UNIVERSITY OF CAMBRIDGE
Faculty of Economics

Institute for New Economic Thinking

CAMBRIDGE WORKING PAPERS IN ECONOMICS
CAMBRIDGE-INET WORKING PAPERS

Capability Accumulation and Conglomeratization in the Information Age

Jun Chen
Renmin University of China

Matthew Elliott
University of Cambridge

Andrew Koh
Massachusetts Institute of Technology

Abstract

The past twenty years have witnessed the emergence of internet conglomerates fueled by acquisitions. We build a simple model of network formation to study this. Following the resource-based view of competitive advantage from the management literature we endow firms with scarce capabilities which drive their competitiveness across markets. Firms can merge to combine their capabilities, spin-off new firms by partitioning their capabilities, or procure unassigned capabilities. We study stable industry structures (stable networks) in which none of these deviations are profitable. We find an upper and lower bound on the size of the largest firm, and show that as markets value more of the same capabilities abrupt increases in these bounds occur.

Reference Details

2069	Cambridge Working Papers in Economics
2020/31	Cambridge-INET Working Paper Series
Published	20 July 2020
Revised	23 March 2023
Websites	www.econ.cam.ac.uk/cwpe
	www.janeway.econ.cam.ac.uk/working-papers

CAPABILITY ACCUMULATION AND CONGLOMERATIZATION IN THE INFORMATION AGE*

Jun Chen†, Matthew Elliott‡ and Andrew Koh§

March 23, 2023

Abstract

The past twenty years have witnessed the emergence of internet conglomerates fueled by acquisitions. We build a simple model of network formation to study this. Following the resource-based view of competitive advantage from the management literature we endow firms with scarce capabilities which drive their competitiveness across markets. Firms can merge to combine their capabilities, spin-off new firms by partitioning their capabilities, or procure unassigned capabilities. We study stable industry structures (stable networks) in which none of these deviations are profitable. We find an upper and lower bound on the size of the largest firm, and show that as markets value more of the same capabilities abrupt increases in these bounds occur.

1 Introduction

Someone in 1980s America might have interacted with two dozen different companies in the course of a typical day. In the near future, however, it is not unthinkable for someone to wake up in an Amazon-sourced apartment[1], check the news on her Facebook Feed and then hail a Google-operated self-driving car on her Apple iPhone to pick up fresh groceries at an Amazon Supermarket. She might then meet some friends for lunch—paid for with Apple Pay—before working from home on her Apple Macbook, collaborating with her co-workers via Google Sheets or on a server hosted on Amazon Web Services. In the evening she might order-in dinner via

*We thank Ben Golub, Sanjeev Goyal, Nicole Immorlica, Matt Jackson, Meg Meyer, Volker Nocke, Marzena Rostek, Larry Samuelson and Rakesh Vohra for helpful comments as well as audiences at Bocconi, Carnegie Mellon, Glasgow, Oxford, Kellogg, MIT, the INFORMS annual meeting, the AEA meetings, the 7th European Meeting on Networks, the 6th Network Science in Economics Conference, the Adansonia Conference and the 72nd European Meeting of the Econometric Society. Jörg Kalbfuss, Sarah Taylor and Luther Yap provided excellent research assistance. Elliott acknowledges financial support from ERC starting grant 757229 and the JM Keynes Fellowships Fund. A previous version of this paper was circulated under the title "Firm capabilities and industry structure."

†Renmin University of China.

‡Cambridge University.

§Massachusetts Institute of Technology.

[1] See e.g. https://www.forbes.com/sites/alyyale/2019/07/23/amazon-enters-the-real-estate-game-launches-smart-tech-heavy-homebuying-program/

Deliveroo[2], chat with her parents over Microsoft's Skype and then unwind over in-house content on Amazon Prime, or with a good book on her Amazon Kindle.

The expansion of internet firms into new markets[3] has been accompanied by their exponential growth far outstripping GDP. Even before the Covid-19 pandemic Apple was valued at nearly 5% of the US's $20.5 trillion GDP,[4] a far larger proportion than AT&T and Standard Oil were at their peeks (Temin and Galambos, 1987). Acquisitions underlie all this. Apple CEO Tim Cook recently told shareholders that Apple acquires companies at a rate of about one every three to four weeks, and that their acquisition strategy is mostly aimed at acquiring technology and talent.[5] Apple is not alone in these aspects. More broadly, we observe the following patterns: (i) internet companies are growing, and rapidly entering a myraid of new (and often unexpected) markets; (ii) this growth has been driven by acquisition of firms that often operate in different markets;[6] (iii) the rationale often given for these acquisitions is that of acquiring key capabilities—typically personnel, technology, and access to active daily users. In this paper we present a simple theory that provides a unified account of these observations.

Our theory is based on the resource-based view of competitive advantage in the management literature, pioneered by Wernerfelt (1984), Prahalad and Hamel (1990) and Barney (1991).[7] This is a hugely influential literature that forms a core part of MBA and executive education syllabi. This matters because the people making and influencing acquisition decisions have typically been exposed to these ideas. The fundamental idea is that different firms have different immutable and scarce resources or core capabilities, and it is these capabilities that deliver competitive advantage and profits. Such capabilities might include, for example, unique forms of human capital, production know-how, patents, a strong brand value, a customer base (especially in the context of network externalities or switching costs), and so on. The immutability of capabilities is crucial. Otherwise, competitors would develop those they are missing and then any competitive advantage would be short-lived. We endow firms with capabilities.

We also endow markets with capabilities to represent which markets value which capabilities. While a given capability will deliver competitive advantage for a firm in some markets, it will not be valued by all markets. For example, a team of molecular biologists might provide a firm with a competitive advantage in biotech markets, but they are unlikely to be a source of competitive advantage for the firm in markets for financial services. The competitiveness of a firm in a given market depends on its *relevant capabilities*—the set of capabilities which it

[2] Deliveroo is a popular food delivery company for which Amazon is the lead investor.

[3] See also a relevant discussion at: `https://www.wsj.com/articles/amazon-is-leading-techs-takeover-of-america-1497653164`

[4] See a recent discussion on how big technology firms have grown even bigger during the Covid-19 pandemic: `https://www.wsj.com/articles/how-big-tech-got-even-bigger-11612587632?mod=tech_lead_pos5`

[5] See `https://www.bbc.co.uk/news/business-56178792`

[6] For example, Bain & Company, a management consultancy, note that 2018 was the first year global M&A activity was dominated by 'scope' deals (taking firms into new lines of businesses) rather than 'scale' deals (within industry mergers).

[7] It is one of the two preeminent theories of firm strategy. The other is due to Porter (2008). The resource-based view of competitive advantage has been deployed very widely in the management literature. At the time of writing the combined Google Scholar citation count for the aforementioned papers stood at over 150,000 citations. Remarkably, these ideas have received relatively little attention in the economics literature.

both possesses, and are valued by the market.

We let firms reorganize their capabilities through *mergers*, *demergers*, *procurements*, and *entries*.[8] We are interested in *stable industry structures* for which there are no profitable mergers, demergers, procurements or entries.[9] Firms pay fixed costs to maintain their capabilities, and we assume these are increasing and convex in the number of capabilities maintained. Combining capabilities can enable synergies to be realized yielding a more competitive firm across multiple markets, but combining unrelated capabilities incurs additional fixed costs without delivering any benefits.

Viewed through the lens of a resource-based view of competitive advantage, it seems that a myriad of markets are beginning to value some of the same capabilities, ones often held by internet firms.[10] For example, the recent acquisition of Whole Foods by Amazon for $13.7B would have made little sense twenty years ago—Amazon and Whole Foods would have had little opportunity to gain competitive advantage by combining their capabilities. This is no longer the case. The merger has allowed Amazon to combine its digital and e-commerce capabilities with the Whole Foods brand and its network of stores. Amazon's proprietary data on the shopping habits and interests of Whole Foods customers can help adverts and offers be better targeted (as is now standard for grocery retailers), while its vast logistical and distribution network are valuable for offering online grocery shopping. A similar phenomenon can be seen in Amazon's recent $8.5B acquisition of MGM to shore up its entertainment business. Other similar examples abound. For instance, the automobile industry, in its race towards driverless cars, has begun to prize computer science talent to complement its engineering mainstays.[11]

Can changes in technology that lead markets to value more of the same capabilities account for the emergence of internet conglomerates? To make progress with the problem we draw on the networks literature and represent the capabilities that firms have and the capabilities markets value as a pair of hypergraphs, where in both cases, each node corresponds to a different capability. The firm hypergraph includes an edge for each firm, corresponding to the set of capabilities that firm holds. Likewise, the market hypergraph has an edge for each market corresponding to the set of capabilities that market values.[12] To the best of our knowledge, we are the first to utilise hypergraphs to model the capabilities firms have and the capabilities markets value. There are only a few papers in the economics literature which use hypergraphs.[13] The closest

[8] We will use the term 'procure' to refer to cases where unassigned capabilities are taken on by a firm. This is as opposed to the term 'acquire', which we use interchangeably with 'merger.'

[9] This approach is standard in the network formation literature. For example, stable financial networks are studied in Cabrales et al. (2017); Farboodi (2017); Erol and Vohra (2018); Elliott et al. (2021) and stable production networks in Carvalho and Voigtländer (2015); Oberfield (2018); Acemoglu and Azar (2020); Elliott et al. (2020); Acemoglu and Tahbaz-Salehi (2020).

[10] For a discussion about why software has become increasingly important see: `https://www.wsj.com/articles/SB10001424053111903480904576512250915629460`.

[11] Requiring both accurate and near-instantaneous image recognition, as well as sophisticated decision making systems, the development of driverless cars has given internet companies a valuable competitive advantage. Tech giants including Google, Amazon have all emerged as competitors in the industry.

[12] Hypergraphs are a generalization of the networks used in the networked-markets literature (e.g., Kranton and Minehart, 2001; Elliott, 2015; Nava, 2015; Condorelli et al., 2016; Bimpikis et al., 2019; Goyal, 2017).

[13] Hypergraphs have mainly been used to represent communication structures among agents and to study the coalitions that form (see e.g., Myerson, 1980; van den Nouweland et al., 1992).

to our paper is Malamud and Rostek (2017) who use hypergraphs to model financial exchanges, and competition across them.[14] Different from their approach of utilising hypergraphs, and the questions they address, our modelling allows us to study aggregate, economy-wide trends: the question of what the set of stable industry structures looks like then becomes a question of which firm hypergraphs are stable for a given market hypergraph. Representing the problem as one of hypergraph formation also allows us to draw on an extensive body of work in discrete probability theory, from which we use both concepts and results. We then provide a simple theory of how industry structures change in response to such changes in the connectedness of markets. Our use of hypergraphs also allows us to capture heterogeneity in how capabilities might variously contribute to firm competitiveness across a range of markets. This, in turn, allows a better understanding of how a merger affects the competitive balance in interconnected markets (which value similar capabilities). This is of interest to antitrust authorities who are starting to look more carefully at conglomerate mergers.

Equipped with this model, we first find that the upper bound on the size of the largest firm in all stable industry structures is given by the size of the largest component in the market hypergraph. This holds very generally, and captures the received wisdom from the finance and management literature that businesses should focus on their core capabilities and not enter unrelated markets. As market connectivity increases, this bound tends to change abruptly and even small changes in the overlap of capabilities valued by different markets can drive severe increases in the upper bound.

The new synergies created by increasingly connected markets affect the relative competitiveness of firms in different markets. While this can create incentives for firms to merge to exploit the new synergies, it can be shown that it is also possible for all firms to instead get smaller (hold fewer capabilities) in response.[15] This raises the question of whether the upper bound is just hypothetical. We show it is not. When the cost of maintaining capabilities is not too convex, the upper bound is *tight*—there exists a stable industry structure in which the size of the largest firm is equal to the upper bound.

A fundamental force driving the sudden transition in the size of the largest firm is the phenomenon that mergers can beget further mergers. For example, many of Google's more recent acquisitions might not have been profitable if not for its preceding ones: if, for instance, it was still narrowly focused on providing a search engine, then the billion dollar acquisition of Waze in 2013, a GPS navigation software app, might not have been a shrewd business decision. However, Google had by then already acquired Zip Dash, Where2 and Keyhole Inc (all in 2004) to develop Google Maps. The acquisition of Waze thus allowed Google to further augment the existing capabilities it had already developed to strengthen its foothold in the mapping business. This example underscores the intuition for the phase transition result: there is a key connectivity threshold for markets (based on the capabilities they jointly value) which, once

[14] See also Rostek and Yoon (2020) who study decentralized exchanges where demand for an asset on a particular exchange can only be conditioned on prices of assets on the same exchange. A market structure in their setting can be viewed as a hypergraph, where assets (nodes) can be traded across multiple venues (edges).

[15] This can be shown via an example. Details are available upon request.

passed, allows merger opportunities to cascade leading to giant firms.[16]

The space of possible industry structures quickly becomes intractable and the set of stable industry structures is hard to get a handle on. Nevertheless, under an additional assumption requiring relevant capabilities for a given market to be complements, we show that our upper bound is achieved in all stable industry structures. Under these conditions large conglomerates are an inevitable feature of industry structure once the capabilities that markets value are sufficiently overlapping.

Related literature. There is relatively little work in economics (in contrast to the management literature) that thinks about firms or markets as being endowed with differing sets of capabilities and some work that is related also studies different questions. Goyal et al. (2008) study R&D collaborations while Sutton (2012) maps firms' capabilities to countries' wealth, and uses this as the basis to study the economics of globalization. Also in an international trade setting, Nocke and Yeaple (2007, 2014) analyze the role of firm heterogeneity in issues such as cross-border mergers, and the international organization of production. An empirical literature on hedonic utilities also seeks to identify the characteristics of firms that matter for a range of outcomes (e.g., Bartik, 1987).

Although the central questions we pose on the emergence of internet conglomerates are contemporary, the analysis of industry structure goes back to at least Chandler (1962). We build on his work, incorporating key forces he identified. The proliferation of conglomeratization in the late 1960s and early 1970s also inspired substantial work. Several explanations for conglomeratization have been put forth, ranging from the overconfidence of managers (Roll, 1986), to empire-building (Jensen, 1986), to transaction costs minimization (Teece, 1982). Our approach differs by emphasizing the value of conglomerate mergers as a means of acquiring complementary capabilities to augment competitiveness across different markets. This is consistent with more recent work on conglomerates: using text-based analysis of product descriptions, Hoberg and Phillips (2017) find that most conglomerates, rather than diversifying across separate lines of business with limited synergies, operate in industries with related products.

Our work also relates to a rich literature in industrial organization on mergers and their regulation. Merger reviews typically focus on the trade-off between the emergence of market power, and potential efficiency gains (Williamson, 1968; Chatterjee, 1986; Larsson and Finkelstein, 1999). Our focus is on production synergies. Previous work considering production synergies has tended to concentrate on integrating the efficiency gains from these synergies into merger evaluations without directly evaluating the size of the efficiency gains (see e.g. Farrell and Shapiro, 1990; Whinston, 2007). We undertake the complementary exercise of providing a theory of such synergies. Empirical work suggests that synergies are an important factor in merger decisions and their success (Larsson and Finkelstein, 1999; Devos et al., 2008; Hoberg and Phillips, 2010; Makri et al., 2010; Bena and Li, 2014).

Finally, our work relates to a new literature on the rise of large firms with market power. Crouzet and Eberly (2019) attribute rising industry concentration to intangible capital such

[16] We use the term giant here because of the link with the technical term giant component from the random graph literature.

as intellectual property, branding, and software. Hsieh and Rossi-Hansberg (2019) document an increase in the number of markets per firm, especially in the services industry which is spurred by the adoption of fixed-cost intensive technologies that lower production costs in all markets the firm operates in. Bessen (2017) and Lashkari et al. (2018) both find that proprietary information technology can explain much of the observed rise in market concentration in US and French firms respectively. These findings are consistent with our theoretical results. In particular, Section 5 demonstrates how scarce capabilities – which proprietary information technology can be – which are valued by many intersecting markets can generate merger synergies precipitating the emergence of a giant firm. Theoretical explanations for these phenomena (e.g. Autor et al., 2020; Aghion et al., 2019; De Ridder, 2019; Luttmer, 2011) have primarily focused on aggregate-level approaches. By contrast, we explicitly model the capabilities held and valued by each firm and market.

The rest of the paper is organized as follows. In Section 2 we introduce a parsimonious baseline model. In Section 3, we obtain a tight upper bound on the size of the largest firm in stable industry structures, and show that this upper bound is also a lower bound under additional assumptions on the complementarity of capabilities. In Section 4, we show that the number of capabilities held by the largest firm can be used to bound the number of markets entered by the largest firm—and this bound undergoes an abrupt transition as markets value more capabilities. In Section 5, we substantially generalize the model, relaxing important simplifications. This generalization allows us to shed light on the important role that scarce capabilities—those which can only be wielded by one firm—play in driving conglomeratization. Section 6 concludes.

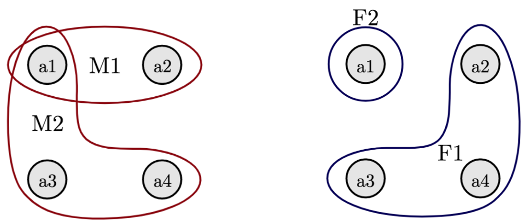
2 Model

2.1 Capabilities, Firms, and Markets. There is a finite set of capabilities $\mathcal{A}$. These are the hard-to-imitate drivers of competitive advantage. There is a finite set of firms $\mathcal{N} = \{1...n\}$. Each firm i is endowed with some set of capabilities which we denote $F_i \subseteq \mathcal{A}$. We assume $\mathcal{A}$ can be partitioned into $(\{F_i\}_{i=1}^n, S)$ where S is the set of unassigned capabilities not held by any firm. This implies that firms hold disjoint capabilities i.e. for any two firms i, i', $F_i \cap F_k = \emptyset$. We remove this assumption in Section 5. There is also a finite set of markets $\mathcal{M} = \{1...m\}$. Each market j is also associated with some set of capabilities $M_j \subseteq \mathcal{A}$. This represents the capabilities which market j values, and are thus a source of competitive advantage in j. We allow markets to value overlapping capabilities i.e. we could have $M_j \cap M_{j'} \neq \emptyset$ for $j \neq j'$. It will be helpful to represent this information with a pair of hypergraphs.[17] Call $H_F := \{\mathcal{A}, \{F_1, F_2, ..., F_n\}\}$ the firm hypergraph and $H_M := \{\mathcal{A}, \{M_1, M_2, ...M_m\}\}$ the market hypergraph. Without loss of generality we let each capability be valued by at least one market. Fixing the capabilities of all firms, let $\theta_{ij} := F_i \cap M_j$ be the capabilities firm i possesses which are relevant to market j.

[17] Technically, the objects we work with are multi-hypergraphs which can contain multiple edges linking the same subset of nodes. For ease of exposition and since there is no ambiguity, we henceforth refer to them as hypergraphs.

Figure 1 below illustrates the model. We have $\mathcal{A} = \{a_1, ..., a_4\}$, $M_1 = \{a_1, a_2\}$ and $M_2 = \{a_1, a_3, a_4\}$. For the firm hypergraph, we have $F_1 = \{a_2, a_3, a_4\}$ and $F_2 = \{a_1\}$ noting that $F_1 \cap F_2 = \emptyset$. Now consider competition in market 2: firm 1 has the relevant capabilities $\theta_{12} = F_1 \cap M_2 = \{a_3, a_4\}$ while firm 2 has the relevant capability $\theta_{22} = \{a_1\}$.

Figure 1: Representation of industry structure



(a) Market hypergraph. (b) Firm hypergraph.

