence is in demonstrating the lack of reverse causality. Headhunters might be more active in countries where the income inequality was higher so they came to the markets to extract higher fee revenues. In this case, the increasing top wages would drive the rise in the headhunter industry, and the mechanism presented in this paper would not be present. However, the results presented in this section suggest that only the future change in top incomes is correlated with the headhunter intensity, so reverse causality can be rejected.

6 Micro evidence

This section presents the empirical analysis of the potential effect of headhunters on the CEO compensation. CEOs constitute a major part of the hires by headhunters, accounting for 20000 hires by headhunters in 2013 in the U.S. alone, and therefore are a good proxy for individual effects of headhunters on the matching between workers and firms.

6.1 Data and Estimation

The data on CEO compensation and the firm level data are obtained from COMPUSTAT dataset. In particular, following Gabaix, Landier and Sauvagnat (2014), the variable TDC1 of EXECUCOMP panel is used to measure CEO compensation. TDC1 includes salary, bonus, restricted stock granted and the Black-Scholes value of stock options granted. Also following Gabaix, Landier and Sauvagnat (2014), four proxies for the firm size will be used: firm value, equity value, sales, and income. All four proxies are constructed from variables obtained from COMPUSTAT yearly dataset²⁴. Industry dummies are constructed using the four-digit SIC industry codes as in Fama and French (1997). A dummy variable for a change of the CEO is constructed such that it is equal to 0 if the CEO is the same as the CEO of the first observation of the company, and 1 otherwise:

$$NewCEO_{i,t} = \begin{cases} 1 & \text{if CEO is different from the first observation of the firm} \\ 0 & \text{otherwise.} \end{cases}$$

Another important variable that will be analyzed is the index of enforceability of non-competition constructed by Garmaise (2011). The index is higher in the states where the non-compete agreements are enforced by courts and low in the states where the non-compete agreements are forbidden. Non-compete agreements restrict job-to-job transitions for workers and therefore limit the activity of headhunters.

The following sample will be used. The time period analyzed is from 1993 to 2013. The analysis will be restricted only to the CEO of every U.S. based company in the dataset.

²⁴Detailed description is in the appendix

If a firm changes the CEO more than once during the sample period, all observations starting from the third CEO are dropped. These restrictions leave 3102 firms with 7.95 average years of observation.

I estimate the following equation:

$$\log(TDC1_{i,t}) = \alpha * NewCEO_{i,t} + \beta * \log(Firm\ size_{i,t}) + FE_t + FE_i + \varepsilon_{i,t},$$

where $TDC1_{i,t}$ is the CEO compensation in firm i and year t , $NewCEO_{i,t}$ is the dummy variable constructed as described above, and $Firm\ size_{i,t}$ is one of the four measures of the firm size of firm i and year t .

6.2 Results

Table 8 presents the results of the estimation using the full sample as well as two sub-samples when headhunter fee revenues were increasing particularly fast (as seen from Figure 5a) - 1993 to 1998 and 2004 to 2007. Columns (1) and (2) present the results of the estimation with firm fixed effects and with or without the year fixed effects for the full sample. The results show that after a company changes the CEO it pays her from 5% to 16% more than to the previous CEO controlling for the firm size. The effect is even stronger if we focus on two sub-samples with the fast industry growth. The coefficient increases from 5% to 9% in the first sub-period (column (3)) and from 5% to 13,6% in the second sub-period (column (4)). This can be viewed as an indirect evidence of a higher use of headhunters during those periods and, therefore, better improvements in the matching between CEOs and firms resulting in higher compensation.

Table 8: CEO Compensation and the Change of the CEO

	Full Sample		1993-1998	2004-2007
Log of compensation	(1)	(2)	(3)	(4)
New CEO	0.1579 (0.0296)	0.0495 (0.0180)	0.0906 (0.0315)	0.1364 (0.0382)
Firm Size Controls	Yes	Yes	Yes	Yes
Year FE	No	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes
R^2	0.66	0.66	0.724	0.785
Number of observations	24673	24673	8304	4505

The important question is what is the channel of this effect, why are firms paying more to new CEOs? One potential explanation can be that the new CEO has a higher bargaining power than the previous CEO, it can be the case especially if the new CEO was hired with the help of a headhunter while the previous CEO was not. To test this channel, I augment the estimated equation with the interaction term between the $NewCEO$ dummy variable and the measures of the firm size. The coefficient of the measure of the firm size in this regression can be viewed as the bargaining power of

the CEO, i.e. how much his compensation increases when the firm is growing, and the interaction term can be viewed as the change in the bargaining power between the new and the previous CEOs. The results (presented in the Appendix) show that the interaction term is negative or not significantly different from 0. These results suggest that the increase of CEO compensation does not come from a higher bargaining power.

In the model, matches are better with headhunters, productivity is higher and drives the wages up. To explore the matching channel, I add the non-compete enforceability index to the analysis. In Table 9 I present the results of the estimation where I add the interaction between the non-compete enforceability index and the new CEO dummy. The interaction term increases the magnitude of the effect of the new CEO dummy and has a negative and significant coefficient by itself. It means that the effect of the change of the CEO on the compensation is higher in the states with low non-compete enforceability index and it decreases with a higher index. It is also interesting to notice that in the states where the index would be 1 (the highest index is 0.9 in Florida) the overall effect of the change of the CEO on the compensation would be negative.

Table 9: CEO Compensation, New CEOs, and the Non-Compete Enforceability Index

	Sample period 1993 - 2013		
Log of compensation	(1)	(2)	(3)
NCEI*New CEO	-0.1685 (0.0475)	-0.2377 (0.0477)	-0.1805 (0.0471)
New CEO	0.1245 (0.0282)	0.0778 (0.0190)	0.0663 (0.0191)
Firm Size Controls	Yes	Yes	Yes
Year FE	No	Yes	Yes
Industry FE	Yes	No	Yes
R^2	0.409	0.381	0.419
Number of observations	24217	24217	24217

6.3 Discussion

The empirical results presented in this section suggest that the matches between CEOs and companies improve over time in the U.S. The improvement in the matching results in higher CEO compensation and larger size of the firms. The fact that the matching is improving over time supports the mechanism discussed in the paper. Headhunters provide better matches for firms and CEOs increasing the firm size and the CEO compensation.

Of course, there are shortcomings in this empirical specification because we don't know which CEOs are hired through a headhunter and which CEOs come from internal promotion or other channels. To address this issue directly, one needs to collect the data on the origins of the CEO and the way she was hired. Such study would be able to analyze the difference between CEO compensation for a CEO coming through headhunters and

not. Most importantly, it would be also able to determine the effect of a CEO hired by a headhunter on the firm performance. However, such datasets are not available at this moment.

To try to overcome the lack of data on identities of the CEOs hired by headhunters, I use non-compete enforceability index as a proxy for the probability to be hired by a headhunter. In the states with a high NCEI, activity of headhunters is limited and, therefore, very few positions are filled by headhunters. The results show, indeed, that in the states with low NCEI the increase in CEO compensation after a CEO change is larger. This suggests that more CEOs are hired by headhunters in the states with low NCEI, so the improvement in matching is stronger and it leads to higher compensation.

Other studies discussing the increase in CEO compensation over the past decades offer various explanations of this phenomena. Gabaix and Landier (2008) and Gabaix, Landier and Sauvagnat (2014) argue that the CEO pay increases because the average company size is increasing. Murphy and Sandino (2010) argue that the CEOs may extract a larger rent from the company by hiring external compensation consultants that follow their interest. Murphy and Zbojnik (2004) show that the nature of CEO skills required to successfully run a company is changing over time and, therefore, more firms hire the CEOs from outside of the firm and have to pay her more.