We will use $|F_{max}|$ to denote the number of capabilities held by the largest firm. It will be helpful to explicitly define components for the market hypergraph H_M. We first define a path between any two nodes $a_1, a_n \in \mathcal{A}$ on H_M as a tuple $(a_1, M_1, a_2, \ldots, a_{n-1}, M_{n-1}, a_n)$, such that for $1 \leq i \leq n-1$, $a_i, a_{i+1} \in M_i$. Then given a market hypergraph H_M, a component of H_M is a subset of the nodes $A \subseteq \mathcal{A}$ such that (i) A and $\mathcal{A} \setminus A$ are disconnected (i.e., there is no path between any pair of nodes $a_1 \in A$ and $a_k \in \mathcal{A} \setminus A$); and (ii) A is self-connected (i.e., for any pair $a_1, a_k \in A$, there exists a path through edges restricted to A). Let $\{C_1 \ldots C_p\}$ be the set of all components and let $\mathcal{P} = \{1 \ldots p\}$. As there is a finite set of capabilities, there exists a component with weakly more capabilities than any other. We denote the number of capabilities in a largest component of a hypergraph by $|C_{max}|$.

2.2 Timing. We consider a two stage model. In the first stage firms rearrange their capabilities through merging, demerging, procurements and entries. In the second stage, firms' capabilities are assumed fixed and they compete across the different markets.

2.3 Second Stage Competition. We will assume for now that firms compete a la Cournot by simultaneously setting quantities in each market. In Section 5 we relax this, as well as other assumptions.

Assumption 1 (Cournot Competition)**.** *Firms compete by simultaneously choosing how much to produce in each market. We make the following assumptions on the inverse demand functions the firms face and how firms' constant marginal costs of production in each market depend on the relevant capabilities they have for the market in question*

(i) (Inverse Demand) Let $P_j(Q_j)$ be the inverse demand function for market j, where $Q_j := \sum_i q_{ij}$ and $q_{ij} \in \mathbb{R}_{\geq 0}$ is the quantity produced by firm i in market j. For all markets j,

(a) *there exists* $\zeta_j > 0$ *such that* $P_j(Q_j) > 0$ *for* $Q_j < \zeta_j$ *and* $P_j(Q_j) = 0$ *for* $Q_j \geq \zeta_j$*;*

(b) P_j *is twice continuous differentiable and there exists* $\alpha_j < 0$ *such that* $P_j'(Q_j) < \alpha_j$ *for all* $Q_j < \zeta_j$*; and*

(c) $2P_j'(Q_j) + Q_j P_j''(Q_j) < 0$

(ii) *(Marginal Cost) The marginal cost faced by all firms in market* j *is* $c_j : 2^{M_j} \to \mathbb{R}_{\geq 0}$ *such that*

(a) $c_j(\emptyset) \geq P_j(0)$ *for all markets* j*; and*

(b) *if* $\theta'_{ij} \supseteq \theta_{ij}$ *then* $c_j(\theta'_{ij}) \leq c_j(\theta_{ij})$ *with strict inequality if the superset is strict.*[18]

Assumption 1(i)(a) requires that the market price is strictly decreasing in market output until it reaches zero, and remains at zero thereafter. Assumption 1(i)(b) and (i)(c) are technical conditions that the inverse demand has derivative bounded above, and that it cannot be too convex. Together, Assumptions 1(i)(a)-(c) guarantee uniqueness of equilibrium in market j (Gaudet and Salant, 1991). Assumption 1(ii)(a) states that firms need at least one relevant capability to profitably enter a market; Assumption 1(ii)(b) states that costs are decreasing in the set order of relevant capabilities.

For all markets j, holding capabilities fixed, each firm i solves the problem

$$\max_{q_{ij} \in \mathbb{R}_{\geq 0}} q_{ij}(P_j(Q_j) - c_j(\theta_{ij})).$$

There is a unique Nash equilibrium output choice for all firms in all markets and we let $\boldsymbol{q}_j^*$ denote the unique Nash equilibrium output choices of firms in market j. Note that $\theta_{ij} \supseteq \theta_{kj}$ implies $q_{ij}^* \geq q_{kj}^*$ with strict inequality if $q_{ij}^* > 0$. We can thus define equilibrium gross profits as a function from the capability space to the real line: firm i's equilibrium gross profits in market j are given by $\pi_{ij} : \Theta_j^n \to \mathbb{R}_{\geq 0}$ where $\Theta_j := 2^{M_j}$ is the set of possible relevant capabilities a firm can have in market j and $\boldsymbol{\theta}_j := \{\theta_{kj}\}_k$ is a tuple of relevant capabilities firms have for market j. We therefore write

$$\pi_{ij}(\boldsymbol{\theta_j}) := q_{ij}^*(\boldsymbol{\theta_j})(P_j(Q_j^*(\boldsymbol{\theta_j})) - c_j(\theta_{ij})).$$

Since $\boldsymbol{\theta_j}$ is implied by the hypergraphs (H_M, H_F), we will often denote equilibrium gross profits in the second stage game with $\pi_{ij}(H_M, H_F)$ which allows us to be precise about changes to the underlying firm or market hypergraphs. We will use $J_{i,\pi>0} := \{j \in \mathcal{M} : \pi_{ij}(\boldsymbol{\theta_j}) > 0\}$ to denote the set of markets firm i operates in. Finally, firm i's (total) gross profits $\sum_{j=1}^{m} \pi_{ij}(H_M, H_F)$ are simply the sum of its gross profits across each individual market.

Assumption 2 (Capability Maintenance Costs)**.** *Firm* i *bears cost* $\kappa(|F_i|)$ *where*

[18] We could instead assume that relevant capabilities increase profits by increasing consumers' willingness to pay, rather than affecting marginal costs, and our results would go through unchanged. Specifically, we could let all firms have identical marginal costs in market j given by $\bar{c}_j$, but generalize the price firm i receives in market j to $P_j(Q_j) + f_j(\theta_{ij})$, where $f_j : 2^{M_j} \to \mathbb{R}_{\geq 0}$ is a function satisfying the following conditions: (i) $\bar{c}_j \geq P_j(0) + f_j(\emptyset)$; (ii) $\bar{c}_j \geq f_j(M_j)$; and (iii) if $\theta'_{ij} \supseteq \theta_{ij}$, then $f_j(\theta'_{ij}) \geq f_j(\theta_{ij})$ with strict inequality if the superset is strict. To see that everything then goes through unchanged consider letting $\bar{c}_j = c_j(M_j)$ and $f_j(\theta_{ij}) = c_j(M_j) - c_j(\theta_{ij})$.

(i) $\kappa(0) = 0$;

(ii) κ *is strictly increasing; and*

(iii) κ *is convex, i.e.* $\kappa(x) - \kappa(x-1) > \kappa(x-1) - \kappa(x-2)$ *for all* $x \geq 2$.

Assumption 2 captures a conglomeratization cost associated with managing many capabilities. It may reflect a wide range of factors including, for example, the scarcity of management time or inability of the firm to tailor its corporate culture towards maintaining specific capabilities when it holds a broad set of capabilities. Condition (iii) states that κ exhibits increasing differences i.e. the cost of maintaining an extra capability is increasing in the number of existing capabilities. We henceforth refer to this as convexity. There are no fixed costs other than capability maintenance costs. Thus holding the capabilities of a firm fixed, a firm i will never choose to operate in a market j by choosing $q_{ij} > 0$ if it would make a loss in that market, and will always make a strictly positive gross profit in all markets it does operate in.[19] Hence, we refer to a firm as operating in a market if it is making strictly positive gross profits in that market. Taking firm i's capability maintenance costs into account, we call $\sum_{j=1}^{m} \pi_{ij}(H_M, H_F) - \kappa(|F_i|)$ firm i's *net* profits.

2.4 First Stage Competition. We now endogenize the industry structure by allowing firms to undertake the following deviations which alter their capabilities:

Definition (Firm Actions). *Firms can reorganize their capabilities through*

(i) ***Procurements.*** *A procurement by firm i lets it procure capabilities $A \subseteq S$. Firm i then has capabilities $F_i \cup A$ and the set of unassigned capabilities shrinks to $S' = S \setminus A$.*

(ii) ***Demergers.*** *A demerger by firm l lets it partition its capabilities among one or more firms $\mathcal{F}$, while simultaneously disposing of unwanted capabilities denoted by D. As such, $F_l = D \cup \bigcup_{i \in \mathcal{F}} F_i$. The set of unassigned capabilities expands to $S' = S \cup D$. If $|\mathcal{F}| = 1$, we call this a* ***disposal****.*

(iii) ***Mergers.*** *A merger between firms i and k combines their capabilities and creates a new firm l where $F_l = F_i \cup F_k$.*

(iv) ***Entries.*** *An entry creates a new firm l endowed with capabilities $F_l \subseteq S$. The set of unassigned capabilities shrinks to $S' = S \setminus F_l$.*

Definition (Stability). *We say an industry structure is stable if there is no strictly net profitable procurement, demerger, merger, or entry. An industry structure is unstable if it is not stable.*

Note that we have given firms considerable freedom to demerge into arbitrary partitions of sub-firms, whereas we have defined mergers more restrictively as only between two firms. All else equal, this tends to make it easier for stable firm hypergraphs to support smaller firms. In Section 3.2 we show that our main results continue to hold when coalitional deviations are permitted.

[19] Consider the case in which firm i is making zero gross profits in market j while choosing $q_{ij} > 0$. This implies that $P_j(Q_j) = c_j(\theta_{ij}) > 0$. Then firm i could obtain strictly positive gross profits in market j by choosing $\hat{q}_{ij} \in (0, q_{ij})$ as the market price is strictly decreasing in Q_j.

2.5 Modeling Choices. Both a strength and weakness of the resource-based view of competitive advantage from the management literature is the broad interpretation of capabilities. Our modelling inherits this. We intend capabilities to capture a broad range of things rather than trying to provide a more descriptively accurate model for a partial set of capabilities. Our goal is to provide a simple parsimonious model that systematically captures some of the key underlying forces that, according to this view, can shape industry structure. For example, in practice it might be possible to license some capabilities, like patents, but not others like production know-how. Human capital can decide to move to a new firm (although complementarities between it and other capabilities may reduce the value of relocation), while a firm's reputation or brand value can not. And so on. Likewise, the strength of the case for convex capability maintenance costs depends on the set of capabilities being considered. For very similar capabilities, there may instead be some economies of scale. Nevertheless, overall, we expect a range of different types of capabilities to matter in markets and view convexity as a reasonable approximation.[20]

For all capabilities, immutability is key. They capture things that drive persistent competitive advantages, and so it must not be easy for another firm to develop the same capability. We capture this in a simple but extreme way by precluding the development of new capabilities. Although we expect the development of capabilities to be important, especially in the longer run, we again view our model as capturing salient forces which might account for the recent explosion of tech acquisitions.

3 Upper Bound on Size of the Largest Firm

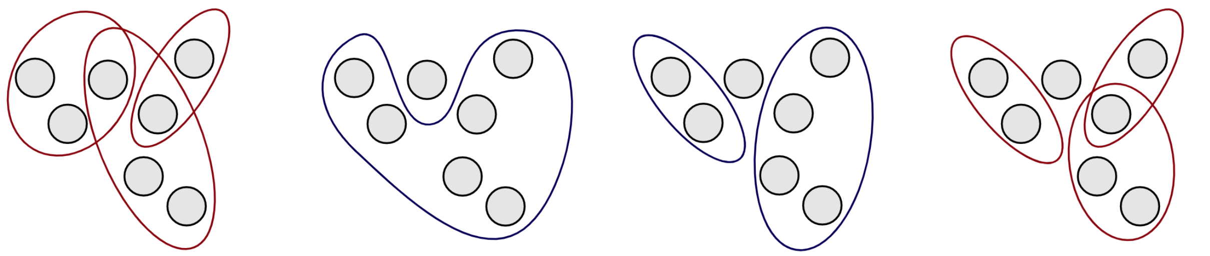
Our goal is to understand how industry structure will evolve in response to markets valuing more of the same capabilities. To do this we first provide a partial characterization of the stable industry structures. The space of possible firm hypergraphs quickly becomes very large, and we do not have enough structure to provide a full characterization of all stable industry structures. As a first step, we show the existence of a stable industry structure and find an upper bound on the size of the largest firm in any stable industry structure in terms of the structure of market hypergraph. This bound is tight in the sense that there exists a stable industry structure with a firm this large in it. We also find conditions under which all stable industry structures have a firm this large. This approach facilitates comparative statics that would otherwise be made difficult by a multiplicity of stable industry structures. In particular, we explore how the bound changes as we allow markets to value more of the same capabilities.

The starting point for our analysis is a simple but powerful observation: firms can never find it optimal to hold a combination of capabilities if there is a way to partition them without destroying any synergies. When such a partition is possible the corresponding demerger generates firms that will obtain exactly the same gross profits in all markets, while the demerger

[20] For example, the corporate cultures of different firms might help them to maintain their respective capabilities and hence maintain their competitive advantages. If so, the ability to tailor a corporate culture to this end will be compromised by as more different capabilities are held, and this can be captured by convex capability maintenance costs. Indeed, consistent with this, different corporate cultures tend to dominate in different industries.

reduces capability maintenance costs and hence is net profitable. This idea is illustrated in Figure 2 where it is net profitable for firm i, shown in panel (b), to demerge into firms k and l, as shown in panel (c). In no markets do both firm k and firm l have relevant capabilities, so at most one of them can compete in each market and they never compete against each other. Moreover, for every market that firm i competed in, either firm k or firm l has exactly the same set of relevant capabilities, and hence generates the same gross profits.

Figure 2: A demerger along the component boundaries of the subhypergraph $\hat{H}_M(F_i)$



(a) H_M (b) F_i (c) F_k, F_l (d) $\hat{H}_M(F_i)$

What are the general conditions under which there exist demergers like this? In order to address this question we need to introduce the concept of subhypergraphs. We say a hypergraph $\hat{H} = \{\hat{A}, \{\hat{E}_i\}_i\}$ is a *subhypergraph* of $H = \{A, \{E_i\}_i\}$ if (i) $\hat{A} \subseteq A$; and (ii) $\hat{E}_i \subseteq \{E_i \cap \hat{A}_i\}$ for all edges $\hat{E}_i \in \{\hat{E}_i\}_i$. We say that a subhypergraph $\hat{H} = \{\hat{A}, \{\hat{E}_i\}_i\}$ of $H = \{A, \{E_i\}_i\}$ is *induced by* the nodes $\hat{A} \subseteq A$ if $\hat{E}_i = E_i \cap \hat{A}$ for all i. We will use the notation $\hat{H}_M(A)$ to denote the market subhypergraph induced by the set of capabilities A. The market subhypergraph of H_M induced by the nodes F_i is illustrated in panel (d) of Figure 2.

Consider again the demerger of firm i into firm k and l illustrated in Figure 2. The demerger occurs along the component boundaries of the induced subhypergraph $\hat{H}_M(F_i)$ shown in panel (d). This provides the means by which the ideas illustrated in Figure 2 can be generalized.

Lemma 1. *Let $\hat{A}_i := \bigcup_{j \in J_{i,\pi>0}} M_j$ denote the set of capabilities valued by markets firm i is operating in. There exists a demerger that firm i can undertake that weakly increases its gross profits (i.e., $\sum_j \pi_{ij}$ weakly increases) if either of the following two conditions hold*

(i) $F_i \supset \hat{A}_i$,

(ii) the market subhypergraph $\hat{H}_M(F_i)$, with all markets (edges) M_j such that $j \notin J_{i,\pi>0}$ then removed, contains more than one component.

We defer the proof of Lemma to Appendix A.2. Lemma 1 provides sufficient conditions for the existence of a profitable demerger because demergers which weakly increase gross profits strictly increase net profits due to the convexity of capability maintenance costs. When either condition (i) or (ii) hold the industry structure cannot be stable. In this way, Lemma 1 allows us to get a first handle on the set of stable industry structures. More specifically, Lemma 1 implies that: (1) in any stable firm hypergraph, firms cannot hold redundant capabilities—those which are not valued by any market it competes in (condition (i)); and (2) that, as discussed earlier, demergers which do not destroy any synergies are always profitable. This resonates with the received wisdom from the finance and management literature—firms should focus on activities aligned with their core capabilities, rather than spread themselves too thin. We will soon employ Lemma 1 to provide an upper bound on the size of the largest firm in any stable firm hypergraph.

It will be helpful to sometimes impose a little more structure on the relationship between capabilities and profits. A first restriction we will sometimes make is that when a firm is already competing in a market, the value of being able to wield an additional unassigned capability valued by that market is always more than the cost of maintaining that capability alone (which is a lower bound on the incremental cost of maintaining that capability). The capabilities valued by a market are intended to represent the key drivers of competitive advantage in that market. Ensuring an additional relevant capability generates a minimum amount of additional value is consistent with this. Similarly, we require that a monopolist with one capability that is relevant for a market will be able to make sufficient gross profits to cover the cost of maintaining that capability.

Definition (Valued Capabilities)**.** *If firm i is already operating in market j ($\pi_{ij}(\theta_{ij}, \boldsymbol{\theta_{-ij}}) > 0$) or no other firms are operating in market j ($\pi_{kj}(\theta_{kj}, \boldsymbol{\theta_{-kj}}) = 0$ for all $k \in \mathcal{N} \setminus \{i\}$), then*

$$\pi_{ij}(\theta'_{ij}, \boldsymbol{\theta_{-ij}}) - \pi_{ij}(\theta_{ij}, \boldsymbol{\theta_{-ij}}) > \kappa(1)$$

for all $\theta'_{ij} \supset \theta_{ij}$ such that $|\theta'_{ij}| - |\theta_{ij}| = 1$.

The next restriction we consider requires capabilities to be complementary in the sense that firms can increase their *gross* profits in a market by combining their capabilities.[21]

Definition (Complementary Capabilities)**.** *For any $\mathcal{F} \subseteq \mathcal{N}$ merging firms $\mathcal{F}$ into a single firm l increases gross profits. So, for all markets j*

$$\pi_{lj}(H_M, H'_F) \geq \sum_{i \in \mathcal{F}} \pi_{ij}(H_M, H_F),$$

where $\theta_{lj} = \bigcup_{i \in \mathcal{F}} \theta_{ij}$, the pre-merger industry structure is (H_M, H_F) and the post-merger industry structure is (H_M, H'_F). Further, the inequality is strict if there exists $i, i' \in \mathcal{F}$ such that $\theta_{ij} \neq \emptyset$, $\theta_{i'j} \neq \emptyset$, and $\pi_{ij}(H_M, H_F) > 0$.

This assumption on complementary capabilities is stronger than the other assumptions we have made. A merger between two firms competing in the same market is not always gross profitable (see, for example, Szidarovszky and Yakowitz, 1982; Salant et al., 1983).[22] Thus to

[21] These capabilities are disjoint since each capability can be held by at most one firm. In Section 5, we relax this assumption.

[22] This is sometimes known as the 'Cournot paradox'.

make all such mergers gross profitable, the synergies have to be sufficiently strong. We note that assuming complementary capabilities does not rule out the possibility that additional relevant capabilities for a given market might have diminishing marginal value. Our definition of complementarity simply requires that the value of holding both together is above that of holding them separately. Finally, in the general model we develop in Section 5, firms can hold multiple copies of the same capability—duplicate capabilities are then (perfect) substitutes.

The assumptions of valued capabilities and complementary capabilities are not maintained—when results depend on these assumptions we state so.

Proposition 1. *The following results tie the size of the largest firm,* $\max_i |F_i|$ *to the size of the largest component on the market hypergraph,* $|C_{max}|$*:*

(i) ***(Upper Bound)*** *In all stable industry structures,* $\max_i |F_i| \leq |C_{max}|$.

(ii) ***(Tightness of Bound)*** *If capabilities are valued then for all* κ *not too convex there exists a stable industry structure in which* $\max_i |F_i| = |C_{max}|$.

(iii) ***(Lower Bound)*** *If capabilities are valued and complementary then for all* κ *not too convex, there is a unique*[23] *stable industry structure (and hence* $\max_i |F_i| = |C_{max}|$*).*

Although the proof of Proposition 1 is fairly rudimentary, it is nevertheless instructive. We defer it until Section 3.1.

Figure 3 provides an illustration of Proposition 1. Panel (a) shows a possible market hypergraph and Panel (b) shows a corresponding firm hypergraph. Part (i) of Proposition 1 implies that there cannot be a firm with more capabilities than the largest firm in the firm hypergraph shown in Panel (b). Part (ii) of Proposition 1 implies that if the costs of maintaining capabilities are not too convex, then there will exist a stable firm hypergraph like the one illustrated in Panel (b) where the largest firm achieves the upper bound. Part (iii) of Proposition 1 implies that if capabilities are also complementary, then all stable firm hypergraphs will have a firm with exactly the size of the largest firm shown in Panel (b). We present this as a lower bound on the size of the largest firm (rather than the exact size of the largest firm) because of how this result generalizes in Section 5.

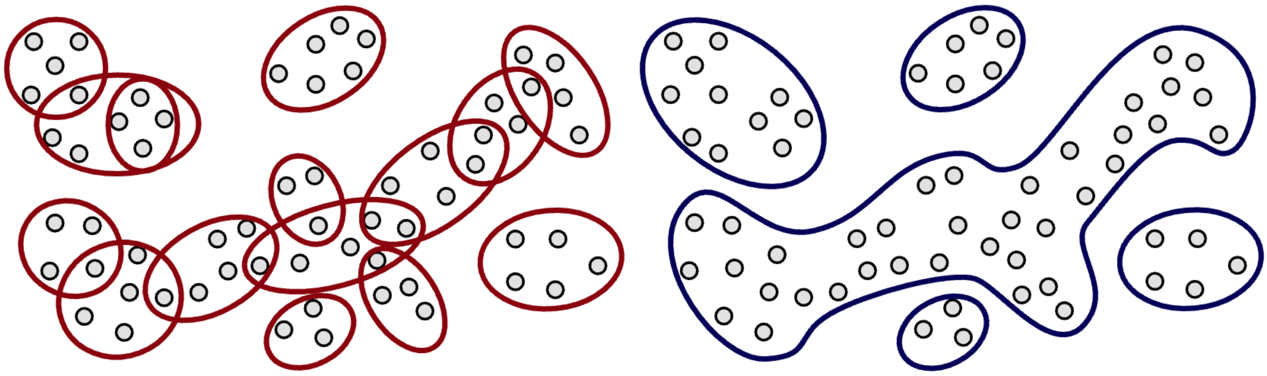
Suppose we take a market hypergraph and adjust it by either (i) adding a new market; or (ii) increasing the capabilities that a market values. In either case, the size of the largest component in the market hypergraph will weakly increase. Indeed, the size of a component will strictly increase if a new market values capabilities from both this component and a different component of the market hypergraph, or if one of its markets expands to value a capability from a different component. This will happen if previously unconnected markets start to value some of the same capabilities.[24] We hypothesize that technological developments have led to such an evolution

[23] Up to relabelling of firms.

[24] A useful benchmark to keep in mind is that if there is a new capability, say the ability to scrutinize large amounts of data, that becomes valued by all markets then regardless of the initial structure of the market hypergraph, after this change there will be a single component containing all markets.

Figure 3: Market hypergraph and implied upper bound on firm size

Notes: Panel (a) illustrates a market hypergraph. Panel (b) shows a firm hypergraph that achieves the upper bound placed on firm size by Proposition 1 (i).



(a) Market hypergraph (b) Firm hypergraph

in the capabilities that markets value. In the introduction we gave the motivating examples of grocery and automobile markets. Another relevant example is healthcare. By collecting large amounts of data on patients including their symptoms, diagnoses and treatment outcomes, it is possible to better tailor treatments to individuals. This makes patient data, as well as the ability to process and analyze this data, newly valuable capabilities in healthcare markets. These newly valued capabilities in healthcare are related to capabilities that are newly valued in other markets. Like image recognition software and artificial intelligence in the case of self-driving cars, and the ability to use data to anticipate consumer demands in the case of grocery markets, these are forms of intangible capital predominantly related to computer science whose growing importance has been documented in macroeconomics (Crouzet and Eberly, 2019; De Ridder, 2019). It is our contention that until fairly recently, grocery markets, automobile markets and healthcare markets would have lived in different components of the market hypergraph, but now they all live in a new giant component of the market hypergraph.

When the size of the largest component in the market hypergraph increases, the upper-bound on firm sizes given in part (i) of Proposition 1 is relaxed and larger firms can exist in stable industry structures. Indeed, Part (ii) of Proposition 1 shows that this increase in firm size is not just hypothetical, but can occur as the stable industry structure evolves in response to changes in capabilities valued by markets. Part (iii) of Proposition 1 gives conditions under which the increase in firm size is inevitable.

While Proposition 1 studies how the largest firm (measured with respect to the number of capabilities held) changes with properties of the market hypergraph, we can extend this result to study the number of markets firms operate in. Denote the markets comprising component r with M_r, and let $|M_{max}| = \max_r |M_r|$ be the maximum number of markets comprising any component.

Corollary. *The following results tie maximum number of markets any firm operates in to* $|M_{max}|$

(i) ***(Upper Bound)*** *In all stable industry structures, all firms operate in at most* $|M_{max}|$ *markets.*

(ii) ***(Tightness of Bound)*** *If capabilities are valued then for all* κ *not too convex there exists a stable industry structure in which there exists a firm operating in* $|M_{max}|$ *markets.*

(iii) ***(Lower Bound)*** *If capabilities are valued and complementary then for all* κ *not too convex, in all stable industry structures, there exists a firm operating in* $|M_{max}|$ *markets.*

The proof of this corollary follows straightforwardly from the construction of stable industry structures presented in the proof of Proposition 1 which we now turn to.

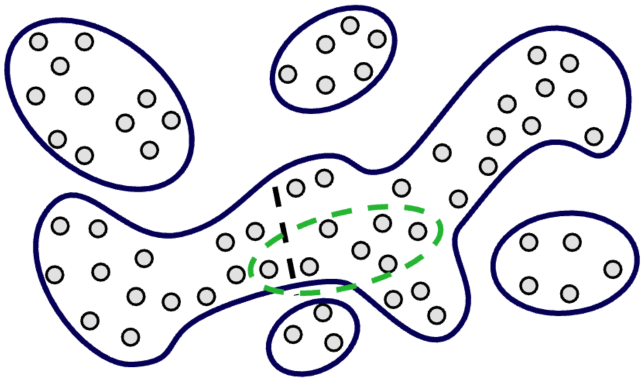
3.1 Proof of Proposition 1. ***Part (i):*** Without loss of generality, suppose that firm i is a largest firm (i.e., $|F_i| = |F_{max}|$). In a stable industry structure no firm, including i, can have a strictly net profitable demerger. Recall that $\hat{A}_i$ is the set of capabilities that are valued by markets i operates in. Hence, by Lemma 1 (i), $F_i \subseteq \hat{A}_i$ otherwise firm i could net profitably dispose of $F_i \setminus \hat{A}_i$, a contradiction. Then by Lemma 1 (ii), $|F_{max}| \leq |C_{max}|$. Otherwise either $F_i \supset \hat{A}_i$ or else $\hat{H}_M(F_i)$ with all markets $j \notin J_{i,\pi>0}$ then removed, contains more than one component.

Part (ii): We proceed by construction. Recall we indexed the components of H_M with $\{1...p\}$. Now for each $i \in \{1...p\}$, let $F_i = C_i$. Namely, for each component of the market hypergraph, we construct a corresponding firm which holds the same capabilities as comprises the component. This generates precisely the firm hypergraph illustrated in Figure 3 (b). We now show that the ensuing firm hypergraph is stable. Note that there are no unassigned capabilities that are valued by at least one market, so there are no possible procurements or entry. Now consider any two firms i, k and note that any merger between them generating firm l is strictly net unprofitable. To see this, note that by Lemma 1, there exists a strictly net profitable demerger of firm l along the component boundaries C_i and C_k which exactly undoes this merger.

Finally, we consider demergers. Pick an arbitrary firm i. Note that (i) firm i is a monopoly in every market $j \in \{j \in \mathcal{M} : M_j \subseteq C_i\}$; and (ii) capabilities are valued, which implies that $J_{i,\pi>0} = \{j \in \mathcal{M} : M_j \subseteq C_i\}$. Consider first a demerger generating the set of firms $\mathcal{F}$ in which no capabilities are disposed of (i.e., $F_i = \bigcup_{i' \in \mathcal{F}} F_{i'}$). Since $J_{i,\pi>0}$ is the set of markets in component C_i, for any non-trivial partition of C_i there will exist two partition elements C_i' and C_i'' and a market j such that $M_j \cap C_i' \neq \emptyset$ and $M_j \cap C_i'' \neq \emptyset$. Figure 4 provides an illustration of this. Thus, for every possible demerger there will exist at least one market j in which gross profits strictly decrease. This is because the initial firm i was (i) a monopolist in market j; and (ii) held all relevant capabilities for j, and so had a strictly lower marginal cost for market j than any resultant firm has after the demerger. Denote the amount of overall gross profits decrease following the demerger by $\varepsilon > 0$. Then for any demerger and any $\varepsilon > 0$, we can lower the convexity of κ until $\kappa(|F_i|) - \sum_{i' \in \mathcal{F}} \kappa(|F_{i'}|) < \varepsilon$. Thus, for all κ not too convex all such demergers are unprofitable.

Figure 4: Possible demerger and lost synergies

Note: A firm hypergraph is represented and a possible demerger of the largest firm illustrated by the dashed line. This would split the capabilities valued by the market shown by the dotted green edge among two firms, thereby destroying some synergies.

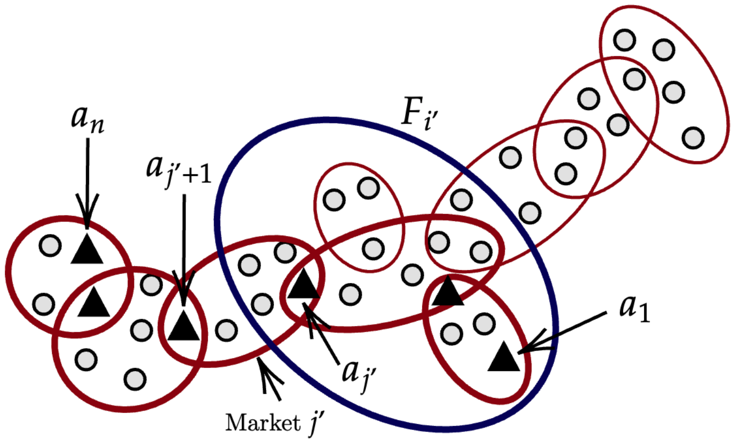


Next, suppose firm i demerges to create firms $\mathcal{F}$ while disposing of capabilities D. Then it is sufficient to consider the reverse process in which firm i is recreated by merging all firms $\mathcal{F}$ and procuring D, and show that overall, this increases net profits for all κ not too convex. Consider first combining all firms $\mathcal{F}$ to create a firm l with capabilities $F_l = \cup_{k\in\mathcal{F}} F_k$. This must weakly increase *gross* profits as after the merger firm l is a monopolist in all markets for which it holds a valued capability. Next consider the procurement of capabilities D by firm l, which generates the original firm i with $F_i = F_l \cup D$. Suppose firm i operates in all markets for which is holds a valued capability. This procurement must then increase gross profits by $|D|\kappa(1)+\varepsilon$ for some $\varepsilon > 0$ because capabilities are valued and every capability in D is deployed in at least one market which firm i subsequently operates in (as $D \subseteq \hat{A}_i$). Finally, for κ with sufficiently low convexity, any additional capability maintenance costs incurred by merging all firms $\mathcal{F}$ and procuring D is outweighed by the additional gross profits, i.e. for any $\varepsilon > 0$, we can reduce the convexity of κ until $\kappa(F_i) - \sum_{i'\in\mathcal{F}} \kappa(|F_{i'}|) < |D|\kappa(1)+\varepsilon$. Therefore, for all κ not too convex, undoing the demerger is strictly net profitable and hence the demerger is strictly net unprofitable.

Part (iii): First observe that by Lemma 1 in a stable industry structure there does not exist any firm $i \in \mathcal{N}$ which spans multiple components of the market hypergraph. Now fix a single component $r \in \{1, \dots p\}$ of the market hypergraph and restrict our attention to all firms which hold a subset of the capabilities in C_r. Denote this set of firms with $\mathcal{F} := \{i \in \mathcal{N} : F_i \subseteq C_r\}$. We will show that in all stable industry structures, there must exist a single firm $i \in \mathcal{F}$ such that $F_i = C_r$. Note that every firm in $\mathcal{F}$ must be making net positive profits, otherwise it can net profitably dispose of all of its capabilities. If $|\mathcal{F}| = 0$, then $C_r \subseteq S$ and since capabilities are valued, for sufficiently low convexity of κ, there exists some strictly net profitable entry. Thus $|\mathcal{F}| \geq 1$.

Now suppose, towards a contradiction, there does not exist a firm i such that $F_i = C_r$. Then consider a firm $i' \in \mathcal{F}$. As $F_{i'} \neq C_r$ and for all firms $i \in \mathcal{F}$ $F_i \subseteq C_r$, there exists a capability $a_n \in C_r \setminus F_{i'}$. Consider any $a_1 \in F_{i'}$. As a_1 and a_n are in the same component of the market

Figure 5: Guaranteed synergies when $F_{i'} \subset C_r$



(a) Example of the path $(a_1, M_1, a_2, \ldots, a_{n-1}, M_{n-1}, a_n)$

hypergraph, there exists a path $(a_1, M_1, a_2, \ldots, a_{n-1}, M_{n-1}, a_n)$ such that $a_i, a_{i+1} \in M_i$ for all $1 \leq i \leq n-1$. We can then iterate backwards from a_n until we find some market j' such that $a_{j'} \in F_{i'}$, $a_{j'+1} \in C_r \setminus F_{i'}$.

Figure 5 shows an example of a path from a capability $a_1 \in F_{i'}$ to a capability $a_n \in C_r \setminus F_{i'}$, represented by the triangles (capabilities) and bold ovals (markets). Note that $a_{j'}, a_{j'+1} \in M_{j'}$, but $a_{j'} \in F_{i'}$ while $a_{j'+1} \notin F_{i'}$. We will show that this is sufficient to guarantee that there must exist either a strictly net profitable merger or procurement.

Either i' operates in market j' or not. If it does, i' has a profitable deviation in which it obtains capability $a_{j'+1}$ for all κ not too convex; if $a_{j'+1}$ is unassigned it procures it increasing its net profits by $\kappa(1) + \varepsilon - (\kappa(|F_{i'}| + 1) - \kappa(|F_{i'}|))$ for some $\varepsilon > 0$, which is strictly positive for all κ not too convex; if $a_{j'+1}$ is held by another firm k', then a merger with firm k' is strictly profitable for all κ not too convex by complementary capabilities.

If i' does not operate in j', then there must exist a firm k' that does operate in market j'—otherwise i would have a profitable deviation to enter market j'. As both firm i' and k' hold at least one capability valued by j' a merger between i' and k' is strictly profitable for all κ not too convex by complementary capabilities.

We have shown that in all stable industry structures, there is a single firm $i \in \mathcal{F}$ such that $F_i = C_r$. But since r was chosen arbitrarily, this uniquely pins down the stable industry structure.

3.2 Coalitional Stability. Proposition 1 (i)-(iii) continues to hold when firms are permitted to undertake a broader set of deviations. Showing this requires a stronger notion of stability.

Definition (Coalitional Stability)**.** *A coalitional deviation by the firms $\mathcal{F} \subseteq \mathcal{N}$ reorganises the capabilities $\bigcup_{i \in \mathcal{F}} F_i$ into a new (possibly empty) set of firms $\mathcal{F}'$ such that $\bigcup_{i' \in \mathcal{F}'} F_{i'} \subseteq$*

$\{S \cup \{\bigcup_{i \in \mathcal{F}} F_i\}\}$.[25] *The industry structure* (H_M, H_F) *is coalitionally stable if there are no strictly profitable coalitional deviations.*

The coalitional deviations we allow are very permissive. Any set of firms can combine their joint capabilities with any unassigned capabilities, and then assign these capabilities in any way they wish among any number of firms, while disposing of any unwanted capabilities. Under these coalitional deviations, fixing the set of capabilities, it is possible to assign them in any way across any number of firms.

Proposition 2. *Proposition 1 (i)-(iii) on the upper bound, tightness, and lower bound on the size of the largest firm in stable industry structures continue to obtain under coalitional stability.*

It should be evident that coalitional stability implies stability, and so the upper bound (Proposition 1 (i)) and lower bound (Proposition 1 (iii)) must continue to obtain since the set of coalitionally-stable industry structures must weakly contract relative to the set of stable industry structures. The existence result of Proposition 1 (ii) extended to coalitional stability is formalised and proved in Appendix A.3.

4 Sensitivity of the Bound on Firm Size

Proposition 1 links the size of the largest firm to the capabilities that markets value and specifically the size of the largest component in the market hypergraph. As markets value more of the same capabilities, both the size of the largest component, as well as the number of markets which comprise it will increase. However, if these changes are only gradual, this mechanism would not offer a satisfactory account of the rapid expansion of internet conglomerates into an ever-increasing array of new markets. Should we expect sudden changes? To explore this question, we need to add some structure to how the market hypergraph evolves. We do this by modelling it as a *random* hypergraph. This provides a natural benchmark and will illustrate how small changes in connectivity can massively relax our upper bound on firm size, as well as the number of markets a firm can enter in equilibrium.

For simplicity, we consider a standard random hypergraph model which yields a neat closed-form characterization though these results hold more broadly. We let each edge (market) of size k in our random market hypergraph occur independently from each other, and independently from edges of other sizes, with probability p_k.[26] We denote the random hypergraph model by $\mathcal{R}(\mathcal{A}, \mathbf{p})$ with $\mathbf{p} = (p_1, p_2, \ldots, p_t)$ where $t < \infty$ is the largest edge size permitted. In a network setting the expected degree, the expected number of nodes a randomly chosen node is directly connected to, provides a key measure of connectivity. In networks a node can be connected to another node through at most one edge, but in a hypergraph a node might be connected to another via multiple edges. There are several ways in which the notion of degree can be

[25] Note that this definition is equivalent to allowing the firms $\mathcal{F}$ to reorganize themselves in any feasible way through any finite sequence of procurements, demergers, mergers, and entries.

[26] Note that this generalises the canonical Bernoulli/Erdős-Rényi random graph model which obtains when $\mathbf{p}$ is such that $p_2 > 0$, and $p_s = 0$ for all $s \neq 2$.

generalized for hypergraphs. We define the degree of capability a as the number of node-edge pairs (a_i, E_i) such that $\{a, a_i\} \subseteq E_i$. This measure is natural for measuring the connectivity of the market hypergraph.[27]

Let $|M_{max}|$ be the largest number of markets any component contains. It turns out that the expected degree of a random hypergraph is a sufficient statistic for determining whether a random market hypergraph drawn according to $\mathbf{p}$ can support large firms or not.