In another paper, Murphy and Zbojnik (2007) provide an empirical evidence on the CEO origins at the moment of her appointment, i.e. whether she is coming from within the company or from outside, and the effect of the origin on the compensation. They study the S&P 500 companies during the period from 1970 to 2005. They show that during the 1970s and the 1980s only 15% and 17% of CEO appointments account for the outside hires, while it increased to 26% in the 1990s and almost 32.7% in the 2000s. Even more importantly, they show that external CEOs receive 14.2% higher compensation on average over the full sample, with the difference being just 6% in the 1970s, 15.9% in the 1980s and 19.6% in the 1990s. Not only the companies rely more and more on the outside CEOs but also the pay difference between the internal and external CEOs is increasing. After reconciling these results with the data that almost all of the outside CEOs are hired by headhunters, results by Murphy and Zbojnik (2007) provide a strong evidence for the mechanism proposed in this paper.

Among other studies closely related to the mechanism studied in this paper, Garmaise (2011) shows that tougher non-compete agreements regulation reduces CEO turnover and compensation. Again, this suggests that the activity of headhunters is limited in the states with higher NCEI²⁵ and, therefore, it reduces opportunities for CEOs to transit between firms and improve the efficiency of matching limiting compensation. Pan (2017) shows the importance of assortative matching between CEOs and firms for determination of the CEO compensation and the firm's performance. However, Pan (2017) doesn't consider the change in the degree of assortative matching over time or geographical differences.

²⁵Garmaise (2011) does not talk about headhunters in his study.

7 Conclusion

This paper introduces the headhunter channel to the standard model of random matching. The fact that headhunters have better information about a worker's skill level and that they can approach workers who are not actively searching for a (new) job at this moment allows for better screening of workers and reduces labor market frictions at the top part of the wage distribution. Moreover, headhunters separate the labor market for high and low-productive firms allowing the high-productive firms to access only the high-skilled workers. Because of worker skill and firm productivity complementarities, the wages of workers hired through headhunters increase more than proportionally to the rest of the workers. Thus, the presence of headhunters generates a fat tail of the wage distribution with a larger wage share of the top 1% and 10% workers.

Quantitative analysis shows that introduction of the headhunter channel in otherwise standard random matching model accounts for 69% of the increase in the top 10% share of wages and 35% of the increase of the top 1% share of wages in the U.S. between 1970 and 2010. The results are robust to the choice of targets related to the headhunter channel. The main effect comes from the improvement in the assortative matching between workers and firms, especially at the top. The pattern and the amplitude of the improvement are comparable to the empirical estimates of the change in assortative matching in the U.S. over the same period. The headhunter channel helps to generate the strong non-linearity in the pattern of matching observed in the data.

The paper also provides the empirical evidence of the joint increase of the use of headhunters by firms and the top income shares. The paper uses cross-country data on headhunter revenues and number of hires through headhunters together with the top income shares to show that normalized hires by headhunters are a good predictor of the future growth of the top income shares in European countries. Then, it also shows that the new CEOs in the U.S. get higher compensations comparing to the previous CEOs in the same companies and this effect is weaker in the states with high non-compete enforceability index, i.e., in the states that potentially limit the activity of headhunters.

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A Appendix

A.1 Model equations

Workers

The value of search through both channels for a high-skilled unemployed worker is:

$$S_{UVH}(e) \equiv f_H(\theta_H) \left(\int_{\hat{p}}^{\bar{p}} (W(e, p) - U(e)) dG(p) - c_{wH}(e) \right) \\ + f_V(\theta_V) (1 - f_H(\theta_H)) \\ \cdot \int_{\underline{p}}^{\hat{p}} (W(e, p) - U(e)) dG(p) \\ - c_{wV}(e).$$

The value of search through both channel for a high-skilled employed worker is defined as:

$$S_{EVH}(e, p) \equiv f_H(\theta_H) \left(\int_{\hat{p}}^{\bar{p}} (W(e, p') - W(e, p)) dG(p') - c_{wH}(e) \right) \\ + f_V(\theta_V) (1 - f_H(\theta_H)) \\ \cdot \int_{\underline{p}}^{\hat{p}} (W(e, p') - W(e, p)) dG(p') \\ - c_{wV}(e),$$

note that in this case the first integral starts always in \hat{p} because it will never be optimal to search through both channels if a worker is already working in a firm that hires through the headhunter channel.

Firms

The value of hiring through the vacancy channel in this case is:

$$V_V(p) = -c_{fV}(p) + \beta \left(V(p) + q_V(\theta_V) \left(\frac{u_V}{u_V + a_V} \int_{\underline{e}}^{\hat{e}} (J(p, e) - V(p)) dU(e) \right. \right. \\ + \frac{u_V}{u_V + a_V} (1 - f_H(\theta_H)) \int_{\underline{e}}^{\bar{e}} (J(p, e) - V(p)) dU(e) \\ + \frac{a_V}{u_V + a_V} \int_{\underline{e}}^{\hat{e}} \frac{\Lambda_V(e, p)}{\Lambda_V(e, \bar{p})} (J(p, e) - V(p)) dL_V(e) \\ \left. \left. + \frac{a_V}{u_V + a_V} (1 - f_H(\theta_H)) \int_{\underline{e}}^{\bar{e}} \frac{\Lambda_{VH}(e, p)}{\Lambda_{VH}(e, \bar{p})} (J(p, e) - V(p)) dL_{VH}(e) \right) \right),$$

where the first part in the summation is the expected value of a match after meeting a low-skilled unemployed worker, the second - a high-skilled unemployed worker, the third - a low-skilled employed worker, and the forth - a high-skilled employed worker.

Similarly, the value of hiring through the headhunter channel is:

$$V_H(p) = -c_{fH}(p) + \beta \left(V(p) + q_H(\theta_H) \left(\frac{u_H}{u_H + a_H} \int_{\underline{e}}^{\bar{e}} (J(p, e) - V(p)) dU(e) \right. \right. \\ + \frac{a_H}{u_H + a_H} \int_{\hat{e}}^{\bar{e}} \frac{\Lambda_H(e, p)}{\Lambda_H(e, \bar{p})} (J(p, e) - V(p)) dL_H(e) \\ \left. \left. + \frac{a_H}{u_H + a_H} \int_{\hat{e}}^{\bar{e}} \frac{\Lambda_{VH}(e, p)}{\Lambda_{VH}(e, \bar{p})} (J(p, e) - V(p)) dL_{VH}(e) \right) \right),$$

where the first part in the summation is the expected value of a match after meeting a high-skilled unemployed worker, the second - a high-skilled employed worker searching only through the headhunter channel, and the third - a high-skilled employed worker searching through both channels.

Now, given the distributions, we can also specify the quit rate of a worker with skill e from a firm with productivity p :

$$s_Q(e, p, \omega) = \begin{cases} f_V(\theta_V) \left(\frac{G(\hat{p}) - G(p)}{G(\hat{p}) - G(\underline{p})} \right) & \text{if } p < \tilde{p}_V(e) \text{ and } e < \underline{e} \\ f_H(\theta_H) \left(\frac{G(\bar{p}) - G(p)}{G(\bar{p}) - G(\hat{p})} \right) & \text{if } \tilde{p}_{VH}(e) < p < \tilde{p}_H(e) \text{ and } e \geq \underline{e} \\ f_H(\theta_H) \left(\frac{G(\bar{p}) - G(p)}{G(\bar{p}) - G(\hat{p})} \right) + & \text{if } p < \tilde{p}_{VH}(e) \text{ and } e \geq \underline{e} \\ + (1 - f_H(\theta_H)) \cdot & \\ \cdot f_V(\theta_V) \left(\frac{G(\hat{p}) - G(p)}{G(\hat{p}) - G(\underline{p})} \right) & \\ 0 & \text{otherwise,} \end{cases}$$

where $\omega = (\theta_V, \theta_H, G)$ is a vector of aggregate labor market variables.

Balances

The inflow from unemployment can be written as:

$$i_U(e, p) = \begin{cases} f_V(\theta_V) \frac{g(p)}{v_V} u(e) & \text{if } e < \hat{e}, p < \hat{p} \\ f_H(\theta_H) \frac{g(p)}{v_H} u(e) & \text{if } e \geq \hat{e}, p \geq \hat{p} \\ (1 - f_H(\theta_H)) f_V(\theta_V) \frac{g(p)}{v_V} u(e) & \text{if } e \geq \hat{e}, p < \hat{p} \\ 0 & \text{otherwise,} \end{cases}$$

where the first condition is satisfied when both workers and firms use only the vacancy channel; the second condition is satisfied when both workers and firms use the headhunter channel; and the third condition is satisfied when a worker searches through both channels and a firm hires through the vacancy channel.