Proposition 3. *There exists a finite constant $\bar{d}$ such that with high probability,*[28]

(i) [subcritical case] if $\mathbb{E}[d(|\mathcal{A}|, \mathbf{p})] < \bar{d}$, then $|C_{max}| = O(\log |\mathcal{A}|)$ and $|M_{max}| = O(\log |\mathcal{A}|)$;

(ii) [supercritical case] if $\mathbb{E}[d(|\mathcal{A}|, \mathbf{p})] > \bar{d}$, then $|C_{max}| = \Omega(|\mathcal{A}|)$ and $|M_{max}| = O(|\mathcal{A}|)$.

Proposition 3 shows there is a critical threshold for the connectivity of markets around which a phase transition in the number of markets a firm can potentially enter occurs. Part (i) of Proposition 3 states that if the expected degree of the random market hypergraph is below this threshold, then all components of the market hypergraph will contain a vanishing proportion of capabilities and markets. The latter imposes a clear upper-bound on the number of markets any single firm can operate in: by Lemma 1, since no firm can span multiple components, the number of markets a firm enters is necessarily constrained by the number of markets comprising the component it is part of. Part (ii) of Proposition 3 states that if the expected degree of the random market hypergraph is above this threshold, then at least one component of the market hypergraph—the component with the most capabilities—will contain a large number of markets. This implies that if some firm holds $|C_{max}|$ capabilities—all those which comprise the largest component—in a stable industry structure, then it must also operate in a large number of markets. Finally, note that this is a threshold phenomenon: the probability that the market hypergraph can sustain giant firms spanning many markets goes from 0 just below the key connectivity threshold $\bar{d}$ to 1 just above it. Even small changes to the connectivity of markets can have a huge impact on the number of markets a firm can enter.

The results on the phase transition of capabilities held by the largest component ($|C_{max}|$) was proven by Schmidt-Pruzan and Shamir (1985) in the random hypergraph literature. We complete the proof of Proposition 3 in Appendix A.1 by employing related techniques to obtain results on the phase transition of the number of markets comprising the largest component

[27] Thus the expected degree of a random hypergraph H with distribution $\mathcal{R}(\mathcal{A}, \mathbf{p})$ is

$$\mathbb{E}[d(|\mathcal{A}|, \mathbf{p})] = \sum_{k=2}^{t} (k-1) \binom{|\mathcal{A}|-1}{k-1} p_k.$$

The binomial coefficient ($|\mathcal{A}|-1$ choose $k-1$) gives the number of different potential edges of size k that include the node in question. Multiplying this by p_k and $k-1$ gives the contribution of edges of size k to the node's expected degree, and summing over k gives the expected degree.

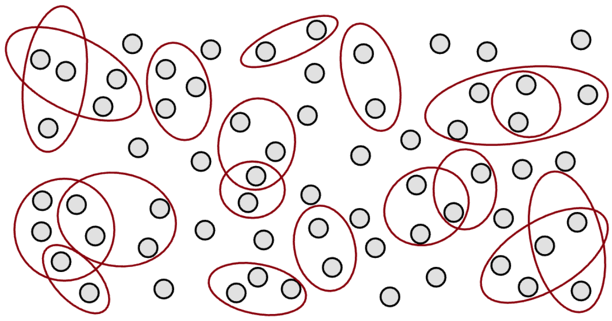
[28] i.e., with probability going to 1 as $|\mathcal{A}|$ goes to infinity. We use standard asymptotic notation where $f(n) = O(g(n))$ if there exists positive real number M such that $|f(n)| \leq M|g(n)|$ for all n and $f(n) = \Omega(g(n))$ if there exists positive real number m such that $|f(n)| \geq m|g(n)|$ for all n.

($|M_{max}|$). While the space of capabilities is abstract and intended to capture a wide variety of things, the phrase transition result on the number of markets a firm can enter in equilibrium is more concrete, and provides predictions consistent with how industry structure has evolved over the past few years.

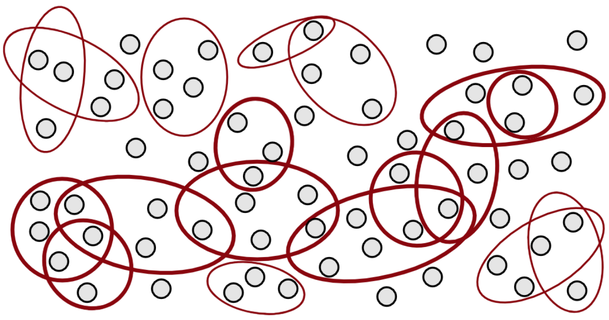
Figure 6 provides an illustration of a random market hypergraph with expected degree just below the threshold in Panel (a), and a random market hypergraph with degree just above the threshold in Panel (b). The markets comprising the giant component in Panel (b) are illustrated in bold. As can be seen, even though here there are a relatively small number of capabilities, the upper and lower bound on the number of markets entered by the largest firm implied by Proposition 1 increases rapidly from 3 in Panel (a) to 10 in Panel (b).

Figure 6: Giant component of the market hypergraph

Notes: The figure illustrates two realizations of random hypergraphs defined over the same space. To generate these hypergraphs we first created a random hypergraph in which each edge contained six nodes. We then randomly removed nodes from these edges according to a threshold that we adjusted to control the expected degree. Thus both hypergraphs have the same number of edges, each edge has a size between 2 and 6 and the hypergraph in (b) can be reached by expanding the size of edges in the hypergraph in (a).



(a) Without a giant component



(b) With a giant component

5 Generalizations and Robustness

We have so far considered a simple baseline model to uncover some of the key forces that could be driving recent conglomeratization. In doing so, we made two crucial simplifications: first, we assumed multiple firms cannot wield the same capability; second, the Cournot setting we worked in, while canonical, is quite special. In this section we relax these assumptions by generalising our baseline model. With some additional machinery, we show that the key economic insights continue to obtain in this broader setting. This will also shed light on the crucial role that *scarce* capabilities—those which can be held by only one firm—play in precipitating conglomeratization.

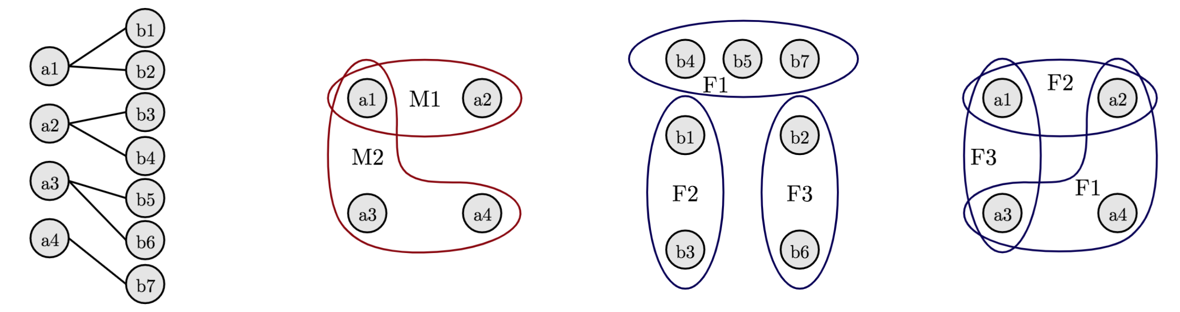
5.1 Capabilities and Competencies. We continue to denote the space of capabilities by $\mathcal{A}$, but will now enrich the model by introducing competencies.[29] We will think of capabilities

[29] This distinction between capabilities and competencies is an abuse of the terminology used in the management literature where the two are typically used as synonyms.

as being abstract and unique, e.g. biotechnology expertise. As before, each market j values capabilities $M_j \subseteq \mathcal{A}$ and the market hypergraph is given by $H_M := \{\mathcal{A}, \{M_1, M_2, ...M_m\}\}$. We continue to denote the set of all components with $\{C_1...C_p\}$ and the size of the largest component with $|C_{max}|$. In contrast, *competencies*, denoted by $\mathcal{B}$, are more specific and represent instances of capabilities—for example, specific human or physical capital like a given biotechnology research team, or intellectual property like pharmaceutical patents. Each firm i is now endowed with *competencies* instead of capabilities, which we denote by $F_i \subseteq \mathcal{B}$. Crucially, multiple firms can possess equivalent competencies corresponding to the same capability. We allow each competency to be deployed across multiple markets and view this as realistic because many competencies—information technology know-how, brands, business processes, databases, patents, supplier networks, customer relationships, etc. can all typically be deployed in one market without substantially diminishing their possible value in others.[30] The firm hypergraph is now defined over the competency space and is given by $H_F := \{\mathcal{B}, \{F_1, F_2, ...F_n\}\}$, and we define $S := \mathcal{B} \setminus \bigcup_{i \in \mathcal{N}} F_i$ as the set of unassigned competencies.

We move between capability and competency spaces with a matching correspondence $\mu : \mathcal{A} \cup \mathcal{B} \rightrightarrows \mathcal{A} \cup \mathcal{B}$, which satisfies the standard conditions: (i) $\mu(a) \subseteq \mathcal{B}$ for all $a \in \mathcal{A}$; (ii) $\mu(b) \subseteq \mathcal{A}$ with $|\mu(b)| = 1$ for all $b \in \mathcal{B}$; and (iii) $b \in \mu(a)$ if and only if $\mu(b) = \{a\}$ for all $b \in \mathcal{B}$ and all $a \in \mathcal{A}$. We also use $\mu(B) := \{a \in \mathcal{A} : \mu(a) \cap B \neq \emptyset\}$ to identify the set of capabilities associated with the set of competencies $B \subseteq \mathcal{B}$ and $\mu(A) := \{b \in \mathcal{B} : \mu(b) \in A\}$ to identify the set of competencies associated with the set of capabilities $A \subseteq \mathcal{A}$. We assume throughout and without loss of generality that $\mu(\mathcal{B}) = \mathcal{A}$, i.e., there is at least one competency associated with each capability. When it is unlikely to cause confusion, we will refer to $\mu(F_i)$—the capabilities corresponding to firm i's competencies—as just firm i's capabilities. Denote the relevant capabilities firm i possesses for market j with $\theta_{ij} := \mu(F_i) \cap M_j$. Finally, we use duplicate competencies to refer to competencies which correspond to the same capability e.g. $b, b' \in \mathcal{B}$ such that $\mu(b) = \mu(b')$ are duplicate competencies.

Figure 7: Representation of market and firm hypergraphs in general model



(a) Matching μ. (b) Market hypergraph. (c) Firm hypergraph. (d) $\mathcal{A}$ representation.

Figure 7 (a) illustrates the mapping $\mu(a_1) = \{b_1, b_2\}, \mu(a_2) = \{b_3, b_4\}, \mu(a_3) = \{b_5, b_6\}, \mu(a_4) = \{b_7\}$. The markets $M_1 = \{a_1, a_2\}$ and $M_2 = \{a_1, a_3, a_4\}$ are represented in Figure 7 (b). The

[30] These competencies can be viewed as forms of intangible capital, whose increasing importance has been documented and studied by, for example, Crouzet and Eberly (2019).

firms $F_1 = \{b_4, b_5, b_7\}, F_2 = \{b_1, b_3\}, F_3 = \{b_2, b_6\}$ are represented in Figure 7 (c). Unlike the market hypergraph, the edges of the firm hypergraph must be disjoint—each competency can be held by at most one firm although multiple firms will hold competencies that are viewed in the same way by markets (i.e., are associated with the same capability). It will sometimes be convenient to represent the firm hypergraph over the capability space rather than the competency space to make direct comparisons with markets possible. In such cases, each edge is defined over the capabilities corresponding to the competencies each firm possesses. For instance, in Figure 7 (d), firm 1 is represented by the edge $\{a_2, a_3, a_4\} = \mu(\{b_4, b_5, b_7\})$. Notice some information is lost in this representation since a single firm might hold multiple competencies corresponding to the same capability. Nevertheless, this representation will be helpful for thinking about profitability and competitiveness across markets.

5.2 Second Stage Competition. Firms' profits in a market j continue to be determined by the distribution of capabilities relevant for that market across the firms. Letting $\boldsymbol{\theta}_{-ij} := \{\theta_{kj}\}_{k \neq i}$ be the tuple of relevant capabilities firms other than i have for market j so that $(\theta_{ij}, \boldsymbol{\theta}_{-ij}) = \boldsymbol{\theta}_j$, firm i's gross profit in market j continues to be given by the function $\pi_{ij}(\theta_{ij}, \boldsymbol{\theta}_{-ij}) : \Theta_j^n \to \mathbb{R}_{\geq 0}$ where, as before, $\Theta_j := 2^{M_j}$. Since $\boldsymbol{\theta_j}$ is still determined by the hypergraph pair (H_M, H_F), as before we denote i's gross profits in market j with $\pi_{ij}(H_M, H_F)$ so that firm i's overall gross profits are $\sum_{j=1}^{m} \pi_{ij}(H_M, H_F)$. Firm i's net profits are $(\sum_{j=1}^{m} \pi_{ij}(H_M, H_F)) - \kappa(|F_i|)$ where κ is identically defined, but is now over the competency space $\mathcal{B}$ i.e., maintenance costs depend on the number of competencies held. Again, $J_{i,\pi>0} = \{j \in \mathcal{M} : \pi_{ij}(H_M, H_F) > 0\}$.

However, we now dispense with Cournot competition and instead impose the following assumption directly on the primitives of profits. These assumptions are consistent with Cournot competition—details available upon request.

Assumption (Primitives on Profits). *Let $I(\boldsymbol{\theta}_j) = \{i \in \mathcal{N} : \pi_{ij}(\theta_{ij}, \boldsymbol{\theta}_{-ij}) > 0\}$ denote the set of firms operating in market j. Let $\boldsymbol{\theta}_{Ij} := \{\theta_{ij}\}_{i \in I}$ denote the capabilities these firms have that are valued by market j. We assume that the profit functions $\pi_{ij}(\theta_{ij}, \boldsymbol{\theta}_{-ij})$ satisfy the following conditions:*

(i) ***Firms with no capabilities make 0 gross profits.***
$\pi_{ij}(\emptyset, \boldsymbol{\theta}_{-ij}) = 0$ *for all* $\boldsymbol{\theta}_{-ij} \in \Theta_j^{n-1}$.

(ii) ***Firms which are not operating in market j do not influence profits in j.***
For all $\boldsymbol{\theta}_j$ and $\boldsymbol{\theta}'_j$ such that (i) $I(\boldsymbol{\theta}_j) = I(\boldsymbol{\theta}'_j)$ and (ii) $\boldsymbol{\theta}_{Ij} = \boldsymbol{\theta}'_{Ij}$, we have that $\pi_{ij}(\theta_{ij}, \boldsymbol{\theta}_{-ij}) = \pi_{ij}(\theta'_{ij}, \boldsymbol{\theta}'_{-ij})$ for all i.

(iii) ***Labels do not matter.***
Fix $\theta' = \theta_{ij}$ and $\boldsymbol{\theta}' = \boldsymbol{\theta}_{-ij}$. For any bijection $b : \{1, \ldots, n\} \to \{1, \ldots, n\}$ (changing the labels of the firms while holding their capabilities fixed), $\pi_{ij}(\theta', \boldsymbol{\theta}') = \pi_{b(i)j}(\theta', \boldsymbol{\theta}')$.

(iv) ***Weaker competitors increase gross profits.***
Let $\boldsymbol{\theta}_{-ikj} := \{\theta_{lj}\}_{l \neq i,k}$. For all $k \neq i$, if $\theta'_{kj} \subset \theta_{kj}$ then $\pi_{ij}(\theta_{ij}, \theta'_{kj}, \boldsymbol{\theta}_{-ikj}) \geq \pi_{ij}(\theta_{ij}, \theta_{kj}, \boldsymbol{\theta}_{-ikj})$ with strict inequality if and only if $\pi_{kj}(\theta_{kj}, \boldsymbol{\theta}_{-kj}) > 0$ and $\pi_{ij}(\theta_{ij}, \theta'_{kj}, \boldsymbol{\theta}_{-ikj}) > 0$.

(v) ***More capabilities increase gross profits.***
If $\theta'_{ij} \supseteq \theta_{ij}$, *then* $\pi_{ij}(\theta'_{ij}, \boldsymbol{\theta}_{-ij}) \geq \pi_{ij}(\theta_{ij}, \boldsymbol{\theta}_{-ij})$ *for all* $\boldsymbol{\theta}_{-ij} \in \Theta_j^{n-1}$.

We view this assumption as providing relatively weak regularity conditions and will maintain it throughout the general model.

5.3 First Stage Competition. We once again endogenize the industry structure by allowing firms to procure, merge, demerge, and enter. Note that since firms hold competencies, these operations are performed on the space $\mathcal{B}$. Procurements, demergers and entry are essentially unchanged, but we allow for a more general definition of mergers to reflect the increased richness of the competency space. In particular, we allow firms to either dispose of duplicate competencies, or to create spin-off firms from them.

Definition (Firm Actions in General Model)**.** *Firms can reorganize their competencies via*

(i) ***Procurements.*** *A procurement by firm* i *lets it procure competencies* $B \subseteq S$. *Firm* i *then has competencies* $F_i \cup B$ *and the set of unassigned competencies shrinks to* $S' = S \backslash B$.

(ii) ***Demergers.*** *A demerger by firm* l *lets it partition its competencies among one or more firms* $\mathcal{F}$, *while simultaneously disposing of unwanted competencies denoted by* D. *As such,* $F_l = D \cup \{\bigcup_{i \in \mathcal{F}} F_i\}$. *The set of unassigned competencies expands to* $S' = S \cup D$. *If* $|\mathcal{F}| = 1$, *we call this a* ***disposal*** *since the demerger in effect generates a single firm.*

(iii) ***Mergers.*** *A merger between two non-empty firms* i *and* k *combines* i*'s and* k*'s competencies, and also permits* i *and* k *to reorganise any duplicate competencies they have by spinning off firms and/or discarding of unwanted duplicate competencies (see conditions (a)-(c) below). Formally, a merger between firms* i *and* k *generates a set of firms* $\{1, \ldots, n\}$ *and set of unassigned competencies* S' *such that*

  (a) $S' \supseteq S$;

  (b) *there exists a firm* $l \in \{1...n\}$ *such that* $\mu(F_l) = \mu(F_i) \cup \mu(F_k)$;

  (c) $\{\bigcup_{l \in \{1...n\}} F_l\} \cup \{S'\} = \{F_i \cup F_k \cup S\}$;

  (d) *firms* $\{F_1, \ldots, F_n\}$ *could not have been generated by a (single) demerger.*[31]

(iv) ***Entries.*** *An entry creates a new firm* l *endowed with competencies* $F_l \subseteq S$. *The set of unassigned competencies shrinks to* $S' = S \setminus F_l$.

Figure 8 provides examples of these actions, illustrating how they affect the firm hypergraph in both the competency space and capability space.

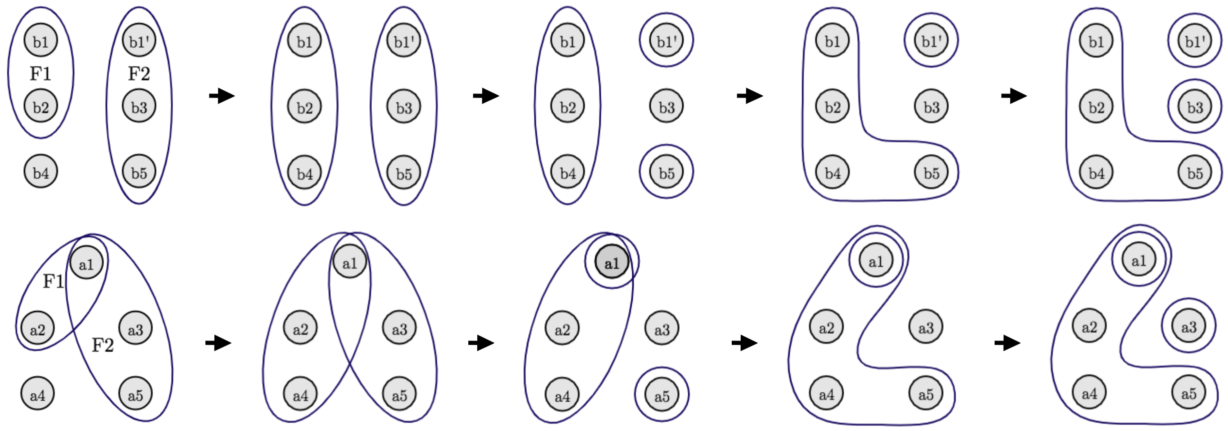
We retain an analogous definition of stability to before.

Definition (Stability)**.** *We say an industry structure is stable if and only if there is no strictly net profitable procurement, demerger, merger, or entry. An industry structure is unstable if it is not stable.*

[31] Any profitable merger that violates condition (d) is either also a profitable demerger (and so already a profitable permitted deviation) or else one firm is sponsoring the unprofitable demerger of the other. Condition (d) rules out such deviations, which we view as inconsistent with antitrust principles.

Figure 8: Sequence of procurements, demergers, mergers, and entry

Note: **Top row**: representation in the competency space $\mathcal{B}$. **Bottom row**: representation in the capability space $\mathcal{A}$. We have matching $\mu(a_1) = \{b_1, b_1'\}$ and $\mu(a_i) = b_i$ for $i = \{2, \ldots, 5\}$. Each action is relative to the previous hypergraph.



(a) Initial (b) Procurement (c) Demerger (d) Merger (e) Entry

5.4 Flexibility of the General Model. The set of competencies is intended to capture anything that might deliver competitive advantage for a firm. This general interpretation of competencies is in line with the management literature. Some concrete examples of competencies that a firm might hold include a brand or reputation, patents, know-how related to a production process, the corporate culture of a firm, human capital of various forms, relationships with suppliers or customers, a distribution network, a customer base, information about customers, and so on. In comparison to the Cournot setting, our general approach does not require any fixed interpretation of how competencies deliver competitive advantages (for example, by reducing marginal costs, or increasing consumers' willingness to pay). There is also considerable flexibility regarding how valuable different competencies are, and under what circumstances. For example, certain combinations of competencies might be especially valuable when held together; the value of a given competency in a market can depend on how many others are deploying the same competency in the market; and so on.[32]

It should be evident that an analogue of Lemma 1 continues to obtain in the general model—if a firm's competencies span multiple components, or include competencies not deployed in any market, then it has a profitable demerger or disposal.

Lemma 2. *Let* $\hat{A}_i := \bigcup_{j \in J_{i,\pi>0}} M_j$ *denote the set of capabilities valued by markets firm* i *operates in. There exists a demerger that firm* i *can undertake that weakly increases its gross profits (i.e.,* $\sum_j \pi_{ij}$ *weakly increases) if any of the following three conditions hold*

(i) $F_i \not\subseteq \hat{A}_i$,

(ii) $|F_i| > |\mu(F_i)|$,

(ii) the market subhypergraph $\hat{H}_M(\mu(F_i))$*, with all markets* $j \notin J_{i,\pi>0}$ *then removed, contains more than one component.*

[32] This flexibility is also in the spirit of the management literature (see e.g. Barney, 1991).

Lemma 2 is almost identical to Lemma 1, but with the additional case where firms hold multiple competencies corresponding to the same capability. It is proved in Appendix A.2.[33]

As before, we will often wish to impose more structure on the relationship between capabilities and profits. While we can directly use the valued capabilities condition previously defined in Section 2, the complementary capabilities condition needs to be adjusted to accommodate the more general environment.

Definition (Complementary Capabilities)**.** *We say capabilities are complementary if for any $\mathcal{F} \subseteq \mathcal{N}$ such that for any $i, i' \in \mathcal{F}$, $\theta_{ij} \cap \theta_{i'j} = \emptyset$, merging firms $\mathcal{F}$ into a single firm l increases gross profits:*

$$\pi_{lj}(H_M, H'_F) \geq \sum_{i \in \mathcal{F}} \pi_{ij}(H_M, H_F),$$

where $\theta_{lj} = \bigcup_{i \in \mathcal{F}} \theta_{ij}$, the pre-merger industry structure is (H_M, H_F) and the post-merger industry structure is (H_M, H'_F). Further, the inequality is strict if there exists $i, i' \in \mathcal{F}$ such that $\theta_{ij} \neq \emptyset$, $\theta_{i'j} \neq \emptyset$, and $\pi_{ij}(H_M, H_F) > 0$.

As before, we note that complementary capabilities do not rule out the possibility that additional relevant capabilities for a given market might have diminishing value; our definition simply requires that the value of holding all of them together is above that from holding them separately across the firms $\mathcal{F}$. We also note that although competencies corresponding to different relevant capabilities are complementary, competencies corresponding to the same capability are (perfect) substitutes. Lemma 2 gives us sufficient conditions for a demerger to be weakly gross profitable. The next lemma offers a converse, and gives conditions under which a firm has no profitable demerger or disposal. It is proved in Appendix A.2

Lemma 3. *Suppose that capabilities are valued and complementary. Then there does not exist a demerger that firm i can undertake that weakly increases its gross profits (i.e., $\sum_j \pi_{ij}$ weakly increases) if all of the following three conditions hold*

(i) $\mu(F_i) \subseteq \hat{A}_i$,

(ii) $|F_i| = |\mu(F_i)|$, *and*

(iii) the market subhypergraph $\hat{H}_M(\mu(F_i))$, with all markets $j \notin J_{i,\pi>0}$ then removed, contains a single component.

5.5 Mapping Results from the Baseline to General Model. It should be evident that Proposition 3 on the phase transition of $|M_{max}|$ continues to hold since the capability space and market hypergraph is defined as before.

[33] In our model, additional competencies corresponding to the same capability cannot be held in a stable industry structure because disposing of them reduces capability maintenance costs without affecting competitiveness in any market. This reflects the static nature of our stability notion. However, a natural question is whether *hoarding* such competencies—holding them to keep potential competitors from procuring them—might be profitable in a dynamic version of our model. We return to this in our concluding discussions (Section 6).

Proposition 1 (i) stating that in all stable industry structures, $|F_{max}| \leq |C_{max}|$ also continues to obtain in our general model. This is because duplicate competencies—multiple competencies corresponding to the same capability—do not strengthen firms' competitiveness, but incur additional maintenance costs.

We now develop an analogue of Proposition 1 (ii).

Proposition 4 (Tightness in General Model)**.** *For any $\mathcal{A}$, $\mathcal{B}$, μ, H_M, for all κ not too convex, if capabilities are valued and complementary, there exists a stable industry structure in which $|F_{max}| = |C_{max}|$. Further, this firm operates in $|M_{max}|$ markets.*

This proposition guarantees the existence of stable industry structures which attain the upper bound in the general model. This is non-trivial because the space of hypergraphs is large and discrete and it is straightforward to show that there are cycles of profitable deviations which makes a proof based on Tarski's fixed point theorem unlikely to work.[34] Further, even in simple examples there remain many possibilities, and these possibilities will depend on the details of the situation that we have not specified—i.e., our assumptions are too weak to be able to say for sure whether one such hypergraph is definitively stable or unstable.

Consider the example shown in Figure 9 below. Panel (a) shows the set of capabilities that exist and the market hypergraph. We assume there are exactly two competencies associated with each such capability. Panels (b) - (d) illustrate 3 of the approximately $700,000$ possible firm hypergraphs.[35] The firm hypergraphs shown in Panels (b) and (c) are necessarily unstable. In Panel (b), firm 2 has a profitable demerger by Lemma 2. In Panel (c), suppose neither firm 2 or 3 has a profitable demerger. This implies that firm 2 competes in markets 1 and 2 and makes non-negative net profits, while firm 3 competes in at least market 3 and makes non-negative net profits. But, as long as capabilities are complementary, firms 2 and 3 will then have a strictly net profitable merger for all κ not too convex. In contrast, Panel (d) shows one of the many firm hypergraphs which *might* be stable—our assumptions are too weak to either rule out or guarantee stability. Nonetheless, even in light of this indeterminacy, Proposition 4 guarantees the existence of at least one stable industry structure for sufficiently low convexity of κ.

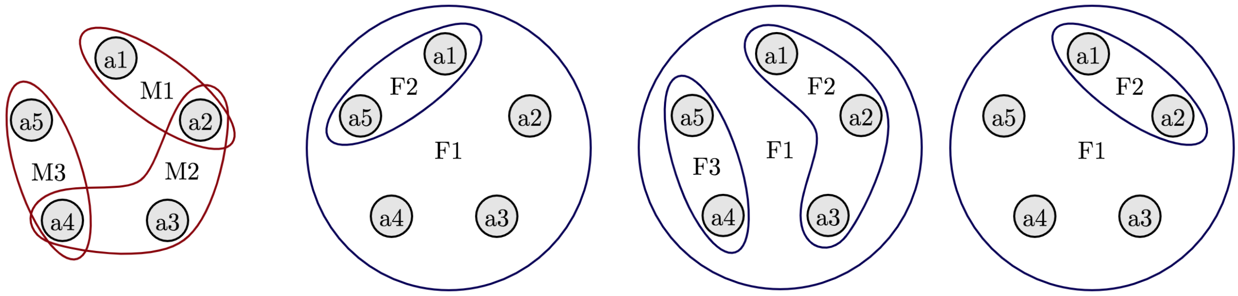
The proof of Proposition 4 is involved and we defer it to Appendix A.4. Briefly, it proceeds by constructing a firm hypergraph via an algorithm which, at each iteration, constructs a giant firm with the set of unassigned competencies, and proceeds to undertake its most profitable demerger. The firm hypergraphs generated by this algorithm exhibit some realistic features. For

[34] See Jackson and Watts (2001).

[35] Use B_n to denote the n-th Bell number, defined recursively such that $B_{n+1} = \sum_{k=0}^{n} \binom{n}{k} B_n$ and $B_0 = B_1 = 1$. This gives the number of ways to partition a set. For 10 competencies, if there are s competencies in the set of unassigned competencies, then there are $\binom{10}{s}$ ways of forming this set. For each such set there are B_{10-s} ways of assigning the remaining competencies to firms. Thus the number of ways to partition the 10 competencies is

$$\sum_{s=0}^{10} \binom{10}{s} B_{10-s} = \sum_{k=0}^{10} \binom{10}{k} B_k = B_{11} = 678,570.$$

Figure 9: A few potential industry structures



(a) H_M (b) Necessarily unstable (c) Necessarily unstable (d) Possibly stable

instance, multiple large conglomerates can co-exist, competing against each other across many markets. As we observed in the introduction, this is characteristic of contemporary industrial organisation. Second, giant conglomerates can co-exist with smaller and more specialized firms.

Finally, we develop a partial lower bound on the size of the largest firm in stable industry structures. To do so we first generalise the notion of complementary capabilities.

Definition (Supermodular Capabilities)**.** *We say capabilities are supermodular if for all i, j*

$$\pi_{lj}(H_M, H'_F) + \pi_{hj}(H_M, H'_F) \geq \pi_{ij}(H_M, H_F) + \pi_{kj}(H_M, H_F)$$

where $\theta_{lj} = \theta_{ij} \cup \theta_{kj}$ and $\theta_{hj} = \theta_{ij} \cap \theta_{kj}$. The inequality is strict if both (i) $\max\{\pi_{ij}, \pi_{kj}\} > 0$; and (ii) $\theta_{ij} \not\subseteq \theta_{kj}$, $\theta_{kj} \not\subseteq \theta_{ij}$

Supermodularity is a standard way of incorporating complementarities. In terms of the profit function it requires that it is profitable to create a very strong firm through a merger and spin-off a second weaker firm from the duplicate competencies.

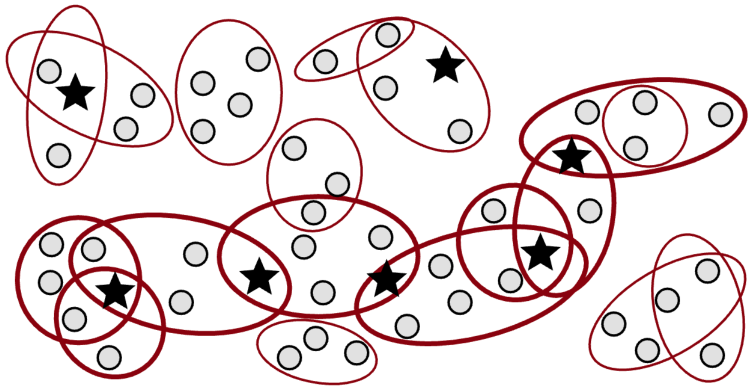
Definition (Scarce Capabilities)**.** *A set of capabilities $A \subseteq \mathcal{A}$ is scarce if for all $a \in A$, $|\mu(a)| = 1$.*

Examples of scarce capabilities abound: some pharmaceutical patents, proprietary algorithms, certain data or human capital, etc. These capabilities in turn offer firms competitive advantages in markets which value them, not simply because they might be valuable, but also because other firms are unable to leverage the same capability. Indeed, firms often go great lengths to ensure such capabilities remain out of reach for their competitors—for example, through protracted and costly patent lawsuits, non-compete clauses for key workers, or resources spent to protect themselves against corporate espionage.

For a set of capabilities $A \subseteq \mathcal{A}$ that are all scarce and path connected on the subhypergraph $\hat{H}_M(A)$, let $\Sigma(A) := \bigcup_{j: A \cap M_j \neq \emptyset} M_j$ be the *scarce-connected capabilties.* We let $\overline{\Sigma}$ denote a largest set of scarce-connected capabilities. Figure 10 shows an example of $\overline{\Sigma}$, where the scarce capabilities are denoted with a star, and $\overline{\Sigma}$ is given by the union of the capabilities valued by the markets in bold.

Figure 10: The largest set of scarce-connected capabilities

Note: Scarce capabilities are represented by stars. The largest set of scarce-connected capabilities ($\overline{\Sigma}$) is given by those capabilities value by the markets in bold.



(a) H_M

The following proposition is a partial analogue to Proposition 1 (iii) and provides a lower bound on the size of the largest firm in stable hypergraphs that is intimately connected with scarcity.

Proposition 5. *Suppose capabilities are valued and supermodular, then for all κ not too convex, any set of scarce-connected capabilities must be held by one firm in all stable industry structures. Thus $|F_{max}| \geq |\overline{\Sigma}|$.*

In the limit, as all capabilities become scarce, our upper and lower bounds coincide:

Corollary. *If all capabilities are scarce then $|\overline{\Sigma}| = |C_{max}|$ and so for all stable industry structures $|F_{max}| = |C_{max}|$ and this firm operates in $|M_{max}|$ markets.*

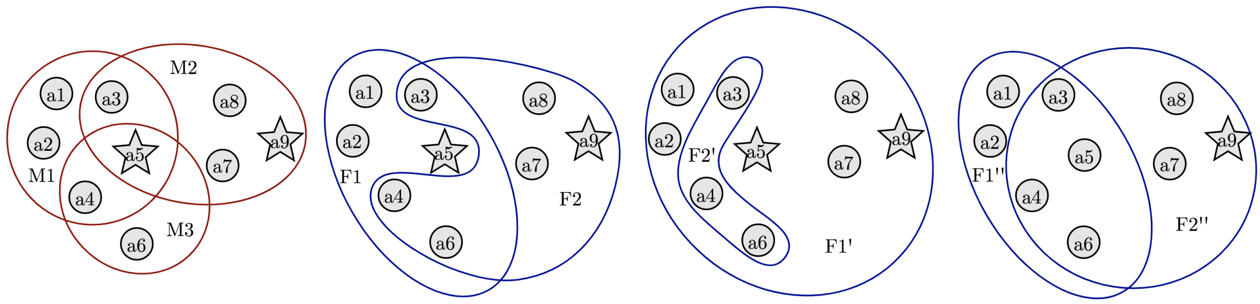
This is unsurprising since our baseline model can be thought of as the special case in which every capability is scarce (so there is a bijection between $\mathcal{A}$ and $\mathcal{B}$). While the upper and lower bound always coincide as all capabilities become scarce, far fewer capabilities need to be scarce for the lower bound to approach the upper bound. Figure 10 provides an illustration in which the lower bound is close to the upper bound even though only 7 out of 50 capabilities are scarce. This figure takes the example of a market hypergraph with a giant component we saw in Figure 6(b) and provides an example of the lower bound when some of those capabilities are scarce (using stars to represent the scarce capabilities). The largest set of scarce-connected capabilities is given by those valued by the markets in bold. Proposition 5 then shows that, when maintenance costs are not too convex and capabilities are both valued and supermodular, then in all stable industry structures there must exist a firm that holds all the capabilities valued by the markets in bold.

The proof of Proposition 5 is in Appendix A.5. Some intuition for Proposition 5 is as follows. Suppose the set $A \subseteq \mathcal{A}$ is both scarce and connected on the subhypergraph $\hat{H}_M(A)$. Now consider some scarce capability $a_1 \in A$ held by firm 1. This capability not only serves as a source of competitive advantage across markets which value a_1, but also as a source of synergies which firm 1 might draw upon to merge with its competitors. It can be shown this implies for any market which values a_1, firm i must hold all capabilities valued by that market, otherwise it must have some profitable procurement or merger. Now consider another scarce capability $a_n \in A$ held by firm n. The same argument must also apply, and firm n must hold all capabilities valued by all markets which value a_n. But since $\hat{H}_M(A)$ is connected and $a_1, a_n \in A$, then a_1 and a_n are path connected such that there exists some path $\{a_1, M_1, a_2, \ldots, a_{n-1}, M_{n-1}, a_n\}$ where for $1 \leq i \leq n-1$, $a_i, a_{i+1} \in M_i$. This then implies there must be a cascade of mergers or procurements along this path such that firm 1 and firm n end up merging.

For concreteness, consider the three markets on the bottom left of Figure 10 which has been reproduced below in Panel (a) of Figure 11. Panel (b) shows a corresponding firm hypergraph where firm 1 holds the scarce capability a_5. Now observe either (i) firm 1 or firm 2 operates in market 2, or (ii) both firms 1 and 2 operate in market 2. In both cases, since neither firm has their capabilities relevant to market 2 nested within the other, this merger generates synergies for market 2. In particular, $\theta_{22} = \{a_3, a_7, a_8, a_9\}$ and $\theta_{12} = \{a_3, a_5\}$. Now consider a merger which generates the union of both firms' capabilities while spinning off the intersection as a competitor firm as shown in Panel (c). This merger must, by the supermodularity of profits, be weakly profitable in markets 1 and 3 (in this case, gross profits in these markets remain constant), and strictly gross profitable in market 2. Hence for sufficiently low convexity of κ, this merger is strictly net profitable.

In contrast, consider an alternate firm hypergraph H''_F shown in Panel (d) where a_5 is not scarce and held by both firms $1''$ and $2''$. Observe that firm $1''$ holds all the capabilities valued by markets 1 and 3, while firm $2''$ holds all the capabilities valued by markets 2 and 3. This implies all mergers between $1''$ and $2''$ create no synergies and will be strictly net unprofitable. This can inhibit the emergence of a giant firm in stable industry structures. However, scarce capabilities are not necessary for the emergence of a giant firm. In this example, a firm could still hold all the capabilities valued by markets 1-3. For instance, we might have multiple copies of $\{a_7, a_8, a_9\}$ which firm $1''$ profitably procures to strengthen its competitiveness in market 2.