Similarly, the inflow from employment can be written as:

$$i_E(e, p) = \begin{cases} f_V(\theta_V) \frac{g(p)}{v_V} \int_{\underline{p}}^{\min\{p, \tilde{p}_V(e)\}} \phi(e, p') dp' & \text{if } e < \hat{e}, p < \hat{p} \\ f_H(\theta_H) \frac{g(p)}{v_H} \int_{\underline{p}}^{\min\{p, \tilde{p}_H(e)\}} \phi(e, p') dp' & \text{if } e \geq \hat{e}, p \geq \hat{p} \\ (1 - f_H(\theta_H)) f_V(\theta_V) \cdot & \text{if } e \geq \hat{e}, p < \hat{p} \\ \cdot \frac{g(p)}{v_V} \int_{\underline{p}}^{\min\{p, \tilde{p}_{VH}(e)\}} \phi(e, p') dp' & \\ 0 & \text{otherwise.} \end{cases}$$

Aggregates

The aggregates that enter the matching functions are determined as follows. The number of unemployed workers searching through the vacancy channel is, simply, the number of unemployed workers:

$$u_V = \int_{\underline{e}}^{\bar{e}} 1 dU(e).$$

The number of unemployed workers searching through the headhunter channel is:

$$u_H = \int_{\hat{e}}^{\bar{e}} 1 dU(e).$$

The number of employed workers searching through the vacancy channel is:

$$a_V = \int_{\underline{e}}^{\hat{e}} \int_{\underline{p}}^{\bar{p}} 1 d\Lambda_V(e, p) + \int_{\hat{e}}^{\bar{e}} \int_{\underline{p}}^{\bar{p}} 1 d\Lambda_{VH}(e, p).$$

The number of employed workers searching through the headhunter channel is:

$$a_H = \int_{\hat{e}}^{\bar{e}} \int_{\underline{p}}^{\bar{p}} 1 d\Lambda_{VH}(e, p) + \int_{\hat{e}}^{\bar{e}} \int_{\underline{p}}^{\bar{p}} 1 d\Lambda_H(e, p).$$

The number of firms using the vacancy channel is:

$$v_V = \int_{\underline{p}}^{\hat{p}} 1 dG(p).$$

And the number of firms using the headhunter channel is:

$$v_H = \int_{\hat{p}}^{\bar{p}} 1 dG(p).$$

A.2 Conditions for Separating Equilibrium

A.2.1 Firms

For separation equilibrium to exist, it must be true that low-productive firms strictly prefer to hire through the vacancy channel while high-productive firms strictly prefer to hire through the headhunter channel. Consider the lowest-productive firm. The lowest-productive firm will successfully hire a worker only if it is matched with an unemployed worker. It prefers to hire through the vacancy channel if $V_V(\underline{p}) > V_H(\underline{p})$. We can rewrite the values of posting a vacancy in each channel for such firm as

$$V_V(\underline{p}) = -c_{fV}(\underline{p}) + \beta \left(V(\underline{p}) + q_V(\theta_V) \frac{u_V}{u_V + a_V} \left(\int_{\hat{e}}^{\bar{e}} (J(\underline{p}, e) - V(\underline{p})) dU(e) \right. \right. \\ \left. \left. + (1 - f_H(\theta_H)) \int_{\hat{e}}^{\bar{e}} (J(\underline{p}, e) - V(\underline{p})) dU(e) \right) \right)$$

and

$$V_H(\underline{p}) = -c_{fH}(\underline{p}) + \beta \left(V(\underline{p}) + q_H(\theta_H) \frac{u_H}{u_H + a_H} \left(\int_{\hat{e}}^{\bar{e}} (J(\underline{p}, e) - V(\underline{p})) dU(e) \right) \right).$$

Condition $V_V(\underline{p}) > V_H(\underline{p})$ holds if

$$\beta \left(q_V(\theta_V) \frac{u_V}{u_V + a_V} \left(\int_{\hat{e}}^{\bar{e}} (J(\underline{p}, e) - V(\underline{p})) dU(e) \right. \right. \\ \left. \left. + (1 - f_H(\theta_H)) \int_{\hat{e}}^{\bar{e}} (J(\underline{p}, e) - V(\underline{p})) dU(e) \right) \right) - > - (c_{fH}(\underline{p}) - c_{fV}(\underline{p})) \\ - \beta \left(q_H(\theta_H) \frac{u_H}{u_H + a_H} \left(\int_{\hat{e}}^{\bar{e}} (J(\underline{p}, e) - V(\underline{p})) dU(e) \right) \right)$$

This condition will be satisfied if at least one of the following holds: 1) the cost of hiring through the headhunter channel is sufficiently higher than the cost of hiring through the vacancy channel - $c_{fH}(\underline{p}) > c_{fV}(\underline{p})$; 2) the matching rate with unemployed workers is

sufficiently higher in the vacancy channel - $q_V(\theta_V) \frac{u_V}{u_V+a_V} > q_H(\theta_H) \frac{u_H}{u_H+a_H}$; 3) there are relatively few unemployed workers above the headhunter threshold; 4) the production of the match increases relatively slow with the worker's skill for this firm.

Consider now the highest-productive firm. The highest-productive firm must strictly prefer to hire through the headhunter channel, $V_H(\bar{p}) > V_V(\bar{p})$. We can rewrite the vacancy values of the highest-productive firm using the fact that any worker matched with this firm will accept the match.

$$V_V(\bar{p}) = -c_{fV}(\bar{p}) + \beta \left(V(\bar{p}) + q_V(\theta_V) \left(\frac{u_V}{u_V+a_V} \int_{\underline{e}}^{\hat{e}} (J(\bar{p}, e) - V(\bar{p})) dU(e) \right. \right. \\ \left. \left. + \frac{u_V}{u_V+a_V} (1 - f_H(\theta_H)) \int_{\hat{e}}^{\bar{e}} (J(\bar{p}, e) - V(\bar{p})) dU(e) \right. \right. \\ \left. \left. + \frac{a_V}{u_V+a_V} \int_{\underline{e}}^{\hat{e}} (J(\bar{p}, e) - V(\bar{p})) dL_V(e) \right. \right. \\ \left. \left. + \frac{a_V}{u_V+a_V} (1 - f_H(\theta_H)) \int_{\hat{e}}^{\bar{e}} (J(\bar{p}, e) - V(\bar{p})) dL_{VH}(e) \right) \right)$$

and

$$V_H(\bar{p}) = -c_{fH}(\bar{p}) + \beta \left(V(\bar{p}) + q_H(\theta_H) \left(\frac{u_H}{u_H+a_H} \int_{\hat{e}}^{\bar{e}} (J(\bar{p}, e) - V(\bar{p})) dU(e) \right. \right. \\ \left. \left. + \frac{a_H}{u_H+a_H} \int_{\hat{e}}^{\bar{e}} (J(\bar{p}, e) - V(\bar{p})) dL_H(e) \right. \right. \\ \left. \left. + \frac{a_H}{u_H+a_H} \int_{\hat{e}}^{\bar{e}} (J(\bar{p}, e) - V(\bar{p})) dL_{VH}(e) \right) \right).$$

Condition $V_H(\bar{p}) > V_V(\bar{p})$ holds if

$$\beta \left(q_H(\theta_H) \left(\frac{u_H}{u_H+a_H} \int_{\hat{e}}^{\bar{e}} (J(\bar{p}, e) - V(\bar{p})) dU(e) \right. \right. \\ \left. \left. + \frac{a_H}{u_H+a_H} \int_{\hat{e}}^{\bar{e}} (J(\bar{p}, e) - V(\bar{p})) dL_H(e) \right. \right. \\ \left. \left. + \frac{a_H}{u_H+a_H} \int_{\hat{e}}^{\bar{e}} (J(\bar{p}, e) - V(\bar{p})) dL_{VH}(e) \right) \right) - \\ - \beta \left(q_V(\theta_V) \left(\frac{u_V}{u_V+a_V} \int_{\underline{e}}^{\hat{e}} (J(\bar{p}, e) - V(\bar{p})) dU(e) \right. \right. \\ \left. \left. + \frac{u_V}{u_V+a_V} (1 - f_H(\theta_H)) \int_{\hat{e}}^{\bar{e}} (J(\bar{p}, e) - V(\bar{p})) dU(e) \right. \right. \\ \left. \left. + \frac{a_V}{u_V+a_V} \int_{\underline{e}}^{\hat{e}} (J(\bar{p}, e) - V(\bar{p})) dL_V(e) \right. \right. \\ \left. \left. + \frac{a_V}{u_V+a_V} (1 - f_H(\theta_H)) \int_{\hat{e}}^{\bar{e}} (J(\bar{p}, e) - V(\bar{p})) dL_{VH}(e) \right) \right) > -(c_{fV}(\bar{p}) - c_{fH}(\bar{p})).$$

This condition will be satisfied, instead, if at least one of the following holds: i) the cost of hiring through the headhunter channel is not much higher than the cost of hiring through the vacancy channel; ii) the absolute matching rate is not much higher in the vacancy channel; iii) there are enough workers accepting the headhunter's calls; iv) the production of the match increases sufficiently fast with the worker's skill for this firm.

A.2.2 Workers

On the worker side, for the separating equilibrium to exist, the highest-skilled worker employed in the lowest-productive firm must agree to consider an offer by a headhunter.

Formally, the value of search only through vacancy channel cannot be optimal for such worker: $S_{EVH}(\bar{e}, \underline{p}) > S_{EV}(\bar{e}, \underline{p})$ and/or $S_{EH}(\bar{e}, \underline{p}) > S_{EV}(\bar{e}, \underline{p})$.

The values of search for such worker are:

$$S_{EVH}(\bar{e}, \underline{p}) = f_H(\theta_H) \left(\int_{\hat{p}}^{\bar{p}} (W(\bar{e}, p') - W(\bar{e}, \underline{p})) dG(p') - c_{wH}(\bar{e}) \right) \\ + f_V(\theta_V) (1 - f_H(\theta_H)) \int_{\underline{p}}^{\hat{p}} (W(\bar{e}, p') - W(\bar{e}, \underline{p})) dG(p') \\ - c_{wV}(\bar{e}),$$

$$S_{EV}(\bar{e}, \underline{p}) = f_V(\theta_V) \int_{\underline{p}}^{\hat{p}} (W(\bar{e}, p') - W(\bar{e}, \underline{p})) dG(p') - c_{wV}(\bar{e}),$$

$$S_{EH}(\bar{e}, \underline{p}) = f_H(\theta_H) \left(\int_{\hat{p}}^{\bar{p}} (W(\bar{e}, p') - W(\bar{e}, \underline{p})) dG(p') - c_{wH}(\bar{e}) \right).$$

Start with the first case. Condition $S_{EVH}(\bar{e}, \underline{p}) > S_{EV}(\bar{e}, \underline{p})$ holds if

$$f_H(\theta_H) \left(\int_{\hat{p}}^{\bar{p}} (W(\bar{e}, p') - W(\bar{e}, \underline{p})) dG(p') - c_{wH}(\bar{e}) - \right. \\ \left. - f_V(\theta_V) \int_{\underline{p}}^{\hat{p}} (W(\bar{e}, p') - W(\bar{e}, \underline{p})) dG(p') \right) > 0$$

This condition will be satisfied if at least one of the following holds: 1) the cost of the headhunter channel is not too high; 2) the matching rate in the vacancy channel is not too high; 3) there are enough firms hiring through the headhunter channel; 4) the production of the match increases sufficiently fast with the firm's productivity for this worker.

The second condition, $S_{EH}(\bar{e}, \underline{p}) > S_{EV}(\bar{e}, \underline{p})$, holds if

$$f_H(\theta_H) \int_{\hat{p}}^{\bar{p}} (W(\bar{e}, p') - W(\bar{e}, \underline{p})) dG(p') - \\ - f_V(\theta_V) \int_{\underline{p}}^{\hat{p}} (W(\bar{e}, p') - W(\bar{e}, \underline{p})) dG(p') - > 0 \\ - (f_H(\theta_H) c_{wH}(\bar{e}) - c_{wV}(\bar{e}))$$

This condition will be satisfied if at least one of the following holds: i) the cost of the headhunter channel is not too high relative to the vacancy channel; ii) the matching rate in the vacancy channel is not too high relative to the headhunter channel; iii) there are enough firms hiring through the headhunter channel; iv) the production of the match increases sufficiently fast with the firm's productivity for this worker.

There is no condition for low-skilled workers in this case because they are excluded from the headhunter channel by assumption.

A.3 Functional Forms for Calibration

The matching function has the standard Cobb-Douglas form:

$$m(u, v) = Mu^\sigma v^{1-\sigma},$$

The production function has the form²⁶:

$$y(e, p) = (e \cdot p)^\gamma,$$

²⁶It is easy to see that this production function is supermodular with $\frac{\partial^2 y(e, p)}{\partial e \partial p} = \frac{\partial^2 y(e, p)}{\partial p \partial e} = \gamma^2 (e \cdot p)^{\gamma-1} > 0$.

with normalization $\gamma = 1$ in the 1970s.

The cost functions have the following form:

$$\begin{aligned} c_{fH}(p) &= c_{fH} \cdot p^{c_f} \\ c_{fV}(p) &= c_{fV} \cdot p^{c_f} \\ c_{wH}(e) &= c_{wH} \cdot e^{c_w} \\ c_{wV}(e) &= c_{wV} \cdot e^{c_w} \end{aligned}$$

with $c_f = 1.5$ and $c_w = 0.5$.

Finally, unemployment benefits are:

$$b(e) = b \cdot e^{b_w}$$

with $b_w = 0.5$.

A.4 Targets Data

The targets for the calibration are taken from the following sources. Top 1% and top 10% wage shares are from Piketty (2003). 90/50 and 90/10 wage ratios are taken from Jencks (2013), 1973 numbers are used for 1970s calibration and 2007 numbers are used for 2010s calibration. Unemployment rate is from BLS. Quit rate is taken from NBER Macroeconomy Database, the data is for 1960s but taken as a proxy for 1970s. Job finding rate is from Shimer (2005). Vacancy costs estimates are from Manning (2011). Headhunter industry targets are from Cappelli and Hamori (2013).

A.5 Idiosyncratic Costs of Headhunters

First, it is more convenient to define two measures for firms with an open position - $G_V(p)$ for the firms using vacancy channel, and $G_H(p)$ for the firms using the headhunter channel.