Figure 11: An example of how scarce capabilities generate synergies



(a) H_M (b) H_F with scarcity (c) H'_F with scarcity (d) H''_F without scarcity

To summarize, Proposition 5 carries two key implications. First, scarce capabilities at the intersection of many markets can function as a crucial source of synergies for conglomeratization. Second, chains of adjacent and scarce capabilities can magnify this force. This implies that a small set of scarce but related and widely valued capabilities can necessitate the presence of a large conglomerate.

6 Conclusions

We have proposed a capability-based explanation for the sudden and unprecedented emergence of what we have termed internet conglomerates. This helps to close a gap between the economics and management literature. Our explanation is that changes in technology have made some capabilities more widely valued across different markets, and that has facilitated the rapid emergence of internet conglomerates. While this is not the whole story, we do think it is a useful framework for making sense of the recent string of acquisitions by internet conglomerates. Of course, antitrust also plays a central role. In our framework, capabilities being more widely valued creates the incentive for conglomerate mergers—whether or not these are permitted is left to antitrust authorities. Often these mergers are between firms operating in different markets, and so consistent with the traditional approach to antitrust they have thus far largely escaped intense scrutiny.[36]

Our framework is relatively simple and versatile. While here we use it to study the forces underlying recent trends toward conglomeratization, it might also be used to address a range of other theoretical and empirical questions. For example, it could underpin a dynamic analysis of the evolution of industry structure. Such an approach would be able to systematically address issues such as hoarding—firms holding multiple competencies corresponding to the same capability. While hoarding is unprofitable in our framework because extra competencies have no myopic value and are costly to maintain, it might be dynamically profitable by suppressing future competition in a similar spirit to the "killer acquisitions" documented by Cunningham et al. (2021).[37] This possibility is important for antitrust policy, particularly regarding conglomerate mergers and regulation of anticompetitive behaviour. Questions about hoarding are inevitably about capabilities—our framework offers a tractable means of putting them at the heart of the analysis. More generally, there are interesting questions on joint ventures and the formation of syndicates to bid on large scale projects that we believe our framework is well suited to analyse.

References

Acemoglu, D. and P. D. Azar (2020): "Endogenous production networks," *Econometrica*, 88, 33–82.

[36] It is possible to incorporate antitrust directly into our model by blocking mergers that result in reduced consumer surplus in any market. Doing this requires defining a consumer surplus function. Supermodularity of these consumer surplus functions is sufficient for our results to go through (details available upon request). With supermodular consumer surplus functions a merger that reduces consumer surplus in a market must be between firms with overlapping capabilities, but then the duplicate capabilities can be spun off into a competitor firm to alleviate antitrust concerns.

[37] Indeed, in simulations we have found hoarding to often be profitable, especially as the market hypergraph becomes more connected.

Acemoglu, D. and A. Tahbaz-Salehi (2020): "Firms, failures, and fluctuations: the macroeconomics of supply chain disruptions," Tech. rep., National Bureau of Economic Research.

Aghion, P., A. Bergeaud, T. Boppart, P. J. Klenow, and H. Li (2019): "A theory of falling growth and rising rents," *NBER Working Paper.*

Autor, D., D. Dorn, L. F. Katz, C. Patterson, and J. Van Reenen (2020): "The fall of the labor share and the rise of superstar firms," *The Quarterly Journal of Economics*, 135, 645–709.

Barney, J. B. (1991): "Firm Resources and Sustained Competitive Advantage," *Journal of Management*, 17, 99–120.

Bartik, T. J. (1987): "The estimation of demand parameters in hedonic price models," *Journal of Political Economy*, 95, 81–88.

Bena, J. and K. Li (2014): "Corporate innovations and mergers and acquisitions," *The Journal of Finance*, 69, 1923–1960.

Bessen, J. E. (2017): "Industry Concentration and Information Technology," *Boston Univ. School of Law, Law and Economics Research Paper.*

Bimpikis, K., S. Ehsani, and R. Ilkiliç (2019): "Cournot competition in networked markets," *Management Science*, 65, 2467–2481.

Cabrales, A., P. Gottardi, and F. Vega-Redondo (2017): "Risk sharing and contagion in networks," *The Review of Financial Studies*, 30, 3086–3127.

Carvalho, V. M. and N. Voigtländer (2015): "Input diffusion and the evolution of production networks," *NBER Working Paper No. 20025.*

Chandler, A. D. (1962): *Strategy and structure: Chapters in the history of the industrial enterprise*, vol. 120, MIT Press.

Chatterjee, S. (1986): "Types of synergy and economic value: The impact of acquisitions on merging and rival firms," *Strategic Management Journal*, 7, 119–139.

Condorelli, D., A. Galeotti, and L. Renou (2016): "Bilateral trading in networks," *The Review of Economic Studies*, 84, 82–105.

Crouzet, N. and J. C. Eberly (2019): "Understanding weak capital investment: The role of market concentration and intangibles," *NBER Working Paper.*

Cunningham, C., F. Ederer, and S. Ma (2021): "Killer acquisitions," *Journal of Political Economy*, 129, 649–702.

De Ridder, M. (2019): "Market power and innovation in the intangible economy," Working paper, London School of Economics.

Devos, E., P.-R. Kadapakkam, and S. Krishnamurthy (2008): "How do mergers create value? A comparison of taxes, market power, and efficiency improvements as explanations for synergies," *The Review of Financial Studies*, 22, 1179–1211.

Elliott, M. (2015): "Inefficiencies in networked markets," *American Economic Journal: Microeconomics*, 7, 43–82.

Elliott, M., C.-P. Georg, and J. Hazell (2021): "Systemic risk shifting in financial networks," *Journal of Economic Theory*, 191, 105157.

Elliott, M., B. Golub, and M. V. Leduc (2020): "Supply network formation and fragility," *Available at SSRN 3525459.*

Erol, S. and R. Vohra (2018): "Network formation and systemic risk," *Available at SSRN 2546310.*

Farboodi, M. (2017): "Intermediation and voluntary exposure to counterparty risk," *Mimeo.*

Farrell, J. and C. Shapiro (1990): "Horizontal mergers: an equilibrium analysis," *American Economic Review*, 107–126.

Gaudet, G. and S. W. Salant (1991): "Uniqueness of Cournot equilibrium: new results from old methods," *The Review of Economic Studies*, 58, 399–404.

Goyal, S. (2017): "Networks and Markets," in *Advances in Economics and Econometrics: Volume 1: Eleventh World Congress*, Cambridge University Press, vol. 58, 215.

Goyal, S., J. L. Moraga-González, and A. Konovalov (2008): "Hybrid R&D," *Journal of the European Economic Association*, 6, 1309–1338.

Hoberg, G. and G. Phillips (2010): "Product market synergies and competition in mergers and acquisitions: A text-based analysis," *The Review of Financial Studies*, 23, 3773–3811.

——— (2017): "Conglomerate industry choice and product language," *Management Science.*

Hsieh, C.-T. and E. Rossi-Hansberg (2019): "The industrial revolution in services," NBER working paper, University of Chicago.

Jackson, M. O. and A. Watts (2001): "The existence of pairwise stable networks," *Seoul Journal of Economics*, 14, 299.

Jensen, M. C. (1986): "Agency costs of free cash flow, corporate finance, and takeovers," *The American Economic Review*, 76, 323–329.

Kranton, R. E. and D. F. Minehart (2001): "A theory of buyer-seller networks," *American Economic Review*, 91, 485–508.

Larsson, R. and S. Finkelstein (1999): "Integrating strategic, organizational, and human resource perspectives on mergers and acquisitions: A case survey of synergy realization," *Organization Science*, 10, 1–26.

Lashkari, D., A. Bauer, and J. Boussard (2018): "Information technology and returns to scale," *SSRN Working Paper.*

Luttmer, E. G. (2011): "On the mechanics of firm growth," *The Review of Economic Studies*, 78, 1042–1068.

Makri, M., M. A. Hitt, and P. J. Lane (2010): "Complementary technologies, knowledge relatedness, and invention outcomes in high technology mergers and acquisitions," *Strategic Management Journal*, 31, 602–628.

Malamud, S. and M. Rostek (2017): "Decentralized exchange," *American Economic Review*, 107, 3320–62.

Myerson, R. B. (1980): "Conference structures and fair allocation rules," *International Journal of Game Theory*, 9, 169–182.

Nava, F. (2015): "Efficiency in decentralized oligopolistic markets," *Journal of Economic Theory*, 157, 315–348.

Nocke, V. and S. Yeaple (2007): "Cross-border mergers and acquisitions vs. greenfield foreign direct investment: The role of firm heterogeneity," *Journal of International Economics*, 72, 336–365.

——— (2014): "Globalization and multiproduct firms," *International Economic Review*, 55, 993–1018.

Oberfield, E. (2018): "A theory of input–output architecture," *Econometrica*, 86, 559–589.

Porter, M. E. (2008): *Competitive advantage: Creating and sustaining superior performance*, Simon and Schuster.

Prahalad, C. and G. Hamel (1990): "The core competence of the corporation," *Harvard Business Review*, 68, 79–91.

Roll, R. (1986): "The hubris hypothesis of corporate takeovers," *Journal of Business*, 197–216.

Rostek, M. J. and J. H. Yoon (2020): "Exchange design and efficiency," *Available at SSRN 3604976.*

Salant, S. W., S. Switzer, and R. J. Reynolds (1983): "Losses from horizontal merger: the effects of an exogenous change in industry structure on Cournot-Nash equilibrium," *The Quarterly Journal of Economics*, 98, 185–199.

Schmidt-Pruzan, J. and E. Shamir (1985): "Component structure in the evolution of random hypergraphs," *Combinatorica*, 5, 81–94.

Sutton, J. (2012): *Competing in capabilities: the globalization process*, Oxford University Press.

Szidarovszky, F. and S. Yakowitz (1982): "Contributions to Cournot oligopoly theory," *Journal of Economic Theory*, 28, 51–70.

Teece, D. J. (1982): "Towards an economic theory of the multiproduct firm," *Journal of Economic Behavior & Organization*, 3, 39–63.

Temin, P. and L. Galambos (1987): *The fall of the Bell system: A study in prices and politics*, Cambridge University Press.

van den Nouweland, A., P. Borm, and S. Tijs (1992): "Allocation rules for hypergraph communication situations," *International Journal of Game Theory*, 20, 255–268.

Wernerfelt, B. (1984): "A resource-based view of the firm," *Strategic Management Journal*, 5, 171–180.

Whinston, M. D. (2007): "Antitrust policy toward horizontal mergers," *Handbook of Industrial Organization*, 3, 2369–2440.

Williamson, O. E. (1968): "Economies as an antitrust defense: The welfare tradeoffs," *American Economic Review*, 58, 18–36.

A Omitted Proofs

A.1 Proof of Proposition 3.

Proof of Proposition 3. As mentioned in the main text, the results on the phase transition of $|C_{max}|$ are from the literature on probabilistic combinatorics:

Proposition Schmidt-Pruzan and Shamir (1985) The upper bound $|C_{max}|$ has the following properties: There exists a finite constant $\bar{d}$ such that with high probability,

(i) [subcritical case] if $\mathbb{E}[d(|\mathcal{A}|)] < \bar{d}$, then $|C_{max}| = O(\log |\mathcal{A}|)$;

(ii) [supercitical case] if $\mathbb{E}[d(|\mathcal{A}|)] > \bar{d}$, then $|C_{max}| = \Omega(|\mathcal{A}|)$.

It remains to show the results on the phase transition of $|M_{max}|$. We first prove the supercritical case (part (ii)). Since every capability in C_{max} is held by some market, and every market can hold at most t capabilities, we have that $|C_{max}|/t \leq |M_{cmax}|$ where M_{cmax} is the set of markets comprising C_{max}. By definition, $|M_{cmax}| \leq |M_{max}|$. Then since $|C_{max}| = \Omega(|\mathcal{A}|)$, $|M_{max}| = \Omega(|\mathcal{A}|)$.

We now prove the subcritical case (part (i)). Our approach will proceed by first showing that with high probability, C_{max} has at most $O(\log |\mathcal{A}|)$ cycles. We then use this to bind the number of markets.

A cycle of length s on a hypergraph $(A, \{E_i\}_i)$ is a tuple $(a_1, E_1, a_2, \dots a_s, E_s, a_1)$ such that for all $1 \leq i, j \leq s$, (i) $E_i \neq E_j$ whenever $i \neq j$; (ii) $E_i \in \{E_i\}_i$; and (iii) $\{a_i, a_{i+1}\} \subseteq E_i$ where we set $a_{s+1} = a_1$. Let CY be the set of distinct cycles on the random hypergraph H with distribution $\mathcal{R}(\mathcal{A}, \mathbf{p})$, and $CY^s \subseteq CY$ be the set of cycles of length s. Observe

$$\mathbb{E}\Big[|CY^s|\Big] = \binom{|\mathcal{A}|}{s}(s-1)!\Big[1 - \prod_{k=2}^{t}(1-p_k)^{\binom{|\mathcal{A}|-2}{k-2}}\Big]^s$$

where $\binom{|\mathcal{A}|}{s}$ is capabilities all cycles of length s can be defined on, and fixing a particular set of capabilities of size s, $(s-1)!$ gives the number of potential cycles which can be realised on those capabilities. Now having fixed this cycle $(a_1, a_2, \dots, a_s = a_1)$, the last term is the probability that this given cycle is realised. Note here that a step from a_i to a_{i+1} can be fulfilled by any hyperedge of size $2 \leq k \leq t$ hence the probability that there is a path from a_i to a_{i+1} is given by the term in the square bracket.

Now observing that

$$\begin{aligned}
\Big(\mathbb{E}[d(|\mathcal{A}|, \mathbf{p})]\Big)^s = \Big[\sum_{k=2}^{t}\binom{|\mathcal{A}|-1}{k-1}(k-1)p_k\Big]^s &= (|\mathcal{A}|-1)^s\Big[\sum_{k=2}^{t}\binom{|\mathcal{A}|-1}{k-1}\Big(\frac{k-1}{|\mathcal{A}|-1}\Big)p_k\Big]^s \\
&\geq \binom{|\mathcal{A}|}{s}s!\Big[\sum_{k=2}^{t}\binom{|\mathcal{A}|-2}{k-2}p_k\Big]^s \\
&\geq \binom{|\mathcal{A}|}{s}s!\Big[1 - \prod_{k=2}^{t}(1-p_k)^{\binom{|\mathcal{A}|-2}{k-2}}\Big]^s = s\mathbb{E}[|CY|^s]
\end{aligned}$$

where the last inequality follows because

$$
\begin{aligned}
1-\sum_{k=2}^{t}\binom{|\mathcal{A}|-2}{k-2}p_k &\leq \exp\Bigg(-\sum_{k=2}^{t}\binom{|\mathcal{A}|-2}{k-2}p_k\Bigg)\\
&= \prod_{k=2}^{t}\exp\Bigg(-\binom{|\mathcal{A}|-2}{k-2}p_k\Bigg)\\
&\leq \prod_{t=2}^{t}(1-p_k)^{\binom{|\mathcal{A}|-2}{k-2}}
\end{aligned}
$$

where the first inequality follows from $1+x \leq \exp(x)$ for all $x \in \mathbb{R}$. To see the last inequality, for each term in the product, take the natural logarithm and use the inequality that $\log(1-x) \geq -x$ for sufficiently small $x > 0$. Here $x = p_k$ and since we are in the subcritical case, $p_k \to 0$ as $|\mathcal{A}| \to \infty$ so the inequality does indeed follow.

As such,

$$
\mathbb{E}[|CY|] = \sum_{s=2}^{\infty}\mathbb{E}[|CY|^s] \leq \sum_{s=2}^{\infty}\frac{\big(\mathbb{E}[d(|\mathcal{A}|,\mathbf{p})]\big)^s}{s} = O(1) \quad \text{whenever } \mathbb{E}[d(|\mathcal{A}|,\mathbf{p})] < 1
$$

which upper bounds the expected number of cycles in the subcritical case.[38] Now define the cycling index of any connected component on the capabilities C and markets M with

$$
CI(C,M) = \Bigg(\sum_{k=1}^{t}(k-1)|M^k|\Bigg) - (|C|-1)
$$

where $M^k \subseteq M$ are the set of markets of size k. The cycling index is useful because the component (C,M) must have at least $CI(C,M)$ cycles—this gives a deterministic lower bound on the minimum number of cycles in terms of the size of the component, and the number of markets of various sizes.

We are now ready to establish the claim in part (i). In the subcritical case, we know H drawn from $\mathcal{R}(\mathcal{A},\mathbf{p})$ has at most $O(1)$ cycles with high probability since by Markov's inequality,

$$
\mathbb{P}(|CY| \geq f(|\mathcal{A}|)) \leq \frac{\mathbb{E}[|CY|]}{f(|\mathcal{A}|)} = 0 \quad \text{as } |\mathcal{A}| \to \infty
$$

for any f for which $f \to \infty$ as $|\mathcal{A}| \to \infty$. But since the number of cycles on the whole hypergraph is weakly greater than the number of cycles on any component, this implies that with high probability, the subhypergraph (C_r, M_r) comprising the capabilities held by component r and its constituent markets has at most $O(1)$ cycles. But since $CI(C_r, M_r)$ gives a lower bound

[38] This is similar to Lemma 3.15 of Schmidt-Pruzan and Shamir (1985) though we give a detailed proof.

on the number of cycles on (C_r, M_r), we must have that with high probability,

$$CI(C_r, M_r) = \left(\sum_{k=1}^{t}(k-1)|M_r^k|\right) - (|C_r| - 1) = O(1).$$

Hence with high probability,

$$|M_r| = \sum_{k=1}^{t}|M_r^k| \leq \left(\sum_{k=1}^{t}(k-1)|M_r^k|\right) \leq |C_{max}| + O(1) = O(\log|\mathcal{A}|)$$

where the last inequality follows because $|C_r| \leq |C_{max}| = O(\log(|\mathcal{A}|)$. Since r was arbitrary, this upper bound applies to every component, completing the proof of part (i). $\square$

A.2 Proof of Lemmas 1-3. We omit the proof of Lemma 1 since it is virtually identical to that of Lemma 2.