Workers

The value functions are the following. For low-skilled unemployed workers:

$$S_U(e) = S_{UV}(e) \equiv f_V(\theta_V) \int_{\underline{p}}^{\bar{p}} (W(e, p) - U(e)) dG_V(p) - c_{wV}.$$

For high-skilled unemployed workers:

$$\begin{aligned} S_U(e) = S_{UVH}(e) \equiv & f_H(\theta_H) (1 - f_V(\theta_V)) \cdot \\ & \cdot \left(\int_{\underline{p}}^{\bar{p}} (W(e, p) - U(e)) dG_H(p) - c_{wH} \right) - c_{wV} + \\ & + f_V(\theta_V) (1 - f_H(\theta_H)) \int_{\underline{p}}^{\bar{p}} (W(e, p) - U(e)) dG_V(p) + \\ & + f_H(\theta_H) f_V(\theta_V) \cdot \\ & \cdot \left(\int_{\underline{p}}^{\bar{p}} \int_{\underline{p}}^{\bar{p}} (\max\{W(e, p), W(e, p')\} - U(e)) dG_H(p) dG_V(p') - c_{wH} \right). \end{aligned}$$

For low skilled employed workers:

$$S_{EV}(e, p) \equiv f_V(\theta_V) \int_p^{\bar{p}} (W(e, p') - W(e, p)) dG_V(p') - c_{wV}.$$

For high-skilled employed workers:

$$S_{EH}(e, p) \equiv f_H(\theta_H) \left(\int_{\max\{\hat{p}; p\}}^{\bar{p}} (W(e, p') - W(e, p)) dG_H(p') - c_{wH} \right),$$

and:

$$\begin{aligned} S_{EVH}(e, p) \equiv & f_H(\theta_H) (1 - f_V(\theta_V)) \cdot \\ & \cdot \left(\int_{\max\{\hat{p}; p\}}^{\bar{p}} (W(e, p') - W(e, p)) dG_H(p') - c_{wV} \right) - c_{wV} + \\ & + f_V(\theta_V) (1 - f_H(\theta_H)) \int_p^{\bar{p}} (W(e, p') - W(e, p)) dG_V(p') + \\ & + f_H(\theta_H) f_V(\theta_V) \cdot \\ & \cdot \left(\int_{\hat{p}}^{\bar{p}} \int_{\underline{p}}^{\bar{p}} (\max\{\max\{W(e, p''), W(e, p')\} - W(e, p); 0\}) dG_H(p') dG_V(p'') - c_{wH} \right). \end{aligned}$$

Firms

The value function of a firm posting a vacancy in this case is:

$$\begin{aligned} V_V(p) = & -c_{fV} \cdot p + \beta \left(V(p) + q_V(\theta_V) \left(\frac{u_V}{u_V + a_V} \int_{\underline{e}}^{\hat{e}} (J(p, e) - V(p, c'_{fN})) dU(e) + \right. \right. \\ & + \frac{u_V}{u_V + a_V} (1 - f_H(\theta_H)) \int_{\hat{e}}^{\bar{e}} (J(p, e) - V(p, c'_{fN})) dU(e) + \\ & + \frac{u_V}{u_V + a_V} f_H(\theta_H) \frac{G_H(p)}{G_H(\bar{p})} \int_{\hat{e}}^{\bar{e}} (J(p, e) - V(p, c'_{fN})) dU(e) + \\ & + \frac{a_V}{u_V + a_V} \int_{\underline{e}}^{\hat{e}} \frac{\Lambda_V(e, p)}{\Lambda_V(e, \bar{p})} (J(p, e) - V(p, c'_{fN})) dL_V(e) + \\ & + \frac{a_V}{u_V + a_V} (1 - f_H(\theta_H)) \int_{\hat{e}}^{\bar{e}} \frac{\Lambda_{VH}(e, p)}{\Lambda_{VH}(e, \bar{p})} (J(p, e) - V(p, c'_{fN})) dL_{VH}(e) + \\ & \left. \left. + \frac{a_V}{u_V + a_V} f_H(\theta_H) \frac{G_H(p)}{G_H(\bar{p})} \int_{\hat{e}}^{\bar{e}} \frac{\Lambda_{VH}(e, p)}{\Lambda_{VH}(e, \bar{p})} (J(p, e) - V(p, c'_{fN})) dL_{VH}(e) \right) \right). \end{aligned}$$

The value function of a firm using the headhunter channel is:

$$\begin{aligned} V_H(p) = & -c_{fH} \cdot p + \\ & + \beta \left(V(p) + q_H(\theta_H) \cdot \right. \\ & \cdot \left(\frac{u_H}{u_H + a_H} (1 - f_V(\theta_V)) \int_{\hat{e}}^{\bar{e}} (J(p, e) - V(p, c'_{fN})) dU(e) + \right. \\ & + \frac{u_H}{u_H + a_H} f_V(\theta_V) \frac{G_V(p)}{G_V(\bar{p})} \int_{\hat{e}}^{\bar{e}} (J(p, e) - V(p, c'_{fN})) dU(e) + \\ & + \frac{a_H}{u_H + a_H} \int_{\hat{e}}^{\bar{e}} \frac{\Lambda_H(e, p)}{\Lambda_H(e, \bar{p})} (J(p, e) - V(p, c'_{fN})) dL_H(e) \\ & + \frac{a_H}{u_H + a_H} (1 - f_V(\theta_V)) \int_{\hat{e}}^{\bar{e}} \frac{\Lambda_{VH}(e, p)}{\Lambda_{VH}(e, \bar{p})} (J(p, e) - V(p, c'_{fN})) dL_{VH}(e) + \\ & \left. \left. + \frac{a_H}{u_H + a_H} f_V(\theta_V) \frac{G_V(p)}{G_V(\bar{p})} \int_{\hat{e}}^{\bar{e}} \frac{\Lambda_{VH}(e, p)}{\Lambda_{VH}(e, \bar{p})} (J(p, e) - V(p, c'_{fN})) dL_{VH}(e) \right) \right). \end{aligned}$$

And the value of an open position is:

$$\tilde{V}(p, c'_{fN}) = \max\{V_V(p); V_H(p) - c'_{fN}\}.$$

The quit rate is:

$$s_Q(e, p, \omega) = \begin{cases} f_V(\theta_V) \left(\frac{G_V(\bar{p}) - G_V(p)}{G_V(\bar{p})} \right) & \text{if } p < \tilde{p}_V(e) \text{ and } e < \underline{e} \\ f_H(\theta_H) \left(\frac{G_H(\bar{p}) - G_H(p)}{G_H(\bar{p})} \right) & \text{if } \tilde{p}_{VH}(e) < p < \tilde{p}_H(e) \\ & \text{and } e \geq \underline{e} \\ (1 - f_V(\theta_V)) \cdot & \text{if } p < \tilde{p}_{VH}(e) \text{ and } e \geq \underline{e} \\ \cdot f_H(\theta_H) \left(\frac{G_H(\bar{p}) - G_H(p)}{G_H(\bar{p})} \right) + & \\ + (1 - f_H(\theta_H)) \cdot & \\ \cdot f_V(\theta_V) \left(\frac{G_V(\bar{p}) - G_V(p)}{G_V(\bar{p})} \right) + & \\ + f_V(\theta_V) f_H(\theta_H) \cdot & \\ \cdot \left(1 - \frac{G_V(p)}{G_V(\bar{p})} \frac{G_H(p)}{G_H(\bar{p})} \right) & \\ 0 & \text{otherwise.} \end{cases}$$

Aggregation

The number of firms using the vacancy channel is:

$$v = \int_{\underline{p}}^{\hat{p}} 1 dG_V(p).$$

The number of firms using the headhunter channel is:

$$h = \int_{\hat{p}}^{\bar{p}} 1 dG_H(p).$$

The number of searching workers is determined as before.

Balance

The aggregate balance equation is, as before:

$$\phi(e, p) (s + s_Q(e, p) (1 - s)) = i_E(e, p) + i_U(e, p),$$

while the inflows now are:

$$i_U(e, p) = \begin{cases} f_V(\theta_V) \frac{g_V(p)}{v_V} u(e) & \text{if } e < \hat{e} \\ f_H(\theta_H) (1 - f_V(\theta_V)) \frac{g_H(p)}{v_H} u(e) + & \text{if } e \geq \hat{e} \\ + (1 - f_H(\theta_H)) f_V(\theta_V) \frac{g_V(p)}{v_V} u(e) + \\ + f_H(\theta_H) f_V(\theta_V) \left(\frac{g_V(p)}{v_V} \frac{G_H(p)}{G_H(\bar{p})} + \frac{g_H(p)}{v_H} \frac{G_V(p)}{G_V(\bar{p})} \right) u(e) \end{cases}$$

and:

$$i_E(e, p) = \begin{cases} f_V(\theta_V) \frac{g_V(p)}{v_V} \int_{\underline{p}}^{\min\{p, \tilde{p}_V(e)\}} \phi(e, p') dp' & \text{if } e < \hat{e} \\ f_H(\theta_H) \frac{g_H(p)}{v_H} \int_{\min\{p, \tilde{p}_H(e)\}}^{\min\{p, \tilde{p}_H(e)\}} \phi(e, p') dp' + & \text{if } e \geq \hat{e} \\ + f_H(\theta_H) (1 - f_V(\theta_V)) \frac{g_H(p)}{v_H} \int_{\underline{p}}^{\min\{p, \tilde{p}_{VH}(e)\}} \phi(e, p') dp' + \\ + (1 - f_H(\theta_H)) f_V(\theta_V) \frac{g_V(p)}{v_V} \int_{\underline{p}}^{\min\{p, \tilde{p}_{VH}(e)\}} \phi(e, p') dp' + \\ + f_H(\theta_H) f_V(\theta_V) \frac{g_V(p)}{v_V} \frac{G_H(p)}{G_H(\bar{p})} \int_{\underline{p}}^{\min\{p, \tilde{p}_{VH}(e)\}} \phi(e, p') dp' + \\ + f_H(\theta_H) f_V(\theta_V) \frac{g_H(p)}{v_H} \frac{G_V(p)}{G_V(\bar{p})} \int_{\underline{p}}^{\min\{p, \tilde{p}_{VH}(e)\}} \phi(e, p') dp' \end{cases}$$