A.2.1 Proof of Lemma 2

Proof. If either $\mu(F_i) \not\subseteq \hat{A}_i$ or $|\mu(F_i)| < |F_i|$, then there exists a competency $b \in F_i$ that firm i does not use it in any market. Disposing of one such competency therefore leaves i's relevant capabilities unaffected for all markets $j \in J_{i,\pi>0}$ and hence firm i's gross profit are unaffected. We now consider the case where $\mu(F_i) \subseteq \hat{A}_i$ and $|\mu(F_i)| = |F_i|$, but $\hat{H}_M(\mu(F_i))$ with all markets $j \notin J_{i,\pi>0}$ then removed, contains more than one component (condition (iii)). Since we are considering demergers, we can restrict our attention to the market hypergraph H'_M which is obtained from H_M by removing all markets $\mathcal{M} \setminus J_{i,\pi>0}$. This is because if firm i did not initially operate in market j, none of the firms generated by the demerger will do so. Next, denote the components of $\hat{H}'_M(\mu(F_i))$ by $\{C_1, C_2, \ldots, C_p\}$ where $p > 1$. Now consider a demerger of firm i along the boundary of these components into the firms $\mathcal{F}_i$ such that $\{\mu(F_k) : k \in \mathcal{F}_i\} = \{C_1, C_2, \ldots, C_p\}$. This demerger is always feasible since $C \cap C' = \emptyset$ for all $C, C' \in \{C_1, C_2, \ldots, C_p\}$. In particular, for each market $j \in J_{i,\pi>0}$, there exists some firm $k \in \mathcal{F}_i$ such that $\theta_{kj} = \theta_{ij}$. By construction, all other firms $k' \in \mathcal{F}_i \setminus \{k\}$ cannot hold capabilities relevant to j, so $\theta_{k'j} = \emptyset$ and hence firm k makes identical gross profits as firm i in market j. Hence $\sum_{j\in J_{i,\pi>0}} \pi_{ij}(H'_M, H_F) = \sum_{k\in\mathcal{F}_i}\sum_{j\in J_{i,\pi>0}} \pi_{kj}(H'_M, H'_F)$ where H'_F is the firm hypergraph after the demerger i.e., the demerged firms make the same gross profits across all the markets as firm i. Therefore, there exists a demerger that firm i can undertake that weakly increases its gross profits. $\square$

A.2.2 Proof of Lemma 3

Proof. Consider a demerger of firm i into a set of firms $\mathcal{F}_i$. If all three conditions hold then $\mu(F_i) \subseteq \hat{A}_i$ and $|\mu(F_i)| = |F_i|$. Hence all possible disposals involve competencies that are used by firm i in at least one market and any disposal will strictly reduce i's gross profits. Further, as $\hat{H}_M(\mu(F_i))$ with all markets $j \notin J_{i,\pi>0}$ then removed, must contain a single component, there is a path between any pair of firm i's capabilities through markets which firm i operates in. Hence all possible partitions of the capabilities into more than one non-empty partition element, will partition the capabilities associated with at least one market firm i originally operates in into more than one non-empty partition element. Thus for a demerger into firms $\mathcal{F}_i$ there will be at least one market $j \in J_{i,\pi>0}$ such that $\{\mu(F_k) \cap M_j\} \subset \{\mu(F_i) \cap M_j\}$ for all firms $k \in \mathcal{F}_i$. For all such markets j, as $|\mu(F_i)| = |F_i|$, the firms $\mathcal{F}_i$ created by the demerger have disjoint capabilities in these markets, and so since capabilities are complementary, the demerger is strictly unprofitable. For all other markets j', either there exists a firm $k \in \mathcal{F}_i$ such that $\mu(F_k) \cap M_{j'} = \mu(F_i) \cap M_{j'}$ or a similar argument as before applies. Hence, gross profits strictly decrease as claimed. $\square$

A.3 Proof of Proposition 2.

Proof. We already argued that the upper and lower bounds continue to obtain under coalitional stability since the set of coalitional stable hypergraphs are subset of stable hypergraphs. For tightness, we proceed as before by construction. Recall we indexed the components of H_M with $\{1...p\}$. Now for each $i \in \{1...p\}$, let $F_i = C_i$ i.e. for each component of the market hypergraph, we construct a corresponding firm which holds the same capabilities which comprises the component. We now show that the ensuing firm hypergraph is coalitionally stable.

Consider any coalitional deviation which reconfigures the firms $\mathcal{F} \subseteq \mathcal{N}$ into the firms $\mathcal{F}'$ such that $\bigcup_{i' \in \mathcal{F}'} F_{i'} \subseteq \left\{S \cup \bigcup_{i \in \mathcal{F}} F_i\right\}$. Note that if any firm $i \in \mathcal{F}'$ generated by the coalitional deviation spans multiple components, by Lemma 2 it has a strictly net profitable demerger. Further, since demerger only entails reducing the total fixed costs borne by the coalition, but leaves the competitive landscape in each market unchanged, this demerger is also strictly net profitable for the whole coalition. As such, this coalition of firms $\mathcal{F}'$ makes strictly less profits than an alternate coalition $\mathcal{F}''$ which is obtained from performing all such demergers until none remain. We will now show that the alternate coalition $\mathcal{F}''$ makes strictly less net profits than the original set of firms $\mathcal{F}$. For each $r \in \{1...p\}$ corresponding to a component of H_M, denote $\mathcal{F}''_r := \{i \in \mathcal{F}'' : F_i \cap C_r \neq \emptyset\}$. Note that by construction $\mathcal{F}''_r \cap \mathcal{F}''_{r'} = \emptyset$ for all $r \neq r'$. Use $\mathcal{C}_r := \{j \in \mathcal{M} : M_j \subseteq C_r\}$ to denote the set of markets which are contained in the component r.

We can exclude the trivial case in which the set $\mathcal{F}''_r$ only contains a single firm identical to the original firm r with capabilities $F_r = C_r$. Note that for all $r \in \{1...p\}$, the set of firms $\mathcal{F}''_r$ collectively make strictly less *gross* profits than the single firm r in the original hypergraph

$$\sum_{j \in \mathcal{C}_r} \pi_{rj}(H_M, H_F) > \sum_{j \in \mathcal{C}_r} \sum_{i \in \mathcal{F}''_r} \pi_{ij}(H_M, H'_F)$$

where H_F is the original hypergraph before the coalitional deviation, and H'_F is the hypergraph after the deviation. To see this, note that since capabilities can only be held by a single firm, there must exist some market $j \in \mathcal{C}_r$ for which there does not exist a firm $r' \in \mathcal{F}''$ such that

$F_{r'} \supseteq M_j$ (otherwise, since components are path-connected, we have a contradiction). For such a market j, the inequality is strict i.e. firm r makes strictly more gross profits than the collective gross profits of the firms $\mathcal{F}''$ because (i) r enjoys a strictly lower marginal cost than any firm in $\mathcal{F}''_r$; and (ii) the presence of additional firms competing in j can only erode the collective profits of the firms $\mathcal{F}''$ in j.

Now since this is true for all r, by the independence of profits across components, we have

$$\begin{aligned}\sum_{j\in\mathcal{M}}\sum_{i\in\mathcal{F}}\pi_{ij}(H_M,H_F) &= \sum_{r\in\{1\dots p\}}\sum_{j\in C_r}\pi_{rj}(H_M,H_F)\\ &> \sum_{r\in\{1\dots p\}}\sum_{j\in\mathcal{C}_r}\sum_{i\in\mathcal{F}''_r}\pi_{ij}(H_M,H'_F) = \sum_{j\in\mathcal{M}}\sum_{i\in\mathcal{F}''}\pi_{ij}(H_M,H'_F)\end{aligned}$$

and for any $\varepsilon > 0$, we can reduce the convexity of κ until $\sum_{i\in\mathcal{F}}\kappa(|F_i|)-\sum_{i\in\mathcal{F}''}\kappa(|F_i|) < \varepsilon$ which then implies the coalitional deviation generating $\mathcal{F}''$ under consideration is strictly dominated by not deviating at all. Then since the deviation generating $\mathcal{F}''$ in turn dominates the one generating $\mathcal{F}'$, the original deviation under consideration is also strictly unprofitable relative to no deviation at all.

$\square$

A.4 Proof of Proposition 4. We show the existence of a stable firm hypergraph for any sets $(\mathcal{A},\mathcal{B})$, any matching μ, and any market hypergraph H_M by construction. Consider

---

Algorithm 1: Stable Firm Hypergraph Construction

---

input: $(\mathcal{A},\mathcal{B})$, $\mu : \mathcal{A}\cup\mathcal{B} \rightrightarrows \mathcal{A}\cup\mathcal{B}$, H_M;

initialize: $H_F^{(0)} = (\mathcal{B},\emptyset)$, $S^{(0)} = \mathcal{B}$, $q = 1$;

while $q = 1$ *or* $H_F^{(q-2)} \neq H_F^{(q-1)}$ **do**

  set $\hat{H}_M(\mu(S^{(q-1)}))$ with components $\hat{C}_r^{(q)}$ where $r \in \mathcal{P}^{(q)} = \{1...p^{(q)}\}$;[a]

  for $r = 1$ **to** $p^{(q)}$ **do**

    set $\mu(F_r) = \hat{C}_r^{(q)}$, $|\mu(F_r)| = |\hat{C}_r^{(q)}|$;[b]

    let F_r undertake its most profitable demerger (which could be the trivial demerger generating just itself);

    if *demerger non-unique (r is indifferent between demerging into the firms* $\{\mathcal{F}_{r1},\mathcal{F}_{r2},\dots\mathcal{F}_{rn}\}$*)* **then**

      execute any demerger $\mathcal{F} \in \{\mathcal{F}_{r1},\mathcal{F}_{r2},\dots\mathcal{F}_{rn}\}$ such that there does not exist $\mathcal{F}' \in \{\mathcal{F}_{r1},\mathcal{F}_{r2},\dots\mathcal{F}_{rn}\}$ for which

      (i) for all $i' \in \mathcal{F}'$, there exists $i \in \mathcal{F}$ such that $\mu(F_{i'}) \subseteq \mu(F_i)$

      (ii) $|\bigcup_{i\in\mathcal{F}} F_i| > |\bigcup_{i'\in\mathcal{F}'} F_{i'}|$;

    end

    denote the set of firms created from component r with $\mathcal{F}_r^{(q)}$

  end

  denote the set of firms created in this iteration with $\mathcal{F}^{(q)} = \bigcup_{r\in\mathcal{P}^{(q)}} \mathcal{F}_r^{(q)}$;

  update $H_F^{(q)} = (\mathcal{B}, \bigcup_{1\le s\le q} \mathcal{F}^{(s)}), S^{(q)} = \mathcal{B} \setminus \{\bigcup_{r\in\mathcal{P}^{(q)}} \bigcup_{i\in\mathcal{F}_r^{(q)}} F_i\}$;

  $q \leftarrow q + 1$

end

---

[a] The order does not matter since the profitability of demergers is independent of each other because by construction, firms $\mathcal{P}^{(q)}$ operate in disjoint sets of markets.

[b] This implies (i) r holds the same capabilities as the component it is matched to; (ii) r never holds more than a single competency corresponding to the same capability.

We will subsequently show that the resultant hypergraph generated by Algorithm 1 is stable.

A.4.1 Notation and Lemmas for Proposition 4

We set out some notation in advance. Denote the last iteration of Algorithm 1 with u. Use $J_l = \{j \in \mathcal{M} : M_j \cap \mu(F_l) \neq \emptyset\}$ to denote the set of markets the firm l has relevant capabilities for. We can bipartition the markets J_l into the disjoint sets $J_{l,\pi>0}^{(q)}$ and $J_{l,\pi=0}^{(q)}$ which are respectively, the set of markets firm l operates in after iteration q, and the set of markets firm l has relevant capabilities for, but does not operate in after iteration q.

The following lemmas are helpful for proving Proposition 4.

Lemma 4 (Concentric property)**.** *Fix any firm hypergraph $H_F^{(u)}$ created by Algorithm 1. Then for any firm $k \in \mathcal{F}^{(q+1)}$, there must exist $i \in \mathcal{F}^{(q)}$ such that $\mu(F_k) \subseteq \mu(F_i)$.*

This is a striking and general feature of the class of hypergraphs generated by Algorithm 1. In particular, it implies

Corollary (To Lemma 4)**.** *For any two firms i, k in any firm hypergraph $H_F^{(u)}$ generated by Algorithm 1, either (i) $\mu(F_i) \cap \mu(F_k) = \emptyset$; or (ii) $\mu(F_i) \subseteq \mu(F_k)$ or $\mu(F_k) \subseteq \mu(F_i)$ or both.*

Lemma 5 (Firms in the same iteration operate in disjoint markets)**.** *For any iteration q of Algorithm 1 and any two firms $i, k \in \mathcal{F}^{(q)}$, for sufficiently low convexity of κ, firms i and k operate in disjoint sets of markets in the industry structure $(H_M, H_F^{(q)})$.*

Lemma 6 (Firms hold maximal feasible capabilities)**.** *For any firm $i \in \mathcal{F}^{(q)}$, if firm i is operating in market j in the industry structure $(H_M, H_F^{(q)})$, then for sufficiently low convexity of κ, $\{\mu(S^{(q)}) \cap M_j\} \subseteq \mu(F_i)$.*

Lemma 7 (Fixed set of operating markets)**.** *For any q and any firm $i \in \mathcal{F}^{(q)}$, $J_{i,\pi>0}^{(q)} = J_{i,\pi>0}^{(u)}$.*

Lemmas 4-7 are subsequently proven in Supplementary Appendix I.

The proof of Proposition 4 relies on Lemmas 4 - 7. While we take these lemmas as given for now, it is worth noting that they rely on each other in a consistent (non-tautological) way. Lemma 5 relies on none of the others; Lemma 6 uses Lemma 5; Lemma 4 uses Lemmas 5 and 6; Lemma 7 uses Lemma 4.

A.4.2 Proof of Proposition 4

Proof. Our proof is constructive, and proceeds by showing all possible deviations – disposals, demergers, procurements, mergers, and entries—undertaken by firms in the industry structure generated by Algorithm 1 cannot be strictly net profitable. The proof considers these deviations sequentially.

(i) No firm has a weakly net profitable disposal.

The essential underlying idea is as follows. In each iteration of Algorithm 1, a firm $i \in \mathcal{F}^{(q)}$ constructed in that iteration undertakes its most profitable demerger. This generates a set of markets that i is operational in. In subsequent iterations, new firms are created. If such firms enter these markets, they must, by construction, find it profitable to do so—otherwise an alternate demerger would have been undertaken. However, by Lemma 6, they must hold fewer relevant capabilities for such markets than firm i. Then since capabilities are valued, each additional capability firm i deploys in such markets generates at least $\kappa(1)$ of gross profits, and so for κ with sufficiently low convexity, firm i continues to find all disposals net unprofitable.

We now make this argument precise and proceed by induction. By construction, all firms in the set $\mathcal{F}^{(u)}$ have no net profitable disposal since they are created in the last iteration of Algorithm 1.[39] We now establish for any iteration $q < u$, if all firms $\mathcal{F}^{(q+1)}$ find it strictly net unprofitable

[39] By the tie breaking rule, if they did have a net profitable disposal, they would have undertaken an alternate demerger rather than the one in fact performed.

to undertake any disposal in the industry structure $(H_M, H_F^{(u)})$, then all firms $\mathcal{F}^{(q)}$ must also find it strictly net unprofitable to undertake any disposal in the industry structure $(H_M, H_F^{(u)})$.

For a given firm $i \in \mathcal{F}^{(q)}$, denote the set of all firms generated in iteration $q+1$ which are nested in i with $\mathcal{F}_{\subseteq i}^{(q+1)} := \{k \in \mathcal{F}^{(q+1)} : \mu(F_k) \subseteq \mu(F_i)\}$. It will be sufficient to show that if all firms in the set $\mathcal{F}_{\subseteq i}^{(q+1)} \subseteq \mathcal{F}^{(q+1)}$ have no net profitable disposal, then neither does firm i. First observe from Lemmas 5 and 7, for all other firms $i' \in \mathcal{F}^{(q)} \setminus \{i\}$ created in iteration q, $J_{i,\pi>0}^{(u)} \cap J_{i',\pi>0}^{(u)} = \emptyset$. The concentric property of Lemma 4 also implies no other firm in the set $\mathcal{F}^{(q+1)} \setminus \mathcal{F}_{\subseteq i}^{(q+1)}$ competes with firm i in overlapping markets. As such, the only firms which compete in the same markets as firm i are the firms $\mathcal{F}_{\subseteq i}^{(q+1)}$ and their nested firms.

We now rule out two trivial cases. First consider the case in which $\mathcal{F}_{\subseteq i}^{(q+1)} = \emptyset$ i.e. firm i has no nested firms. This might obtain because, having generated the firm i in iteration q, all interim firms generated in iteration $q+1$ which intersect firm i face strong competition from the firms generated before iteration q, and so find it profitable to dispose of all competencies. If so, then by the above observation, there are no firms generated after iteration q which operate in the markets $J_{i,\pi>0}^{(u)}$. By our regularity assumption (iv), this implies firm i's profits in all such markets must remain unchanged between iteration q and iteration u. Since firm i found it net unprofitable to undertake any disposal in iteration q, then this must also be the case in industry structure $(H_M, H_F^{(u)})$. Next consider the case in which there exists some $k \in \mathcal{F}_{\subseteq i}^{(q+1)}$ such that $\mu(F_i) = \mu(F_k)$. We know Algorithm 1 never creates firms with multiple competencies corresponding to the same capability and since firms i and k have identical capabilities, they must make the same profits in all markets, and bear the same maintenance costs. Then since firm k, by the induction hypothesis, finds all disposals strictly net unprofitable, so too must firm i. The remaining non-trivial case to consider is therefore when $\mu(F_k) \subset \mu(F_i)$ for all firms $k \in \mathcal{F}_{\subseteq i}^{(q+1)}$.

To rule out the remaining case, we will consider potential disposals undertaken by firm i, and show that i cannot find it net profitable to dispose of capabilities relevant to markets subsequent firms operate in, or capabilities only relevant to markets no subsequent firms operate in.

Consider some disposal of the capabilities $D \subseteq \mu(F_i)$ undertaken by i. We first rule out the possibility that $D \cap \{\bigcup_{j \in J_{k,\pi>0}^{(u)}} M_j\} \neq \emptyset$ for some $k \in \mathcal{F}_{\subseteq i}^{(q+1)}$. First observe $D = \mu(F_i) \cap \{\bigcup_{j \in J_{k,\pi>0}^{(u)}} M_j\}$ i.e. disposing all capabilities relevant to markets k operates in must be strictly dominated by an alternate disposal retaining the capabilities $\mu(F_k)$ for sufficiently low convexity of κ, since by the induction hypothesis, firm k does not find it net profitable to dispose of any capability and so it must be making strictly net positive profits. Further, for sufficiently low convexity of κ, this alternate disposal retaining $\mu(F_k)$ must also dominate one which retains only a strictly subset of the capabilities $\mu(F_k)$ since by the induction hypothesis, firm k does not find it net profitable to dispose of any capability. Next, observe each capability in the set $\left\{\mu(F_i) \cap \{\bigcup_{j \in J_{k,\pi>0}^{(u)}} M_j\}\right\} \setminus \mu(F_k)$ is valuable on the margin since capabilities are valued, and so for sufficiently low convexity of κ, the disposal retaining capabilities $\mu(F_k)$ is in turn dominated by a disposal which retains all capabilities $\mu(F_i) \cap \{\bigcup_{j \in J_{k,\pi>0}^{(u)}} M_j\}$. We can therefore rule out any disposal D which disposes of any capability relevant to any market in the set

$\bigcup_{k\in\mathcal{F}^{(q+1)}_{\subseteq i}} J^{(u)}_{k,\pi>0}$.

Finally, we consider disposals of capabilities relevant only to the markets $J_i \setminus \left\{\bigcup_{k\in\mathcal{F}^{(q+1)}_{\subseteq i}} J^{(u)}_{k,\pi>0}\right\}$, i.e. markets in which no firm in the set $\mathcal{F}^{(q+1)}_{\subseteq i}$ operates. We have already observed that the only firms created during or after iteration q competing in the same markets as firm i are the firms $\mathcal{F}^{(q+1)}_{\subseteq i}$ and their nested firms. By definition, such firms do not operate in the markets under consideration. Further, by regularity assumption (ii) on profit functions, firms which do not operate in a market do not influence the profitability of those which do. As such, for any such disposal of capabilities D by firm i, the decrease in gross profits in markets $J_i \setminus \left\{\bigcup_{k\in\mathcal{F}^{(q+1)}_{\subseteq i}} J^{(u)}_{k,\pi>0}\right\}$ as well as the decrease in capability maintenance costs must be identical in iteration q and u. Then since this disposal was strictly net unprofitable in iteration q, it must continue to be so in the industry structure $(H_M, H_F^{(u)})$.

(ii) No firm has a weakly net profitable demerger.

We first show for κ not too convex, any demerger which disposes of any capabilities must be strictly dominated by a demerger which does not. We then show for κ not too convex, any partition (demerger without disposals) of a firm's capabilities must in turn be strictly dominated by no demerger at all.