A.6 Wage Bargaining

In this extension, wages are determined by period by period wage bargaining between the worker and the firm. This might change the implication of the model because headhunters will affect the outside options of both parties. They improve the value of the vacancy for the firm, so improving firm's bargaining position and driving the wages of top earners down, potentially dampening the effect of better matching. But at the same time, they facilitate job search for high-skilled workers improving their bargaining position and increasing their wages even more. Moreover, the bargaining position of medium-skilled workers worsens because they lose the possibility to move to better matches, therefore decreasing their wages.

As in standard Nash bargaining, wage in a match between a worker with skill e and a firm with productivity p is a solution of the Nash bargaining problem:

$$w(e, p) = \max_w \left(\hat{W}(e, p, w) - U(e) \right)^\gamma \left(\hat{J}(e, p, w) - V(p) \right)^{1-\gamma},$$

where γ is the bargaining power of the worker.

The FOC:

$$\begin{aligned} & \gamma \left(\hat{W}(e, p, w) - U(e) \right)^{\gamma-1} \left(\hat{J}(e, p, w) - V(p) \right)^{1-\gamma} \frac{\partial \hat{W}(e, p, w)}{\partial w} = \\ & - (1 - \gamma) \left(\hat{W}(e, p, w) - U(e) \right)^\gamma \left(\hat{J}(e, p, w) - V(p) \right)^{-\gamma} \frac{\partial \hat{J}(e, p, w)}{\partial w}, \end{aligned}$$

or simply

$$\gamma \left(\hat{J}(e, p, w) - V(p) \right) \frac{\partial \hat{W}(e, p, w)}{\partial w} = - (1 - \gamma) \left(\hat{W}(e, p, w) - U(e) \right) \frac{\partial \hat{J}(e, p, w)}{\partial w}.$$

From the value functions we can find that:

$$\frac{\partial \hat{W}(e, p, w)}{\partial w} = - \frac{\partial \hat{J}(e, p, w)}{\partial w} = 1,$$

so the equilibrium wage for every match must satisfy the standard sharing rule:

$$\gamma \left(\hat{J}(e, p, w) - V(p) \right) = (1 - \gamma) \left(\hat{W}(e, p, w) - U(e) \right).$$

Start with the model without headhunters. RHS of the sharing rule can be written as:

$$\begin{aligned} & \gamma (\mathbf{y} - \mathbf{w} + \beta ((s + \mathbf{s}_Q (1 - s)) V' + (1 - \mathbf{s}_Q) (1 - s) \mathbf{J}') - \\ & - (-c_{fV} \cdot p + \beta (V' + q_V E_{e'|V} [P(A) (J' - V')]))), \end{aligned}$$

and the LHS can be written as:

$$(1 - \gamma) (\mathbf{w} + \beta (sU' + (1 - s) (\mathbf{W}' + \mathbf{S}'_E))) - (b + \beta (U' + S'_U)).$$

If the worker does not search on-the-job the expressions simplify. For the RHS:

$$\gamma (\mathbf{y} - \mathbf{w} + \beta (sV' + (1 - s) \mathbf{J}') - (-c_{fV} \cdot p + \beta ((1 - q_V) V' + q_V E_{e'|V} J'))),$$

and for the LHS:

$$(1 - \gamma) (\mathbf{w} + \beta (sU' + (1 - s) \mathbf{W}')) - (b + \beta (U' + S'_U)).$$

We can solve for \mathbf{w} and apply the sharing rule for the next period to get:

$$\mathbf{w} = \gamma (\mathbf{y} + c_{fV} \cdot p) + (1 - \gamma) b + \beta \gamma (q_V V' - q_V E_{e'|V} J') + (1 - \gamma) \beta S'_U.$$

Without worker/firm heterogeneity this expression collapses to the standard wage equation - equilibrium value of tomorrow search will be equal to equilibrium value of a job, that in turn will be equal to the expected cost of a vacancy posted (κ/q).

Now consider the case when the worker searches on-the-job. We can solve for the wage, \mathbf{w} , from the initial sharing rule, applying the sharing rule of the next period when needed to obtain the following expression for the wage:

$$\begin{aligned} \mathbf{w} = & \gamma (\mathbf{y} + c_{fV} \cdot p) + (1 - \gamma) b - (1 - \gamma) \beta ((1 - s) \mathbf{S}'_{\mathbf{E}} - S'_U) \\ & - \beta \gamma (\mathbf{s}_{\mathbf{Q}} (1 - s) (\mathbf{J}' - V') + q_V E_{e'|V} [P(A) (J' - V')]). \end{aligned}$$

This expression doesn't change in the case of the model with the headhunter channel (except the expectation operator). What changes with the headhunters are the values of the search for the worker, both from unemployment and employment, the value of a vacancy for the firm, and the quit rate. Effects of headhunters on wages are heterogeneous across different matches and depend dramatically on the bargaining power. For example, for the match between the top-ranked worker and the top-ranked firm, where the worker doesn't search on-the-job and the quit rate is equal to 0, the headhunter channel increases both, the outside option of the worker, S'_U , and the outside option of the firm. They have opposite effects on the wage, and which one will be stronger depends fully on the bargaining power. For other matches, the effect is even more complicated. On top of the opposing effects of outside options, there is also an effect on the worker's on-the-job search. With headhunters, the worker doesn't lose the possibility to continue search on-the-job and receive offers from better firms. This puts downward pressure on wages because the worker agrees to the match easier. Moreover, there is an interaction between the worker's search and the value of a vacant position for the firm through the quit rate. The value of a vacant position increases with headhunters, putting upward pressure on wages, but because quit rate increases at the same time, this effect is decreased leaving the overall effect ambiguous.

Numerical simulations show that the overall effect on individual wages, and, especially, on the wage distribution is ambiguous and depends crucially on the choice of the bargaining power. With a high bargaining power of the worker, the effect of headhunters on top wages is higher than in the benchmark model, while with a very low bargaining power the effect is even the opposite, with headhunters reducing wage inequality (even though the value of the bargaining power is not realistic in such simulations). When bargaining power is set to the levels used in the literature, the overall effect is close to the benchmark results.

A.7 Headhunters as Profit-Maximizers

In this section, I extend the model to add headhunters as additional agents choosing the fee and the screening standards in order to maximize the profits. To choose the screening standard, the headhunters need to compare the expected payoff from firms willing to use headhunters with the given standard to the cost of screening. Headhunters have correct expectations about the number and the productivity of firms that will use headhunters

with each screening standard. The headhunters solve the following problem:

$$\max_{\hat{e}, c_{fH}} \left[\int_{\hat{p}(\hat{e}, c_{fV})}^{\bar{p}} c_{fV} \cdot p dG(p) - \int_{\hat{p}(\hat{e}, c_{fV})}^{\bar{p}} c_H(\hat{e}) dG(p) \right],$$

where the first part is the fee revenues from firms using headhunters, and the second term is the total cost of screening the workers. Headhunter balance between the fee and the screening standard. When the screening standard is very high, many firms will want to participate and pay a high fee for it, but the cost of screening for headhunters will be also high, reducing the profits. And when the screening standard is low, firms' willingness to use headhunters decreases, so the headhunter has to reduce the fee, and the profits drop. Solution to this problem crucially depends on the form of the screening cost function.

Modeling headhunters explicitly and calibrating the cost function to match the screening standard and the optimal fee would be equivalent to directly calibrating the standard and the fee, as in the benchmark experiments of this paper. This would change, however, if we studied a dynamic version of the model, but this is the question for future research. Another issue with modeling headhunters is the choice of the market structure. Is it a competitive market, monopoly, or monopolistically competitive market? This question is also left for the future research.

A.8 Robustness of Quantitative Results

To check how much the magnitude of the increase of top wages depends on the choice of the skill threshold for the headhunter channel, I do similar experiments for top 1%, 2%, 3%, 7%, or 10% of workers being eligible for the headhunter channel. The results are presented in Table 10. Not surprisingly, a higher threshold increases the wage share of the top 1% of the workers but decreases the share of the top 10%. This happens because, with a higher threshold, the most efficient workers are more concentrated in the top firms, for example, they all work in top 2% of the firms instead of top 5%. Their wages increase even more due to complementarities, so the top 1% wage share increases more. Instead, for the workers in the 10-1% bracket, the probability of working in the best firms decreases with a higher threshold. Workers in 5-2% are excluded from the headhunter channel and many of them end up in bad or average firms, so the top 10% wage share drops relative to the baseline calibration despite the top 1% wage share increase. The overall effect of the sorting mechanism is still striking - it explains at least 36% of the actual increase in the top 1% share of wages and 62% of the top 10% wage share (together with SBTC fixed at the baseline calibration level).

Table 10: Top Wage Shares in the Model and Data for Different Skill Thresholds

Model	Share	Top 1%	Top 10%	Δ 90/50
Without HH	0%	4.82%	26.18%	0
With HH on top 5% (baseline)	5.25%	7.91%	35.21%	0.39
With HH on top 10%	11.32%	6.94%	36.55%	0.68
With HH on top 7%	7.85%	7.81%	36.20%	0.39
With HH on top 3%	3.36%	8.61%	33.69%	0.39
With HH on top 2%	2.05%	9.32%	32.97%	0.39
With HH on top 1%	1.18%	9.68%	31.71%	0.39

Another target that doesn't have a properly estimated empirical counterpart is the share of firms using the headhunters. In the baseline calibration, it is chosen to fit the estimates by AESC. I redo the experiment with different shares to see how sensitive are the results depending on the choice of the target. I set the share of the firms using the headhunter channel to be 20%, 40%, 60%, 80%, or 100%. The results are presented in Table 11. The increase of the top 10% wage share is decreasing with a lower share of firms using headhunters, but the major part of the effect is still there even if every 5th firm is allowed to use the headhunter channel every period. In this case, the model is still able to explain 82% of the increase in the top 10% wage share (again, together with SBTC). Even when the share of firms using headhunters is set to the most conservative estimate, the model is still able to predict a large share of the increase in top wages. Interestingly, the top 1% wage share is non-linear with respect to the share of firms using the headhunter channel with the baseline calibration being around the minimum.

Table 11: Top Wage Shares in the Model and Data for Different Headhunter Intensity

Model	Top 1%	Top 10%	Δ 90/50
Without HH	4.82%	26.18%	0
With HH, baseline	7.91%	35.21%	0.39
With HH, 100%	8.45%	36.64%	0.39
With HH, 80%	8.52%	36.06%	0.39
With HH, 60%	8.05%	35.42%	0.39
With HH, 40%	8.03%	34.66%	0.39
With HH, 20%	8.18%	33.47%	0.39

A.9 Additional Cross-Country Evidence

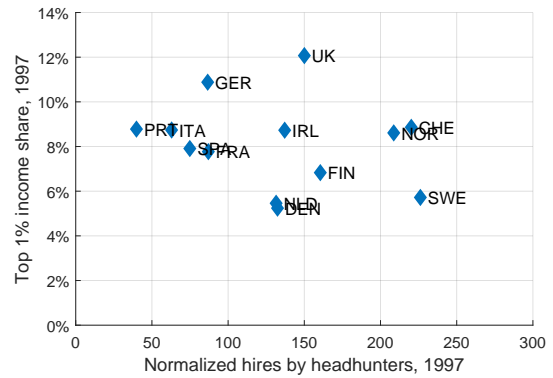
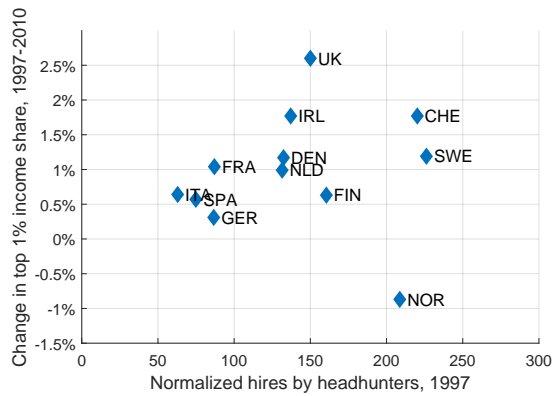
Figure 7 repeats the analysis presented in Section IV for the case of the top 1% income shares. Figures 8 and 9 plot similar relations for normalized fee revenues. It is evident from the figures that the pattern stays the same regardless of the measure used in the analysis. Both measures of headhunter intensity predict future growth in both the top 1% and the top 10% income shares while there is no positive correlation between these measures and the level of top incomes or previous top income growth.

A.10 COMPUSTAT Data

The four firm size proxies are constructed as follows. First, firm value is constructed as the sum of the market value of equity, defined as a number of shares outstanding multiplied by the end-of-fiscal-year stock price, and the book value of debt, defined as total assets minus the sum of the book value of equity and deferred taxes. Second, equity value constructed as the number of shares outstanding multiplied by the end-of-fiscal-year stock price. Third, the sales variable from the COMPUSTAT. Fourth, the income is measured as earnings before interest and taxes.

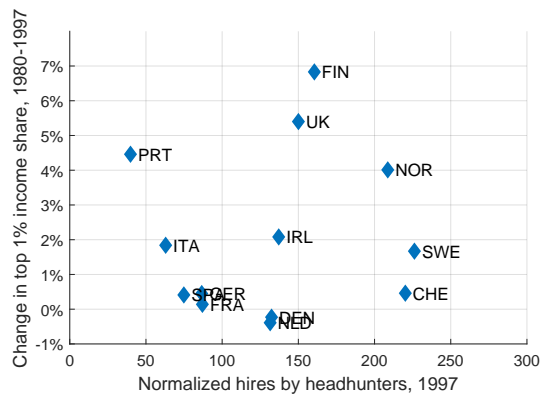
A.11 Additional Micro Evidence

Table 12 presents the results for individual measures of the firm size. Columns (1) and (2) present the results for the firm value measure and columns (3) and (4) for the equity



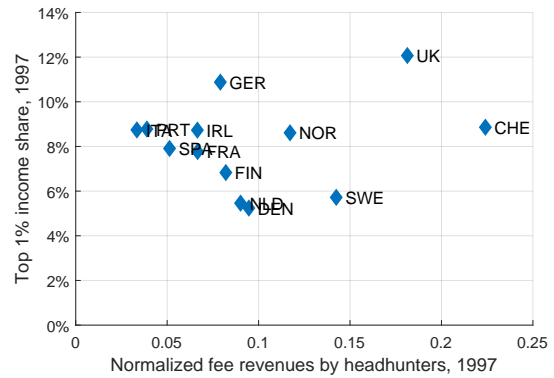
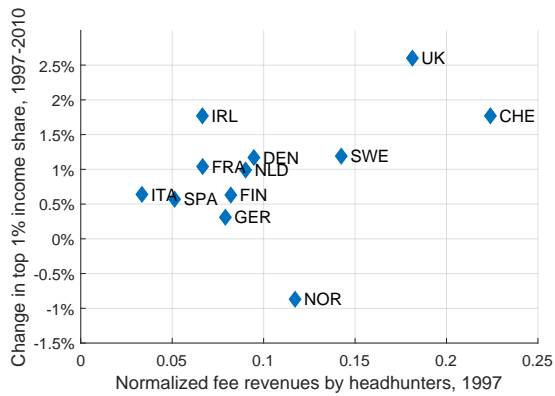
(a) Top 1% Income Share Growth, 1997-2010, and Normalized Hires by Headhunters, 1997