For any iteration q, and any firm $i \in \mathcal{F}^{(q)}$, consider the demerger of firm i into the firms $\mathcal{F}_i$ such that

$$\mu(F_i) = \left\{\bigcup_{i'\in\mathcal{F}_i} \mu(F_{i'})\right\} \cup D,$$

where D is the set of capabilities which are disposed. We have already established in the special case where $|\mathcal{F}_i| = 1$ and $D \neq \emptyset$ these demergers are strictly net unprofitable (since they are simply disposals).

We now show any demerger with $|\mathcal{F}_i| > 1$ and $D \neq \emptyset$ is strictly unprofitable. Consider an alternative demerger of $\mathcal{F}_i$ into the single firm l where $\mu(F_l) = \{\bigcup_{i'\in\mathcal{F}_i} \mu(F_{i'})\}$. This is a pure disposal of capabilities, and so by the above argument, it must be strictly unprofitable and satisfies

$$\sum_{j\in\mathcal{M}} \left(\pi_{ij}(H_M, H_F^{(u)}) - \pi_{lj}(H_M, H_F'^{(u)})\right) > \left(\kappa(|F_i|) - \kappa(|F_l|)\right), \tag{1}$$

where $H_F'^{(u)}$ is the firm hypergraph after iteration u but with firm i replaced with firm l. Since capabilities are complementary, firm l must make at least as much gross profits in every market as the firms $\mathcal{F}_i$, so

$$\sum_{j\in\mathcal{M}} \left(\pi_{lj}(H_M, H_F'^{(u)}) - \sum_{k\in\mathcal{F}_i} \pi_{kj}(H_M, H_F''^{(u)})\right) \geq 0, \tag{2}$$

where $H_F''^{(u)}$ is the firm hypergraph after iteration u but with firm i replaced with the firms $\mathcal{F}_i$. Combining inequalities (1) and (2),

$$\sum_{j\in\mathcal{M}}\left(\pi_{ij}(H_M,H_F^{(u)})-\sum_{k\in\mathcal{F}_i}\pi_{kj}(H_M,H_F''^{(u)})\right)>\left(\kappa(|F_i|)-\kappa(|F_l|)\right).$$

Since for any $\epsilon>0$, we can reduce the convexity of κ until

$$\left(\kappa(|F_i|)-\sum_{k\in\mathcal{F}_i}\kappa(|F_k|)\right)-\left(\kappa(|F_i|)-\kappa(|F_l|)\right)<\epsilon,$$

for all κ not too convex,

$$\sum_{j\in\mathcal{M}}\left(\pi_{ij}(H_M,H_F^{(u)})-\sum_{k\in\mathcal{F}_i}\pi_{kj}(H_M,H_F''^{(u)})\right)>\left(\kappa(|F_i|)-\sum_{k\in\mathcal{F}_i}\kappa(|F_k|)\right).$$

Thus, the demerger under consideration is strictly net unprofitable.

We conclude by showing any demerger with $|\mathcal{F}_i|>1$ and $D=\emptyset$ must also be strictly net unprofitable. First observe that, by the construction of firm i, $|\mu(F_i)|=|F_i|$. Next, recall that firm i was created in iteration q. By construction of the algorithm, firm i cannot have had a strictly profitable demerger at the end of iteration q. By Lemma 2, it must therefore have been the case that: (i) $\mu(F_i)\subseteq\hat{A}_i^{(q)}$ where $\hat{A}_i^{(q)}=\bigcup_{j\in J_{i,\pi>0}^{(q)}}M_j$; and (ii) the market subhypergraph $\hat{H}_M(\mu(F_i)\cap\hat{A}_i^{(q)})$ must contain a single component. By Lemma 7, $J_{i,\pi>0}^{(q)}=J_{i,\pi>0}^{(u)}$, we thus conclude: (i) $\mu(F_i)\subseteq\hat{A}_i^{(u)}$; and (ii) the market subhypergraph $\hat{H}_M(\mu(F_i)\cap\hat{A}_i^{(u)})$ must contain a single component. We have shown that the three conditions of Lemma 3 are satisfied. This implies that the demerger of firm i into firms $\mathcal{F}_i$ must strictly reduce gross profits. For any $\epsilon>0$, we can reduce the convexity of κ until $\kappa(F_i)-\sum_{k\in\mathcal{F}_i}\kappa(|F_k|)<\epsilon$. Hence for all κ not too convex, the proposed demerger is strictly net unprofitable.

(iii) No firm has a strictly net profitable merger.

Without loss of generality, suppose $i\in\mathcal{F}^{(q)}$ and $k\in\mathcal{F}^{(p)}$ where $q\le p\le u$. We want to show i and k have no strictly net profitable mergers. Observe by the concentric property of Lemma 4, there must exist some firm $k'\in\mathcal{F}^{(q)}$ such that $\mu(F_k)\subseteq\mu(F_{k'})$. In the special case $q=p$, $k=k'$. Use $\{\hat{C}_r^{(q)}\}_{r=1}^{p}$ to denote the set of all components of the subhypergraph $\hat{H}_M(S^{(q)})$. By Lemma 2, if $i\in\mathcal{F}^{(q)}$, then there must exist $r\in\{1...p\}$ such that $\mu(F_i)\subseteq C_r^{(q)}$ and so let $\mu(F_i)\subseteq C_r^{(q)}$ and $\mu(F_{k'})\subseteq C_{r'}^{(q)}$.

First suppose $k'=i$, then $\mu(F_k)\subseteq\mu(F_i)$ so k is nested within i. Then i and k have no feasible non-trivial merger.

Now suppose $k'\neq i$ and $r\neq r'$. This implies i and k' are on different components of $\hat{H}_M(S^{(q)})$ and so by Lemma 3, there cannot be a strictly net profitable merger between i and k'. Since $\mu(F_k)\subseteq\mu(F_{k'})\subseteq C_{r'}^{(q)}$, this is also the case between i and k.

Finally, suppose $k'\neq i$ and $r=r'$, i.e. i and k' are subsets of the same component $C_r^{(q)}$. This implies i and k' were demerged from the same interim firm with capabilities corresponding to a component of $\hat{H}_M(S^{(q)})$. We first show there cannot be synergies between i and k', and then show this implies there cannot be synergies between i and k. Observe by construction $\mu(F_i) \cap \mu(F_{k'}) = \emptyset$ and recall $J^{(u)}_{i,\pi>0}$ is the set of markets in which firm i makes strictly positive profits under $(H_M, H_F^{(u)})$. Now by Lemma 5, $J^{(u)}_{i,\pi>0} \cap J^{(u)}_{k',\pi>0} = \emptyset$ and so there are no synergies among markets either firm operates in. We also need to rule out markets which neither i nor k' operates in, but the resultant firm generated by a merger between i and k' does. Suppose, towards a contradiction, there exists some $j \in \mathcal{M} \setminus \{J^{(u)}_{i,\pi>0} \cup J^{(u)}_{k',\pi>0}\}$ such that the firm with capabilities $\mu(F_l) = \mu(F_i) \cup \mu(F_{k'})$ can make strictly positive gross profits. By Lemma 6, there does not exist a firm $l \in \mathcal{F}^{(q)} \setminus \{i, k'\}$ such that $j \in J_{l,\pi>0}$. This then implies for sufficiently low convexity of κ, the demerger could not have been optimal in iteration q of the algorithm. This is because an alternate demerger generating a single firm with capabilities $\mu(F_i) \cup \mu(F_{k'})$ in lieu of firms i and k' would make strictly greater gross profits across markets, while leaving the gross profits of all firms $l \in \mathcal{F}^{(q)} \setminus \{i, k'\}$ unchanged. Then for sufficiently low convexity of κ, this alternate demerger must be strictly more profitable than the one in fact undertaken, a contradiction.

We now show that there cannot be synergies between i and k. Since $\mu(F_k) \subseteq \mu(F_{k'})$, $J^{(u)}_{i,\pi>0} \cap J^{(u)}_{k,\pi>0} = \emptyset$. Now suppose, towards a contradiction, there exists some market $j' \in \mathcal{M} \setminus \{J^{(u)}_{i,\pi>0} \cup J^{(u)}_{k,\pi>0}\}$ in which the firm with capabilities $\mu(F_i) \cup \mu(F_k)$ can make strictly positive gross profits. Since $\mu(F_k) \subseteq \mu(F_{k'})$, then $J^{(u)}_{k,\pi>0} \subseteq J^{(u)}_{k',\pi>0}$. We have already observed $j' \notin \mathcal{M} \setminus \{J^{(u)}_{i,\pi>0} \cup J^{(u)}_{k',\pi>0}\}$ since there cannot be synergies between i and k'. This implies $j' \in \{J^{(u)}_{k',\pi>0} \setminus J^{(u)}_{k,\pi>0}\}$ i.e. j' must be a market firm k' but not k operates in. But since the firm with capabilities $\mu(F_i) \cup \mu(F_k)$ can operate in market j' although firms i and k cannot individually do so, this implies $\theta_{ij} \neq \emptyset$ and $\theta_{kj} \neq \emptyset$ and so i or k' cannot have held the maximal set of capabilities for j in iteration q which, by Lemma 6, is a contradiction. As such, a merger between i and k cannot be strictly net profitable.

(iv) No firm has a strictly net profitable procurement.

Consider firm $i \in \mathcal{F}^{(q)}$ generated in iteration q, and denote the market component i was demerged from with $\hat{C}_r^{(q)} \supseteq \mu(F_i)$. We use $\mathcal{F}_r^{(q)} \subseteq \mathcal{F}^{(q)}$ to denote the set of firms generated in iteration q from the component $\hat{C}_r^{(q)}$. Now suppose, towards a contradiction, that firm i has a strictly net profitable procurement $A \subseteq \mu(S^{(u)})$ from the capabilities unassigned after Algorithm 1 terminates.

First observe firm i must find it strictly net unprofitable to undertake any procurement of capabilities outside $\hat{C}_r^{(q)}$. Call such capabilities $A' \subseteq A$. If a standalone firm with capabilities A' can make strictly net positive profits, we have a contradiction since $A' \subseteq A \subseteq \mu(S^{(u)})$ and so Algorithm 1 could not have terminated in iteration u, a contradiction. If it cannot, then since A' exists on another component on $\hat{H}_M(\mu(S^{(q)}))$ and $\mu(S^{(q)}) \supseteq \mu(S^{(u)})$, there are no synergies and such a procurement is strictly dominated by an alternate procurement of capabilities $A \setminus A'$.

We thus restrict our attention to possible procurements from the set $\hat{C}_r^{(q)} \cap \mu(S^{(u)})$. Denote the set of markets which intersect this component with $J_r := \{j \in \mathcal{M} \,:\, M_j \cap \hat{C}_r^{(q)} \neq \emptyset\}$.

Decompose the capabilities A such that $A = A_{other} \cup A_{not}$ and $A_{other} \cap A_{not} \neq \emptyset$ where (i) $A_{other} \subseteq \{\bigcup_{k \in \mathcal{F}_r^{(q)} \setminus \{i\}} \mu(F_k)\}$ is the set of capabilities held by *other* firms generated in iteration

q; and (ii) $A_{not} \subseteq \{\hat{C}_r^{(q)} \setminus \bigcup_{k \in \mathcal{F}_r^{(q)}} \mu(F_k)\}$ is the set of capabilities which are not.

We can first rule out $A = A_{other}$ i.e. only capabilities held by other firms generated in the same iteration are procured. To see this, observe that if gross profits only strictly increase in markets $\bigcup_{k \in \mathcal{F}_r^{(q)} \setminus \{i\}} J_{k,\pi>0}^{(u)}$, then since by Lemma 6, firm i cannot possess capabilities relevant to these markets and κ is convex, the standalone firm with capabilities A must also be strictly net profitable. But if this is so, then since $A \subseteq \mu(S^{(u)})$, the algorithm could not have terminated in iteration u, a contradiction. Alternatively, if the procurement generates strictly positive gross profits in any market $j \in \{J_r \setminus \bigcup_{k \in \mathcal{F}_r^{(q)}} J_{k,\pi>0}^{(u)}\}$, this implies there were synergies between the firms $\mathcal{F}_r^{(q)}$. Now consider an alternate demerger into the single firm l where $\mu(F_l) = \bigcup_{k \in \mathcal{F}_r^{(q)}} \mu(F_k)$. The difference in gross profits is

$$\sum_{j \in J_r} \left(\pi_{lj}(H_M, H_F^{(q)}) - \sum_{k \in \mathcal{F}_r^{(q)}} \pi_{kj}(H_M, H_F^{(q)}) \right)$$
$$= \underbrace{\sum_{j \in \left\{ J_r \setminus \bigcup_{k \in \mathcal{F}_r^{(q)}} J_{k,\pi>0}^{(q)} \right\}} \pi_{lj}(H_M, H_F'^{(q)})}_{>0} + \underbrace{\sum_{j \in \left\{ \bigcup_{k \in \mathcal{F}_r^{(q)}} J_{k,\pi>0}^{(q)} \right\}} \left(\pi_{lj}(H_M, H_F'^{(q)}) - \sum_{k \in \mathcal{F}_r^{(q)}} \pi_{kj}(H_M, H_F^{(q)}) \right)}_{\geq 0}$$

where $H_F'^{(q)}$ is identical to $H_F^{(q)}$ with the exception that the firms $\mathcal{F}_r^{(q)}$ are replaced by the single firm l and we used the fact that the set of operating markets are unchanged between iterations q and u (Lemma 7). Hence for all κ not too convex, the demerger in fact undertaken in iteration q could not have been optimal, a contradiction.

We can also rule out $A = A_{not}$, recalling these are capabilities not held by other firms generated in the same iteration q. To see this, observe that this procurement could only have increased gross profits in markets $\{J_r \setminus \bigcup_{k \in \mathcal{F}_r^{(q)}} J_{k,\pi>0}^{(u)}\}$, otherwise by Lemma 6, at least one firm is not holding the maximum feasible capabilities, a contradiction. By our regularity assumption on profits (iv), competition across all markets could have only increased between iteration q and u, implying if firm i finds it strictly net profitable to procure A_{not} under $(H_M, H_F^{(u)})$, this must also be so under $(H_M, H_F^{(q)})$. And since $A_{not} \subseteq \mu(S^{(u)}) \subseteq \mu(S^{(q)})$, the demerger of the interim firm with capabilities $\hat{C}_r^{(q)}$ which was in fact undertaken in iteration q could not have been optimal, a contradiction.

Finally, consider the mixed procurement with $A_{not} \neq \emptyset, A_{other} \neq \emptyset$. As before, gross profits in markets $\bigcup_{k \in \mathcal{F}_r^{(q)}} J_{k,\pi>0}^{(u)}$ must remain unchanged, otherwise by Lemma 6, at least one firm is not holding the maximum feasible capabilities, a contradiction. Now recall we assumed the procurement was strictly net profitable, hence the inequality

$$\sum_{j \in \left\{ J_r \setminus \bigcup_{k \in \mathcal{F}_r^{(q)}} J_{k,\pi>0}^{(u)} \right\}} \left(\pi_{i'j}(H_M, H_F'^{(u)}) - \pi_{ij}(H_M, H_F^{(u)}) \right) > |A|\kappa(1)$$

must be fulfilled, where i' has capabilities $\mu(F_{i'}) = \mu(F_i) \cup A$, and $H_F'^{(u)}$ is identical to $H_F^{(u)}$

with the exception that firm i is replaced by firm i'. Then construct firm l such that $\mu(F_l) = \{\bigcup_{k\in\mathcal{F}_r^{(q)}} \mu(F_k)\} \cup \{A_{not}\}$. Since $\mu(F_l) \supseteq \mu(F_{i'})$, then profits across markets $\{J_r \setminus \bigcup_{k\in\mathcal{F}_r^{(q)}} J^{(u)}_{k,\pi>0}\}$ must increase by at least $|A|\kappa(1) \geq |A_{not}|\kappa(1)$ and for all $\epsilon > 0$, we can reduce the convexity of κ until $\kappa(|F_l|) - \sum_{k\in\mathcal{F}_r^{(q)}} \kappa(|F_k|) - |A_{not}|\kappa(1) < \epsilon$. Since $A_{not} \subseteq \mu(S^{(u)}) \subseteq \mu(S^{(q)})$, the demerger undertaken in iteration q cannot have been optimal, a contradiction.

(v) No firm has a strictly net profitable entry.

We want to show no firm i with capabilities $\mu(F_i) \subseteq \mu(S^{(u)})$ can be strictly net profitable for the industry structure $(H_M, H_F'^{(u)})$ where $H_F'^{(u)}$ is reached from $H_F^{(u)}$ by adding firm i. Suppose there exists such firm i with $\mu(F_i) \subseteq \mu(S^{(u)})$. Recall $\{\hat{C}_r^{(u)}\}_{r=1}^p$ is the set of components of the subhypergraph $\hat{H}_M(\mu(S^{(u)}))$, and $\mathcal{P}^{(u)}$ is the index of these components. If firm i can make strictly net positive profits, then at least one firm i' with capabilities $\mu(F_{i'}) \in \{\mu(F_i) \cap \hat{C}_r^{(u)} : r \in \mathcal{P}^{(u)}\}$ must be making strictly net positive profits by Lemma 2. But if so, then since $\mu(F_{i'}) \subseteq \{\hat{C}_r^{(u)} \cap \mu(S^{(u)})\}$ for some $r \in \mathcal{P}^{(u)}$, Algorithm 1 could not have terminated in iteration u, a contradiction.

Finally, since we have ruled out strictly net profitable disposals, demergers, procurements, mergers, and entries for all firms in $(H_M, H_F^{(u)})$, the industry structure created by Algorithm 1 is stable. □

A.5 Proof of Proposition 5.

Proof. The following lemma, subsequently proven in Supplementary Appendix I, will be helpful.

Lemma 8. *For a given market hypergraph H_M, if there exists some scarce capability $a \in \mathcal{A}$, then for sufficiently low convexity of κ, in all stable industry structures, there must exist some firm $i \in \mathcal{N}$ such that $\bigcup_{j:a\in M_j} M_j \subseteq \mu(F_i)$.*

Now observe if $a, a' \in \mathcal{A}$ are scarce and adjacent such that there exists $j \in \mathcal{M}$ such that $a, a' \in M_j$, then for sufficiently low convexity of κ, in all stable industry structures there must exist some firm $i \in \mathcal{N}$ such that $\bigcup_{j:\{a\cup a'\}\cap M_j\neq\emptyset} M_j \subseteq \mu(F_i)$. To see this, observe from Lemma 8 there must exist some firm $i \in \mathcal{N}$ such that $\bigcup_{j:a\in M_j} M_j \subseteq \mu(F_i)$ and some firm $i' \in \mathcal{N}$ such that $\bigcup_{j:a'\in M_j} M_j \subseteq \mu(F_{i'})$. Then since a and a' are scarce and adjacent, i and i' must be the same firm.

Now notice that if the scarce capabilities A are connected on the subhypergraph $\hat{H}_M(A)$, then for any pair of capabilities $a_1, a_n \in A$, there must exist some path on $\hat{H}_M(A)$ $\{a_1, M_1, a_2, \ldots, a_{n-1}, M_{n-1}, a_n\}$ where for all $1 \leq i \leq n-1$, $a_i, a_{i+1} \in M_i$. The rest of the result follows immediately since there must exist some firm $i \in \mathcal{N}$ such that $\bigcup_{j:\{a_i\cup a_{i+1}\}\cap M_j\neq\emptyset} M_j \subseteq \mu(F_i)$ for all $1 \leq i \leq n-1$ implying $|F_i| \geq \mu(F_i) \geq |\bigcup_{j:A\cap M_j\neq\emptyset} M_j| = |\bar{\Sigma}|$. □