(b) Top 1% Income Share, 1997, and Normalized Hires by Headhunters, 1997



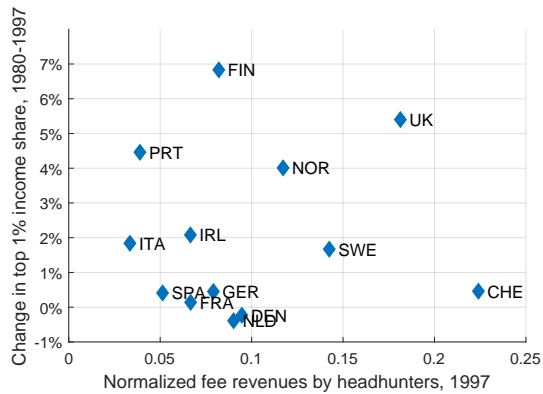
(c) Top 1% Income Share Growth, 1980-1997, and Normalized Hires by Headhunters, 1997.

Figure 7: Top 1% Income Share and Normalized Hires by Headhunters



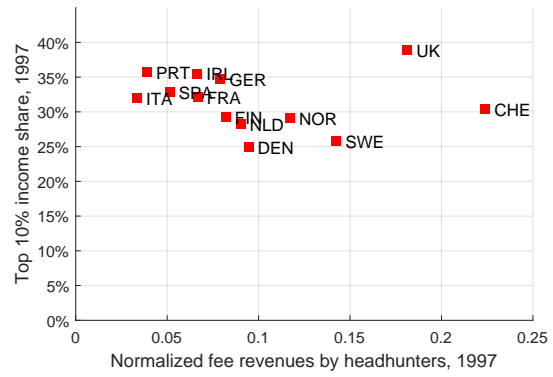
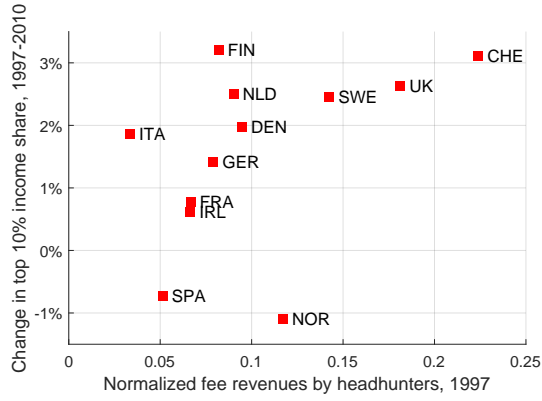
(a) Top 1% Income Share Growth, 1997-2010, and Normalized Fee Revenues by Headhunters, 1997

(b) Top 1% Income Share, 1997, and Normalized Fee Revenues by Headhunters, 1997



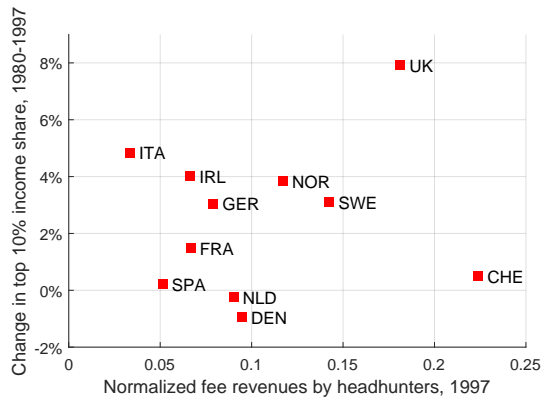
(c) Top 1% Income Share Growth, 1980-1997, and Normalized Fee Revenues by Headhunters, 1997

Figure 8: Top 1% Income Share and Normalized Fee Revenues by Headhunters



(a) Top 10% Income Share Growth, 1997-2010, and Normalized Fee Revenues by Headhunters, 1997

(b) Top 10% Income Share, 1997, and Normalized Fee Revenues by Headhunters, 1997



(c) Top 10% Income Share Growth, 1980-1997, and Normalized Fee Revenues by Headhunters, 1997

Figure 9: Top 10% Income Share and Normalized Fee Revenues by Headhunters

value measures. The results are consistent with the results for the full sample.

Table 13 presents the results of the estimation using an interaction term between measures of firm size and the new CEO dummy (as a proxy for the bargaining power). Columns (1) and (2) present the results for the firm value as a proxy for the firm size and columns (3) and (4) present the results for the equity value of the firm.

To evaluate the matching channel, I test the effect of a change of the CEO on the firm size. Table 14 presents the results, columns (1) and (2) show the effect on the firm value and columns (3) and (4) show the effect on the equity value. As it can be seen from the table, the effect of the change of the CEO on the firm size is positive. These results possibly indicate the presence of the channel related to the productivity of the match between the new CEO and the firm.

Table 15 presents the results of the regression including just the enforceability index, but not the new CEO dummy. The results confirm the well-known result that CEO compensation is lower in the states that enforce the non-compete agreements.

Table 12: CEO Compensation and the Change of the CEO, Individual Firm Size Measures

Sample period 1993-2013				
Log of compensation	(1)	(2)	(3)	(4)
New CEO	0.0437 (0.0177)	0.1389 (0.0297)	0.0420 (0.0171)	0.1876 (0.0314)
Log of Firm Value	0.4311 (0.0180)	0.4620 (0.0193)	- -	- -
Log of Equity Value	- -	- -	0.3442 (0.0161)	0.3572 (0.0191)
Year FE	Yes	No	Yes	No
Firm FE	Yes	Yes	Yes	Yes
R^2	0.66	0.653	0.66	0.654
Number of observations	24673	24673	24673	24673

Table 13: CEO Compensation and the Change of the CEO, Bargaining Power

Sample period 1993 - 2013				
Log of compensation	(1)	(2)	(3)	(4)
New CEO	0.3590 (0.1293)	0.3145 (0.1277)	0.4079 (0.1318)	0.3168 (0.1302)
Log of Firm Value	0.4746 (0.0173)	0.4466 (0.0183)	- -	- -
Log of Equity Value	- -	- -	0.3720 (0.0159)	0.3629 (0.0164)
Log of FV*New CEO	-0.0269 (0.0165)	-0.0332 (0.0165)	- -	- -
Log of EV*New CEO	- -	- -	-0.0291 (0.0182)	-0.0366 (0.0184)
Year FE	No	Yes	No	Yes
Firm FE	Yes	Yes	Yes	Yes
R^2	0.654	0.659	0.654	0.662
Number of observations	24673	24673	24673	24673

Table 14: CEO Compensation and the Change of the CEO, Match Efficiency

	Firm Value		Equity Value	
	(1)	(2)	(4)	(5)
New CEO	0.4784 (0.0399)	0.6138 (0.0450)	0.4023 (0.0399)	0.5208 (0.0448)
Year FE	No	Yes	No	Yes
Industry FE	Yes	Yes	Yes	Yes
R^2	0.200	0.231	0.095	0.132
Number of observations	24673	24673	24673	24673

Table 15: CEO Compensation and the Non-Compete Enforceability Index

	Sample period 1993 - 2013		
Log of compensation	(1)	(2)	(3)
NCEI	-0.0222 (0.0028)	-0.0286 (0.0029)	-0.0221 (0.0029)
Firm Size Controls	Yes	Yes	Yes
Year FE	No	Yes	Yes
Industry FE	Yes	No	Yes
R^2	0.410	0.384	0.420
Number of observations	24217	24217	24217